Book of abstracts

PGMO DAYS 2022

November 29th and 30th, 2022

EDF Labs Paris-Saclay

Preface

This volume contains the extended abstracts of the talks presented at the conference PGMO-DAYS 2022 held on November 29th – 30th, 2022 at EDF Labs Paris-Saclay, celebrating the tenth anniversary of PGMO.

We especially acknowledge the support of EDF, FMJH, and Labex LMH. We thank CNRS, Institut polytechnique de Paris, Université Paris-Saclay and INRIA. We also thank ROADEF, SMAI-MODE, SMF, GdR MOA, GdR RO, and GdR Jeux. We are grateful to the plenary speakers and to the organizers of the invited sessions.

November 3, 2022 The organizers of the PGMO days
Palaiseau

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Organizers of Invited Sessions

Claudia D'Ambrosio: *Energy4Climate*
Luce Brotcorne and Hélène Le Cadre: *Equilibrium problems and decomposition methods in energy*
Jean-Baptiste Caillau and Walid Djema: *Optimal control applied to life sciences*
Michel De Lara: *The ℓ_0 pseudonorm – Theory and Applications*
Antonio Frangioni: *Bundle Methods: Methodology and Applications to Energy Optimization*
Cédric Josz: *First-order method and o-minimal structures*
Guilherme Mazanti and Laurent Pfeiffer: *Mean Field Games, Control, and Applications*
Joseph Mikael: *Quantum Operations Research: Overview and latest advances*
Clément Royer: *Optimization for engineering and scientific computing*
Mateusz Skomra: *Polynomial Optimization*
Cheng Wan: *Pricing problems in energy management*
Hasnaa Zidani: *New developments in optimal control theory*

Table of Contents

A Glance at Nonsmooth Automatic Differentiation

Jérôme Bolte

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I will recall the fundamental role of nonsmooth automatic differentiation as the core learning mechanism in modern AI. I will then show how a recent idea we developed with E. Pauwels — "Conservative gradients" — helps to understand fundamental phenomena, such as the convergence of learning phases in deep learning, the optimization of learning parameters, the nonsmooth cheap gradient principle, or the differentiation of algorithms.

Recent aspects of mean field control

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Mean field control consists in the optimal control of a large population of devices: it is motivated for instance by the optimal control of a fleet of vehicles or a fleet of drones, or the optimal charging of batteries,. . . Very often the system is simplified into the optimal control of a partial differential equation of McKean-Vlasov type. In this talk I will present a series of works (obtained with S. Daudin, J. Jackson and P. Souganidis) in which we investigate quantitatively the validity of this approximation.

Dyadic Linear Programming

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A rational number is dyadic if it is an integer multiple of $1/2^k$ for some nonnegative integer k. Dyadic numbers are important for numerical computations because they have a finite binary representation, and therefore they can be represented exactly on a computer in floating-point arithmetic. When real or rational numbers are approximated by dyadic numbers on a computer, approximation errors may propagate and accumulate throughout the computations. So it is natural to ask when linear programs have dyadic optimal solutions. We address this question in joint work with Ahmad Abdi, Levent Tuncel, and Bertrand Guenin.

Applications of Interior Point methods: from Sparse Approximations to Discrete Optimal Transport

Jacek Gondzio

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A variety of problems in modern applications of optimization require a selection of a 'sparse' solution, a vector with preferably few nonzero entries. Such problems may originate from very different applications in computational statistics, signal or image processing or compressed sensing, finance, machine learning and discrete optimal transport, to mention just a few. Sparse approximation problems are often solved with dedicated and highly specialised first-order methods of optimization.

In this talk I will argue that these problems may be very efficiently solved by the more reliable optimization techniques which involve some use of the (inexact) second-order information as long as this is combined with appropriately chosen iterative techniques of linear algebra, such as for example methods from the Krylov-subspace family. Two particular classes of methods, the Newton Conjugate Gradient and the Interior Point Method will be interpreted as suitable homotopy type approaches and will be applied to solve problems arising from: compressed sensing, multi-period portfolio optimization, classification of data coming from functional Magnetic Resonance Imaging, restoration of images corrupted by Poisson noise, classification via regularized logistic regression, and discrete optimal transport. In all these cases, the performance of the proposed methods will be compared against competitive first-order methods. Computational evidence will be provided to demonstrate that specialized second-order methods compare favourably and often outperform the cutting-edge first-order methods.

For more details, see:

References

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[2] F. Zanetti and J. Gondzio, A sparse interior point method for linear programs arising in discrete optimal transport, Tech Report (22 June 2021). https://arxiv.org/abs/2206.11009

Bilevel Optimization Under Uncertainty

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Bilevel optimization problems can serve as a powerful tool for modeling hierarchical decision making processes. The resulting problems are highly challenging to solve—both in theory and practice. Fortunately, there have been significant algorithmic advances in the field of bilevel optimization so that we can solve much larger and also more complicated problems today compared to what was possible to solve two decades ago.

This results in more and more challenging bilevel problems that researchers try to solve today. In this talk I will give an overview of one of these more challenging classes of bilevel problems: bilevel optimization under uncertainty. We will discuss classical ways of addressing uncertainties in bilevel optimization using stochastic or robust techniques. Moreover, the sources of uncertainty in bilevel optimization can be much richer than for usual, single-level problems, since not only the problem's data can be uncertain but also the (observation of the) decisions of the two players can be subject to uncertainty. Thus, we will also discuss bilevel optimization under limited observability, the area of problems considering only near-optimal decisions, and intermediate solution concepts between the optimistic and pessimistic cases.

The talk is based on the article by Yasmine Beck, Ivana Ljubic, Martin Schmidt: A Survey on Bilevel Optimization Under Uncertainty, https://optimization-online.org/2022/06/8963/

Some splitting methods from a nonsmooth optimizer's perspective

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For large-scale optimization, popular approaches such as the ADMM and the Progressive Hedging algorithm exploit separable structures by solving in parallel individual sub-problems which are then coordinated by performing a simple algebraic step (typically, a projection onto a linear subspace)

While parallelism is the strength of all Douglas-Rachford-like splitting methods, their weak points are the adjustment of certain proximal parameter and the lack of a fully implementable stopping test. These difficulties, which date back to Spingarn's method of partial inverses, stem from the very design of these approaches, which were created to find zeroes of operators.

