Postdoc proposal : Wave turbulence for wave equations

Anne-Sophie de Suzzoni

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Wave turbulence is the study of the evolution of the law of a solution of a wave equation, when the initial data is a random variable or when the equation admits a random forcing term. It was introduced by [7] in the context of crystals and was popularized by Zakharov, one of whose major contributions was the introduction of the now-named Kolmogorov–Zakharov spectrum, which models an energy transfer between spatial scales, see [8]. This is a very active field in mathematical physics, as evidenced by books such as [9], [6] or [5].

Starting in the late 2000s, the mathematical community began to take an interest in weak turbulence, a subfield of wave turbulence in which the initial data are assumed to be small. A major discovery in the work of the above-mentioned physicists was the possibility of describing the statistics of certain solutions of wave equations by so-called kinetic equations. The aim is to derive, i .e. to obtain by a rigorous mathematical process, or to study kinetic equations governing or emanating from the study of the statistics of solutions of wave equations. Important works are the ones by Deng and Hani [1, 3, 2, 4].

In a work in progress, Annalaura Stingo, Arthur Touati and I are deriving for short times an equation by studying the statistical evolution of the solution of a system of two Klein-Gordon equations. The resulting dynamics is not kinetic: for systems, the invariance mechanisms that allow the trivial resonances to be ignored for Schrödinger or primitive equations no longer exists and we can observe a nonlinear and nontrivial evolution arising from trivial resonances. However, an appropriate choice of non-linearity makes this evolution trivial.

The aim of this postdoc is to adapt the techniques we used – for instance, they heavily rely on the use of normal forms and on an adapted diagrammatic – to the case where the evolution is trivial and push the analysis to higher time scales where a kinetic regime would be observed. Other problems at hand include the study of the dynamics of the equation we are deriving in our on-going project with Stingo and Touati, but also the long-time derivation of this equation, following the ideas of [4].

The postdoc would also benefit from the proximity of the Laboratoire de Physique des Plasmas, and its experts in wave turbulence, and from the proximity in centre de mathématiques Laurent Schwartz of experts in random matrices applied to wave turbulence and in kinetic equations.

An appropriate candidate would have a PhD in dispersive PDEs.

References

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[5] Sébastien Galtier. *Physics of Wave Turbulence*. Cambridge University Press, 2022.

[6] Sergey Nazarenko. *Wave turbulence*, volume 825 of *Lecture Notes in Physics*. Springer, Heidelberg, 2011.

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[9] Vladimir E Zakharov, Victor S L'vov, and Gregory Falkovich. *Kolmogorov spectra of turbulence I: Wave turbulence*. Springer Science & Business Media, 2012.