We discuss how to endow some splitting methods with an optimization perspective that introduces knowledge about the objective function throughout the iterative process. The resulting new family of methods à la bundle incorporates a projective step to coordinate parallel information while allowing the variation of the proximal parameters and the construction of a reliable stopping test. A Bundle Progressive Hedging algorithm, derived from the general theory, illustrates the interest of proposal. Credit to co-authors will be given during the talk

PGMO 2022 PhD Prizes

The Gaspard Monge Program for Optimization, Operations Research, and their Interactions with Data Sciences, awards every year, under the scientific patronage of ROADEF and SMAI-MODE, two PhD prizes (exaequo). All the fields in Optimization, and Operations Research, including their Interfaces, are eligible. The applicants must have been defended their PhD in France, during the previous civil year.

The two 2022 PGMO PhD prizes were awarded to

- *Étienne Boursier*, for his PhD at ENS Paris-Saclay, under the supervision of Vianney Perchet, on "Sequential Learning in a strategical environment".
- *Adèle Pass-Lanneau*, for her PhD at Sorbonne University, under the cosupervision of Pascale Bendotti, Philippe Chrétienne and Pierre Fouilhoux, on "Anchored solutions in robust combinatorial optimization".

The 2022 PhD prize committee was chaired by Samir Adly (University of Limoges). It was composed of the following researchers:

- Charles Dossal (INSA)
- Bruno Escoffier (Sorbonne Université)
- Chloé Jimenez (Université de Brest)
- Laetitia Jourdan (Université de Lille)
- Edouard Pauwels (Université Toulouse 3 Paul Sabatier)
- Ruslan Sadykov (INRIA)
- Samuel Vaiter (Université Côte d'Azur)
- Guillaume Vigeral (Université Paris Dauphine)
- Bruno Escoffier (Sorbonne Université)

A simple city equilibrium model with an application to teleworking

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Keywords: spatial equilibria, teleworking, optimal transport.

We propose a simple semi-discrete spatial model where rents, wages and the density of population in a city can be deduced from free-mobility and equilibrium conditions on the labour and residential housing markets. We prove existence and (under stronger assumptions) uniqueness of the equilibrium. We extend our model to the case where teleworking is introduced. We present numerical simulations which shed light on the effect of teleworking on the structure of the city at equilibrium.

A mean field model for the interactions between firms on the markets of their inputs

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Keywords: capital distribution, heterogeneous agents, mean field equilibrium.

We consider an economy made of competing firms which are heterogeneous in their capital and use several inputs for producing goods. Their consumption policy is fixed rationally by maximizing a utility and their capital cannot fall below a given threshold (state constraint). We aim at modeling the interactions between firms on the markets of the different inputs on the long term. The stationary equilibria are described by a system of coupled non-linear differential equations: a Hamilton-Jacobi equation describing the optimal control problem of a single atomistic firm; a continuity equation describing the distribution of the individual state variable (the capital) in the population of firms; the equilibria on the markets of the production factors. We prove the existence of equilibria under suitable assumptions.

The Unit Commitment problem with integration of Local Energy Communities

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Keywords: Energy, renewable energies, economic dispatch, optimization

For the last five decades, the Unit Commitment (UC) problem is one of the tools that have been used to optimize energy generation. With the rise of renewable energy, collective organizations such as Local Energy Communities (LECs) play a key role in the transition to more sustainable energy generation. However, the production of energy at minimum cost is still a topic of interest.

In this work, the UC problem is extended to a LEC schema. For modeling purposes, LECs are considered as sets of units with power demands over a given horizon and the objective is to minimize the total operation costs. Additional features such as power sharing between communities, external power purchasing and power storage are also considered and evaluated using benchmark instances. This work is developed under the framework of the SEC-OREA project.

A Rank-Based Reward between a Principal and a Field of Agents: Application to Energy Savings

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Keywords: Ranking games, Principal-Agent problem, Mean-field games, Energy savings

An energy retailer has incentives to generate energy consumption saving at the scale of its customer portfolio. For example in France, since 2006, power retailers have a target of a certain amount of Energy Saving Certificates[1] to hold at a predetermined future date (usually 3 or 4 years). If they fail to obtain this number of certificates, then they face financial penalties.

There is evidence from behavioral economy that energy consumption reductions can be motivated by providing a financial reward and/or information on social norms or comparison to customers, and electricity providers are aware of this lever to make energy savings. In fact, contracts offering bonus/rewards in compensation of reduction efforts appear, see e.g. the offers of "Plüm énergie" or "OhmConnect"[2]. The interest of this kind of solutions is reinforced in the current situation of gas and power shortage where many countries intend to diminish their global energy consumption.

In this presentation, we analyze games with large populations of heterogeneous agents (consumers) interacting through their empirical distribution. We focus on interactions based on the *rank* of each players: in our context, the rank measures the reduction effort of a consumer compared with the rest of the population, and is computed as the cumulative probability associated with his consumption. A rank $r \in [0, 1]$ indicates that the consumer is among the r percent of the population with the highest consumption reduction. The Principal (retailer) will then design a rank-based reward function to incentive agents to reduce their consumption.

We first explicit the equilibrium for the mean-field game played by the agents, and then characterize the optimal reward in the homogeneous setting. For the general case of a heterogeneous population, we develop a numerical approach, which is then applied to the specific case study of the market of Energy Saving Certificates.

An article based on this work is available as a preprint [1].

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[2]https://plum.fr/cagnotte/,https://www.ohmconnect.com/

No Self-Concordant Barrier Interior Point Method Is Strongly Polynomial

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Keywords: interior points methods, self-concordant barriers, tropical geometry

Self-concordant barrier interior point methods (IPMs) solve minimization problems over a polyhedron with linear cost $x \mapsto \langle c, x\rangle$ by considering instead the cost function $x \mapsto f(x)+\eta\langle c, x\rangle$ for a parameter $\eta > 0$ and f a self-concordant barrier. Self-concordant barriers are strictly convex functions which blow up at the border of the feasible set and whose Hessian locally does not change much. For each $\eta > 0$, the parametric problem has a unique minimizer $x(\eta)$, and the curve $\eta \mapsto x(\eta)$ is the *central path.* As $\eta \to +\infty$, the central path converges to an optimal value of the initial linear problem. IPMs follow the central path to find an approximated solution.

One of the major open problems in computational optimization, known as Smale's ninth problem, asks to find a *strongly polynomial algorithm* for linear programming. In this context, making any substantial progress on the understanding of the worst-case number of iterations performed by IPMs is a notorious open question.

Our contribution generalizes the result in [1] which shows that IPMs based on the logarithmic barrier are not strongly polynomial. The authors study parametric families of linear programs with their central paths by looking at their *tropicalization*, *i.e.* the limit of their log-image. The tropicalization of the central paths is a piecewise linear curve called the tropical central path.

We present our results from [2] which show that *no* self-concordant barrier IPM is strongly polynomial. To achieve this, we show that any self-concordant barrier behaves essentially like the logarithmic barrier. We use this to prove that IPMs draw polygonal curves in a multiplicative neighborhood of the central path whose log-limit coincides with the tropical central path, independently of the barrier. We provide an explicit parametric linear program that falls in the same class as the Klee–Minty counterexample, *i.e.* whose feasible set is a combinatorial cube. Its tropical central path is composed of $2^n - 1$ segments. When the parameter is large, the log-image of the trajectory of the IPM approximates the tropical central path, meaning that the IPM must perform that many iterations, thus breaking strong polynomiality.

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Vector-borne Disease Outbreak Control via Instant Vector Releases

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Keywords: Vector-borne diseases, sterile insect technique, Wolbachia, optimal epidemic vector control, dengue.

Vector-borne diseases have a significant impact on human health worldwide, accounting for 17% of all infectious diseases. These diseases can be caused by parasites, bacteria or viruses and can be transmitted by different types of vectors, such as ticks, fleas or mosquitoes. In this talk, we present the study of optimal vector release strategies for vector-borne diseases such as dengue, Zika, chikungunya and malaria. Two techniques are considered:

- The sterile insect technique (SIT), which involves the release of sterilised male vectors among wild vectors in order to disrupt their reproduction.
- The use of Wolbachia (currently used mainly for mosquitoes), which consists on the release of vectors infected with a bacterium that limits their ability to transmit the pathogen in order to replace the wild population.

In each case, an epidemiological model including humans and vectors is considered. The temporal dynamics of the populations is modelled by a system of ordinary differential equations. However, to simplify the study and given that the duration of the releases is very small compared to the duration of the experiment, the releases are considered instantaneous. Mathematically they are represented by linear combinations of Dirac measures with positive coefficients determining their intensity. We model the question of the optimal mosquito release strategy by an optimal control problem that we solve numerically using ad hoc algorithms, based on writing first-order optimality conditions characterising the best combination of Dirac measures. The objective expressed by the cost function is to reduce the amount of human infections in a fixed time horizon. We then discuss the results obtained, focusing in particular on the complexity and efficiency of the optimal controls and comparing the strategies obtained for the two techniques.

References

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Intelligent Pricing in Electric Networks

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Keywords: Games with correlated constaints; Normalized equilibrium, scalable billing

The model. There are M pricing programs. The price corresponding to each m consists of a fix subscribing fee plus a congestion cost which is a function of the total electricity that is used in that pricing class. There are N players where player n has a demand d(n). A player n has to decide how much electricity x(n,m) to buy from each m of the M pricing classes. The cost for player i for buying a unit of electricity from class m is given by a function f of the total amount of electricity that is bought in program m. It thus has the form of a function f of the player i and of x(m), which is defined (with some abuse of notation) to be the sum of flows x(n,m) over all players N. We call f(i,x(m)) the cost density for player i at class m. Unless stated explicitely, the cost density of a player i will be assumed not to depend on the player and i is then dropped from the notation. We assume that the cost for each player n is additive, i.e the overall cost for player n is the sum of the cost densities f(x(m)) weighted by the flow x(n,m). The cost C(n,x) for player n is thus given by the sum over classes m of x(n,m)f(x(m)).

Definition of constrained games and equilibria. Define y(x) to be a vector whose n-th component y(n,x) is the best response of player n to x[-n] (i.e. it minimizes C(n,x)). When the minimization is over all x that are nonnegative and satisfy the demand constraints for all players ; then this defines a standard Nash equilibrium. When the minimization is with respect to a restricted subset of flows then this defines a constrained Nash equilibrium. Consider in particular capacity constraints : for each class m, the total flow using classs m is bounded by a constant d(m). Hence the sum over m of x(n,m) is bounded by d(m). These are correlated constraints in the sense of Rosen [1] and the game is a special case of the so called generalized equilibrium. For any player n, and any x, y(n,x) is a best response against x if and only if the corresponding KKT conditions hold. At equilibrium, the best response is optimal at n and x if the prices are chosen as the Lagrange multipliers. We call this a KKT pricing. A fixed point is shown to exist and it therefore has stability properties. It corresponds to an equilibrium in the constrained game. A desirable property of the equilibrium is that the Lagrange multipliers be strongly scalable for billing purposes in the following sense. The price (or the Lagrange multiplier) that corresponds to the cost for player n when using program m depends only on m and on x(m). Hence for strongly scalable billing it is not needed to inspect how much elecricity originates from each source and which player requested a given amount of Amperes. In a strongly scalable equilibrium, billing is done in a fully symmetric way.

Scalable billing and normalized equilibrium. We study in the full paper also the case of several classes of players. We show that there exist Lagrange multipliers that are the same for all players within each given class. The resulting equilibrium is called weakly-scalable. A scalable (strong or weak) pricing is a special case of "normalized equilibrium" defined in [1]. We finally identify cost functions that extend convexity to a game setting ; they are said to satisfy the strict diagonal convexity property which is shown to imply the existence and uniqueness of a normalized equilibrium (for each weight vector defining the costs) and hence the existence of a scalable pricing.

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We thank the support of the FMJH Program PGMO and the support to this program by EDF.

A branch-and-bound method for single-leader-multi-follower games with energy management applications

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Keywords: Bilevel optimization, single-leader-multi-follower games, SOS-1 approach, industrial eco-parks.

A single-leader-multi-follower game (SLMFG) is a bilevel problem in which the lower level is composed of a non-cooperative (generalized) Nash game between a finite set of agents, called the followers. A classic approach to solve SLMFGs consists in replacing the followers' optimisation problem by their Karush–Kuhn–Tucker conditions, thus leading to the resolution of a mathematical problem with complementarity constraints (MPCC). It is well known that these MPCCs are quite hard to solve due to the non-convexity of the resulting constraints set, which, additionally, has a non-empty interior.

As an alternative to Big-M-based techniques, in this work we explore the applicability of the *Special-Ordered-Set of Type 1* approach to SLMFGs with linear lower-level problems. Applications to two energy management problems will be considered as well. On the one hand, the optimal design of industrial eco-parks [1] and, on the other hand, carbon tax policy for freight transport in Ecuador [2].

References

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[2] D. Aussel and D. Villamar, *An SOS1 approach for the optimal design of tax policy for freight transport model*, submitted, (2022).

Multi-parameter data for energy optimisation: the Energy4Climate building-based demonstrators

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Keywords: Smart building, Energy management, optimisation, multi-parameter data

According to the International Energy Agency (IEA), the buildings and construction sector accounted for 36% of final energy use and 39% of energy and process-related carbon dioxide (CO2) emissions in 2018. Almost 40% of CO2 emissions come from the building industry and space heating is the most important end-use in the residential sector (68% in Europe).

The current energy transition towards low-carbon and renewable sources (with more electric uses, as heat pumps and electric vehicles) is expected to come from the aggregation of basic low voltage power supply grids, known as microgrids, which associate local energy production with storage capacities and energy consumers. Buildings equipped in such way are then expected to become 'Smart' actors in terms of local energy management and service supply inside their walls but also outside (through exchanges with their neighborhood). In terms of energy, Smart buildings could actively dispatch its power generation, control storage capacities and also manage its energy demand. Indeed, efficiency could be gained through consumption reduction and self-consumption rate could increase through techniques such as load shifting, to better match the demand with the renewable power generation. The role of the occupants, that is the consum'actors, could be key to achieve these goals.

The service improvement thanks to Smart Buildings should be evaluated by cost-benefit analyses through performance indicators [1]. These indicators can be of different nature : economic (for instance through the electrical bill change), environmental (the carbon foot print of the used electricity), related to services to the grid (such as load shedding during peak periods) or to well-being measures (such as user satisfaction, comfort or new user services). Data availability is key in order to be able to optimise for a given service, and this data can be of different types: weather forecasts, energy consumption, energy production, room temperature, the grid energy mix, to mention a few.

In order to enable the development and test of new optimisation solutions for Smart Buildings, the Energy4Climate center builds building-size demonstrators of different kinds (tertiary, residential, 100% electric, electric-thermic) and collects multi-parameter data that are available for the scientific and industrial communities.

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Structured learning for vehicle routing problems

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Vehicle routing and vehicle scheduling problems are extensively studied in the literature, and extremely efficient algorithms are now available to tackle them. Both exact and heuristic approaches exist to solve these “pure” vehicle routing and scheduling problems. For example, the Vehicle Scheduling Problem (VSP) can be exactly solved by a flow linear program, and the Capacitated Vehicle Routing Problem (CVRP) can be exactly solved using Branch-and-Price, or heuristically on larger instances using the state-of-the-art Hybrid Genetic Search (HGS) from Vidal 2021.

However, algorithms for variations of these “pure” problems are often not as efficient, and have difficulty to scale on large instances. An example of this setting is the *Stochastic Vehicle Scheduling Problem* (StoVSP), a variation of the VSP where travel and task times are randomly perturbed after the vehicle routes are chosen, the objective beeing to minimize the average total delay of the tasks. StoVSP can be solved with column generation on instances with up to 25 tasks, but then fails to scale and generalize to larger instance sizes. Another example is the *Dynamic Capacitated Vehicle Routing Problem with Time Windows* (Dynamic-CVRPTW), a multi-stage variation of the CVRP, for which the HGS does not apply anymore.

Using “pure” vehicle routing and scheduling problem algorithms as *Combinatorial Optimization* (CO) layers in *Machine Learning* (ML) pipelines can be a way to generate good algorithms for their variations. We present such hybrid pipelines for the two variations presented above, implemented by leveraging `InferOpt.jl`[1] (see Dalle et al. 2022), our open source Julia package gathering several state-of-the-art methods for integrating CO algorithms in ML pipelines. For the StoVSP, we learn a pipeline containing a VSP flow algorithm, and are able to generalize and obtain solutions with small gaps on instances with up to 1000 tasks. For the Dynamic-CVRPTW, we learn a pipeline containing a modified HGS as a CO layer, and use it at each stage to retrieve a good policy.

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Stability and optimization of the chemostat system including a linear coupling term between species

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Keywords: Population model, Optimal control, Dynamical system, Global stability, Pontryagin Maximum Principle

In this presentation, we consider the chemostat system with $n \geq 1$ species, one limiting substrate, and mutations between species. Such a system is commonly used in population models to describe the behavior of n consumers competing for a same resource (the substrate). Based on results of [1], we shall first give stability properties of the resulting dynamical system. In particular, global stability at the unique equilibrium point (apart the wash-out) is proved for a constant (sufficiently small enough) dilution rate value. Next, we will introduce feedback controls called *auxostats* in biotechnology. These controllers allow the regulation of the substrate concentration in the process. We prove that such feedback controls globally stabilize the resulting closed-loop system near a desired equilibrium point. This result is obtained by combining the theory of asymptotically autonomous systems and an explicit computation of solutions to the limit system (see [2]). The performance of such controllers will be finally illustrated on an optimal control problem of Lagrange type which consists in maximizing the production of species over a given time period with respect to the dilution rate chosen as control variable.

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Hybrid optimal control including a regionally constant parameter and applications

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Keywords: Optimal control, stratified problems, hybrid maximum principle, regionally non-permanent control.

In this presentation, we consider a Hybrid optimal control problem of Mayer type in which the controlled system is defined over a partition of the Euclidean space into a countable number of strata. In addition, we suppose that the dynamics depends on a *regionally switching parameter*. This means that the parameter should remain constant in every stratum (but not necessarily over the time period). The motivation behind this framework comes from multiple applications related to aerospace engineering or epidemiology, typically when a loss of control occurs depending on the position of the system in the state space. Our goal is to derive the necessary optimality conditions in this new framework in the form of a Pontryagin principle. This is made possible thanks to "regional" needle perturbations of a nominal control in this hybrid setting and a careful sensitivity analysis, see [1]. Next, based on [2], we will highlight these optimality conditions by presenting several examples in which a loss of control occurs (depending on the stratum in which the system belongs).

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Regional decarbonization studies: a first step to assess how to share decarbonization effort at the scale of a country. The example of Centre-Val de Loire

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Keywords: Decarbonization, Sectorial modelling, Regional analysis

Based on engineers quantified methodologies, regional decarbonization studies proposed at EDF R & D allows assessing how the activation of sectorial "levers" (e.g., building insulation in the building sector, or electric mobility in the transportation sector) can lead to decrease the regional CO2 emissions. The global methodology and used tools, as well as typical results – here in the case of Centre-Val de Loire region – will be presented. Also, it will be explained how such modelling exercise could be used to address the question of sharing decarbonization efforts at the scale of a country.

NOMA-based Power Control for Machine-Type Communications: A Mean Field Game Approach

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Keywords: NOMA, Mean Field Theory, Machine-Type Communications, power control.

The drastic growth in the number of wireless devices and the inclusion of the Internet of things (IoT) in the future sixth generation (6G) networks emerge various interesting challenges. Particularly, the operators have to provide massive connectivity and ensure higher spectral efficiency for densely deployed networks. Furthermore, orthogonal resource allocation to Machine Type Devices (MTDs) is not sustainable anymore especially, in presence of a large population. In this vein, Non-Orthogonal Multiple Access (NOMA) has been conceived to deal with the increasing number of users by allowing multiple users to access the same resources while an additional domain is exploited, namely power domain or code domain, in order to perform multiplexing of users' signals. Meanwhile, game theory has been widely adopted as a greatly appealing mathematical framework to model and analyze the cooperative and competitive behaviors among rational users under NOMA scenarios. Particularly, Mean field game (MFG) has increasingly gained attention in wireless communication networks, especially when a large number of users are involved.

In this work, we address dense scenarios in which users are divided into NOMA coalitions and we aim to investigate the impact of the interference effects on the decision-making process of users in a large population. Firstly, the power control problem is modeled as a differential game. Then, we extend the formulated game to an equivalent tractable MFG by considering the effect of the collective behavior of devices. In doing so, the mathematical complexity of the proposed scheme is drastically reduced since it relies on only two equations, namely Hamilton- Jacobi-Bellman (HJB) and Fokker-Planck-Kolmogorov (FPK) equations. Furthermore, we proposed a distributed algorithm with the aim of iteratively achieving the mean field equilibrium (MFE) by enabling each device to autonomously adjust its transmit power in response to brief information received from the BS, instead of worrying about the actions of all its opponents. Simulation results illustrate the equilibrium behaviors of the formulated game and accentuate the robustness of the proposed power control approach against the interference effects compared to existing works in the literature [1].

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Overlapping decomposition in column generation

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Keywords: Column Generation, Variable Splitting, Unit Commitment Problem,

To solve large scale integer programs, decomposition techniques such as column generation or Lagrangian relaxation deeply rely on the chosen structure induced by the dualized constraints. The overlapping decomposition (also referred to as Lagrangian Decomposition or Variable Splitting [1, 2]) is to simultaneously consider several substructures within the same decomposition method. Managing several substructures can be done by duplicating variable subsets which will then be generated through two distinct pricing methods, and linking the corresponding variables using equalities in the master problem.

In this work, an overlapping decomposition is proposed to provide strong linear relaxation values for two distinct problems within the framework of Dantzig-Wolfe decomposition: the Reliable Facility Location Problem [5] and the Unit Commitment Problem (UCP) [3, 4, 5].

To obtain an evaluation before the column generation convergence, we extend the classical lower bound computation to the overlapping decomposition context. For the UCP, we propose additional reinforcement inequalities.

Experimental results show the importance of designing the constraints linking the duplicated variables as inequalities rather than equalities: such inequalities are an important leverage to speed up convergence. The repartition of the dual cost between the two pricing subproblems also improves the convergence. We propose a way to simultaneously use these two concepts of linking inequalities and cost repartition.

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Mean Field Games with Incomplete Information

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Keywords: Mean Field Games, Structure of Information, Master Equation

This talk is concerned with mean field games in which the players do not know the repartition of the other players. First a case in which the players do not gain information is studied. Results of existence and uniqueness are discussed. Then, a case in which the players observe the payments is investigated. A master equation is derived and partial results of uniqueness are given for this more involved case. This talk is based on the paper [1].

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Stochastic Subgradient Descent Escapes Active Strict Saddles on Weakly Convex Functions

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Keywords: non-smooth optimization, SGD, avoidance of traps, Clarke subgradient, stratification

Strengthened projection formula. In the first part of this work, We refine the so-called projection formula of [1] to the case of definable in a o-minimal structure, locally Lipschitz functions by establishing a Lipschitz-like condition on the (Riemannian) gradients of two adjacent stratas.

Escaping active strict saddles. In the second part of this work, we analyze the *stochastic subgradient descent* (SGD), that produces the iterates as follows:

$$x_{n+1} \in x_n - \gamma_n \partial f(x_n) + \gamma_n \eta_{n+1} , \tag{1}$$

where (γ_n) is a sequence of step-sizes, (η_n) are the perturbations, $f : \mathbb{R}^n \to \mathbb{R}$ is a locally Lipshitz continuous function and $\partial f(x_n)$ is the Clarke subgradient of f at x_n.

In this context, an *active strict saddle*, a notion introduced in [2], is a Clarke critical point x^* that is lying on a manifold M such that: *i)* f varies smoothly on M, *ii)* f restricted to M admits a direction of negative curvature, *iii)* f varies sharply outside of M. A typical example of such a point is the origin for the function $(y, z) \mapsto -y^2 + |z|$. In [2] the authors have shown that *generically* (in the sense of linear perturbations) the only Clarke critical points that a semialgebraic (or more generally definable), weakly convex function might have are local minima and active strict saddles.

We introduce two additional assumption on the active manifold M: the Verdier and the angle condition. Under these, and assuming that the perturbation sequence (η_n) is omnidirectional, we show that $\mathbb{P}[x_n \to x^*] = 0$, where x^* is an active strict saddle. Furthermore, we show that both of our conditions are generic in the class of weakly convex, semialgebraic functions. As a consequence, we can interpret our results as the fact that the SGD on a generic, semialgebraic and weakly convex function converges to a local minimum.

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An exact method for a problem of time slot pricing

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Keywords: time slot pricing, transportation problem, dynamic programming

We consider a market where a population of customers is willing to purchase a service offered at different times $y_1, y_2, \dots, y_n$ under limited capacities. Our task is to anticipate and to implement a pricing profile $(p_1, \dots, p_n)$ that will optimally spread the expected hourly demand for the service. The time slot pricing problem is looked upon from the perspective of the service provider, in a competitive environment where profit maximization is a prime and vital concern for the company. Each customer has a preferred time $x \in \mathbb{R}$ for receiving the service, and the population is fully characterized by an absolutely continuous measure μ, with $\mu(A)$ being the volume of customers planning service during time interval A. In our model every customer is expected to either choose a time slot j that minimizes their total incurred cost $p_j + d(x - y_j)$, where p_j is the price paid for the service and the strictly convex function d is an expression of the inconvenience of rescheduling from their preferred time; or to reject the service if no time slot looks satisfying. The service plan induced by a pricing profile $(p_1, \dots, p_n) \in P^n$ is modeled by a collection $\tau_1, \dots, \tau_n$ of measures on $\mathbb{R}$, where $\tau_j(A)$ denotes the volume of the customers willing to be served during interval A and eventually getting the service at time y_j. Our goal is to implement a pricing profile in P^n which maximizes the profit of the service provider, defined as $p_1\tau_1(\mathbb{R}) + \cdots + p_n\tau_n(\mathbb{R})$, subject to maximal demand $\tau_1 + \cdots + \tau_n \leqslant \mu$ and to capacity limitations $\tau_j(\mathbb{R}) \leqslant \nu_j$ for $j = 1, \dots, n$. The considered setting gives rise to a bi-level decision problem which shares similarities with the optimal transport framework. With these analogies in mind, we develop an exact method of solution based on dynamic programming that produces a profit-maximizing pricing profile in $\text{poly}(n, |P|)$ basic operations if $|P|$ price values are available at each time slot for decision making. The proposed algorithm can accommodate distinct classes of customers with a computational overhead linear in the number of distance functions. Extensions to differenciated or random service times are straightforward. Our approach also applies for $P \equiv \mathbb{R}$ through discretization of the pricing space, then producing near-optimal solutions together with lower and upper bounds for the optimal value of the initial continuous-price problem. Under strongly convex d, pointwise convergence of these bounds towards the optimum typically occurs at rate $O(\delta)$ linear in the discretization step δ, slowing down to $O(\sqrt{\delta})$ in some irregular cases where ϵ-optimality of the profit can still be guaranteed within polynomial computation time. Numerical experiments are under way for a problem of time slot pricing at electric vehicle charging stations.

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Neural Networks for First Order HJB Equations

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Keywords: neural networks, deterministic optimal control, dynamic programming principle, first-order Hamilton Jacobi Bellman equations, state constraints, front propagation

Abstract

We consider a deterministic optimal control problem, in a finite horizon context, and propose deep neural network approximations for Bellman's dynamic programming principle [2], corresponding also to some first-order Hamilton-Jacobi-Bellman (HJB) equations. This work follows the lines of Huré *et al.* [1] where algorithms are proposed in a stochastic context. However, we need to develop a completely new approach in order to deal with the propagation of errors in the deterministic setting, where no diffusion is present in the dynamics. Our analysis gives precise error estimates in an average norm. The study is then illustrated on several academic numerical examples related to front propagations models in the presence of obstacle constraints (modelized by an optimal maximum running cost problem) showing the relevance of the approach for average dimensions (e.g. from 2 to 8), even for non-smooth value functions.

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Subgradient Sampling in Nonsmooth Nonconvex Minimization

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Keywords: Subgradient sampling, stochastic gradient, online deep learning, conservative gradient, path-differentiability, o-minimal structure

We consider a stochastic minimization problem in the nonsmooth and nonconvex setting which applies for instance to the training of deep learning models. A popular way in the machine learning community to deal with this problem is to use stochastic gradient descent (SGD). This method combines both subgradient sampling and backpropagation, which is an efficient implementation of the chain-rule formula on nonsmooth compositions. Due to the incompatibility of these operations in the nonsmooth world, SGD can generate artificial critical points in the optimization landscape which does not guarantee the convergence of the method. We will explain in this talk how the model of Conservative Gradients [1] is compatible with subgradient sampling and backpropagation, allowing to obtain convergence results for nonsmooth SGD. By means of definable geometry [2], we will emphasize that functions used in machine learning are locally endown with geometric properties of piecewise affine functions, allowing to interprete the chain-rule as a projection formula. In this setting, chain-ruling nonsmooth functions and sampling subgradients output conservative gradients, justifying the use of SGD in deep learning. As an additional consequence of the definable setting, subgradient sequences are generally gradient sequences, and artificial critical points are hardly attained when performing SGD in practice.

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Error estimates of a θ-scheme for second-order mean field games

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Keywords: Mean field games, numerical method, finite-difference scheme, error estimates

We introduce and analyze a new finite-difference scheme, that we call θ-scheme, for solving monotone second-order mean field games. These games consist of a coupled system of the Fokker-Planck and the Hamilton-Jacobi-Bellman equation. We use the θ-method for discretizing the diffusion terms, that is to say, we approximate them with a convex combination of an implicit and an explicit term. On contrast, we use an explicit centered scheme for the first-order terms. Assuming that the running cost is strongly convex and regular, we first prove the monotonicity and the stability of our θ-scheme, under a CFL condition. Taking advantage of the regularity of the solution of the continuous problem, we estimate the consistency error of the θ-scheme. Our main result is a convergence rate of order $\mathcal{O}(h^r)$ for the θ-scheme, where h is the step length of the space variable and $r \in (0,1)$ is related to the Hölder continuity of the solution (and some of its derivatives) of the continuous problem.

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Quantum Operation Research*

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Quantum optimization is a new and attractive area with potential significant implication in operation research. The basic concepts have been introduced early by Richard Feynman (Preskill, 2021) in the early 70s and the first algorithmic contributions have been introduced in the early 90 ': P. Shor published in 1996 a polynomial time algorithm for factorization and Grover the famous algorithm that gives a quadratic speedup for finding some market solutions into an unsorted finite set. In the last few years several quantum metaheuristics have been introduced coming from the quantum physic community, defining a family of quantum approximate algorithms that encompasses for example the sequel Adiabatic based Algorithms that provides an approximate solution of the Schrödinger equation.

Minimization problems can now be investigated using quantum metaheuristics with the promise of a strongly effective approach avoiding trapping into local minima that standard local search and satisfiability problem can be now investigating taking advantages of superposition.

The presentation will try to explain first where are the promises of quantum and why theoretical considerations and the capability in numerical experiment should push the researchers of the OR community to initiate researches in the topic.

Second we will try to present the various initiatives of the community around quantum including sessions, conferences and experience sharing all around the community.

Thirdly we will describe the French ecosystem around quantum initiatives and emphasis the scientific and financial support promoted by the various scientific organizations.

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*: this presentation is achieved on behalf of the GT ROQ that is a working group of the GDR RO

Learning with sign-dependent combinatorial optimization layers: a multiplicative perturbation approach

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Keywords: combinatorial optimization, machine learning, perturbed maximizer.

Combinatorial optimization (CO) layers in machine learning (ML) pipelines are a powerful tool to tackle data-driven decision tasks. Nonetheless, the solution of a CO problem often behaves as a piecewise constant function of its objective parameters. Given that ML pipelines are typically trained using stochastic gradient descent, the absence of slope information is very detrimental. Berthet et al. propose a framework to address this difficulty. They introduce an additive perturbation to differentiate through argmax. Nonetheless, this perturbation does not prevent the sign of the cost vector from changing, which can be detrimental. For instance, Dijkstra's algorithm only works with non-negative costs. In a recent work, we derive a multiplicative perturbation, which allows us to learn with sign-dependent combinatorial optimizers. We do so both in a learning by imitation and learning by experience settings. We emphasize the performance and utility of our method on the task of learning shortest paths on Warcraft terrain images (Figure 1), a benchmark problem introduced by Vlastelica et al.. To the best of our knowledge, it is the first time a learning by experience pipeline is used for this application.

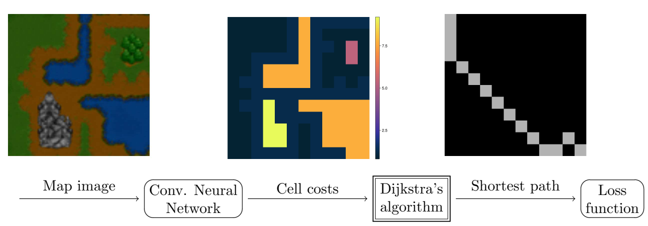

Figure 1: Pipeline for computing shortest paths on Warcraft maps – data from Vlastelica et al.

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Asynchronous Optimization of Relatively Smooth Objectives

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Keywords: Distributed optimization, asynchronous algorithms, proximal algorithms, Bregman divergence, Poisson inverse problems

Motivation. Many machine learning problems write as the minimization of a finite sum of loss functions over training examples. Oftentimes, the training data isn't stored locally but in remote servers, parallel cores, user-owned devices, and centralizing the data isn't always an option.

Setting. The *asynchronous centralized* setting is one of the possible approaches to the optimization of a distributed objective. In this setting, the objective function is split among a set of worker nodes that can all communicate with a central node. Each worker performs local computations based on its own data and information received from the central node.

Challenges. The data might be *unbalanced* which translates to ill-represented and/or slower nodes. There might also be *communication limitations* which could delay some workers' updates, which in turn might bias the minimization towards the objective of workers less slowed down.

Proposition. An asynchronous variant of (proximal) gradient algorithms which can cope with arbitrarily long delays has already been made under the assumption of *smooth local objectives* [1]. In this talk, based on [2], I present an asynchronous variant of the Bregman proximal-gradient method [3], which only requires *relative smoothness*. I then illustrate its behavior on a distributed Poisson linear inverse problem which takes the form $\min_{x\in\mathbb{R}^n_+} \sum_i f_i(x) + \lambda\|x\|_1$ with $\lambda \geq 0$ and

$$f_i(x) = \sum_{j\in J(i)} \langle a_j, x\rangle \log\langle a_j, x\rangle - (\log b_j + 1)\langle a_j, x\rangle + b_j$$

where each worker i possesses a few rows $J(i)$ of the matrix $A \in \mathbb{R}^{m\times n}_+$ and of the vector $b \in \mathbb{R}^m_{++}$.

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Mean-field Optimization regularized by Fisher Information

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Keywords: Mean field optimization, Nonlinear Schrodinger operator, Gradient flow, Monte Carlo method, Neural network)

Recently there is a rising interest in the research of mean-field optimization, in particular because of its role in analysing the training of neural networks. In this talk, by adding the Fisher Information (in other word, the Schrodinger kinetic energy) as the regularizer, we relate the mean-field optimization problem with a so-called mean field Schrodinger (MFS) dynamics. We develop a free energy method to show that the marginal distributions of the MFS dynamics converge towards the unique minimizer of the regularized optimization problem. We shall see that the MFS is a gradient flow on the probability measure space. Finally we propose a Monte Carlo method to sample the marginal distributions of the MFS dynamics.

Subset Sampling for Low-Discrepancy Point Sets

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Keywords: Discrepancy, Computational geometry, Mixed-Integer Linear Programming

Discrepancy measures are metrics designed to quantify how well a point set is spread in a given space. Among the different measures, the star discrepancy is the most important, measuring the largest deviation between the volume of any box $[0, q)$, $q \leq 1$, and the proportion of points that are inside this box.

Point sets of low discrepancy are used in many contexts, including Quasi-Monte Carlo integration, financial mathematics, computer vision and simulation-based optimization. Low-discrepancy constructions have been extensively studied asymptotically, such as Sobol' and Hammersley sequences, which are known to have low discrepancy values when the number of points tends to infinity [3]. However, in practice, applications requiring low-discrepancy point sets will only use a finite, much smaller number of points.

In this talk, we introduce the Star Discrepancy Subset Selection Problem [1], which consists in choosing a subset P_m from a set of n points, $m \leq n$, such that the L_∞-star discrepancy of P_m is minimized. The aim of this approach is to provide point sets better tailored to practical applications. Given that the complexity of calculating the star-discrepancy is $W[1]$-hard [2], it is not surprising that we are able to show that this problem is NP-hard. We provide two exact algorithms, respectively based on Mixed Integer Linear Programming and Branch-and-Bound, to tackle this problem. For dimensions two and three and n not too large, our algorithms provide point sets of much smaller L_∞-star discrepancy than for point sets taken directly from usual low-discrepancy sequences. We also extend this approach to higher dimensions with heuristics, providing point sets of better discrepancy for all dimensions for which the star discrepancy can be computed, answering open questions on the inverse star discrepancy [4].

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Pricing Bundles for Airline Revenue Management

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Keywords: Dynamic pricing, Bundling, Airlines, Revenue Management, Choice modelling

Airline companies use revenue management techniques in their daily operations to a great extent, but they frequently overlook ancillaries revenue [1]. Ancillaries are non-core products, such as on-board (i.e. Wi-Fi, additional baggage) or post-flight (i.e. insurance, car rental) services, that account for an increasing share of airlines revenue. Most of the time, these products have a quasi-zero marginal cost, so the production cost of one additional item is negligible. An important aspect of ancillaries is that they are lost if they are not bought. For instance, food will eventually expire, and services like additional baggage are lost when the flight departs. Hence, in this context, it's very meaningful to make use of revenue management. These additional products are also seen as very important by some customers. As a consequence, the latest developments in the airline industry have the objective of pushing their purchase. Ancillaries can be combined into a single commodity called bundle, which will offer a set of ancillaries at a single price. This bundling strategy increases demand and overall profit by sacrificing the individual ancillary profit margin. In particular, web sales are well adapted to the proposal of a few bundles to the client.

The main challenge of this approach lies in the pricing and assortment of the bundles offered for sale. The difficulty arises from the structure of the problem, since only 3 or 4 bundles of a combinatorially large number can be proposed to purchase. An important aspect to take into account is the diversity of the proposed bundles. Ideally, we want to propose to customers a set of diverse and relevant bundles. Another challenge that we face is the destroy ancillary value effect. This effect occurs if we don't take into consideration that ancillaries can be sold individually at a later time. This loss of value is linked to the observation that customers' price sensitivity is lower at check-in time than at selling time. It could be because they have forgotten how much they have already spent or due to unforeseen circumstances when they purchased the ticket.

In this work, we present a revenue management model based on a Nested Multinomial Logit customer choice model that does not destroy ancillaries value. Therefore, we introduce a two stage booking process where we take into account the selling time and the check-in time.

Numerical experiments highlight the efficiency and diversity of the suggested bundles of the proposed model.

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Colored Petri-Nets and Synchronous Dataflow Graphs to optimize a medical evacuation

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Keywords : Colored Petri Net, Synchronous DataFlow, Timed Weighted Event Graph

MEDICAL evacuation is a key element in a military operation. This study proposes a new way to dimension medical evacuation (MEDEVAC), such as wounded transportation and medical teams. Each wounded type has its own medical protocol, with different steps and due dates. Dimensioning MEDEVAC corresponds to find the proper number of transport teams and medical teams that allows a scheduling saving every one.

The formalism from literature called Synchronous Dataflow Graphs (SDF) [1] is a Petri-net capable of representing a deterministic flow of tokens on a graph. The SDF formalism is a timed event graph with production and consumption weights on the arcs. Each task in the SDF graph is defined by an execution time. A task is executed if the number of tokens present on each of its incoming arcs is greater or equal than the corresponding consumption weights. At the end of its execution, this task produces on its outgoing arcs a number of tokens equal to the corresponding weights.

Our new formalism enriches the Synchronous Dataflow Graphs model with colored tokens, which depict the severity of the wounded and the medical teams they must go, and with a constraint on weights such that no loss or gain of tokens happens after a task's execution. Other properties were studied: dimensions of tasks which allow multiple execution of the same task with different starting time.

Our new formalism inherits from SDF properties, which allows a simple estimation of consistency and liveness [2]. Consistency determines whether a graph is well constructed. Liveness determines the absence of deadlock during its execution. This formalism also provides various approaches to determine a lower bound of the throughput.

This presentation introduces the Colored-Conservative SDF (Co2SDF) formalism, the consistency verification and the possibilities to evaluate and optimize a MEDEVAC with a Co2SDF.

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A static game model for analyzing country carbon emissions

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Keywords: Carbon emissions Game theory Public policy

In this work, we study a game in which the players are the governments, the action of a player is the carbon emission level, and the utilities implement a tradeoff between the benefit for emitting which is the gross domestic product and the damage caused by climate changes. We discuss the choice of the benefit function, the damage function, and the choice of the geophysical model. Compared to existing works, our approach integrates the existence of a dynamics for the global earth temperature and CO2 concentrations and considers the damage functions obtained from the DICE model.

Mean-field Control under State Constraint: Optimality Conditions and Mean-field Limit

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Keywords: Mean-field Control, State-Constrained Optimization ,Mean-field Game,

We present a stochastic control problem where the probability distribution of the state is constrained to remain in some region of the Wasserstein space of probability measures. Reformulating the problem as an optimal control problem for a (linear) Fokker-Planck equation, we derive optimality conditions in the form of a mean-field game system of partial differential equations. The effect of the constraint is captured by the presence, in this system, of a Lagrange multiplier which is a non-negative Radon measure over the time interval. Our main result is to exhibit geometric conditions on the constraint, under which this multiplier is bounded and optimal controls are Lipschitz continuous in time. As a consequence we prove, in a second time, that the stochastic control problem with constraints in law, arises as limit of control problems for large number of interacting agents subject to almost-sure constraints. The first part of the presentation is based on the results of [1].

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Hidden Convexity in the ℓ_0 Pseudonorm

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Keywords: generalized convexity, sparse optimization, ℓ_0 pseudonorm and rank

The so-called ℓ_0 pseudonorm counts the number of nonzero components of a vector. It is a standard notion in sparse optimization problems. However, as it is a discontinuous and nonconvex function, the ℓ_0 pseudonorm cannot be satisfactorily handled with the Fenchel conjugacy. In this talk, we review a series of recent results on a class of Capra (Constant Along Primal Rays) conjugacies that reveal hidden convexity in the ℓ_0 pseudonorm [3, 1, 2, 4].

First, we present the Euclidean Capra-conjugacy. We show that it is suitable to analyze the ℓ_0 pseudonorm, as this latter is "convex" in the sense of generalized convexity (equal to its biconjugate). We immediately derive a convex factorization property (the ℓ_0 pseudonorm coincides, on the unit sphere, with a convex lsc function) and variational formulations for the ℓ_0 pseudonorm. We present mathematical expressions of the Capra-subdifferential of the ℓ_0 pseudonorm, and graphical representations.

In a second part, we provide different extensions. We introduce the class of Capra-conjugacies defined by means of norms — especially strictly-orthant monotonic norms (including the Euclidean norm) — or, more generally, of 1-homogeneous nonnegative functions. We show that such Capra-conjugacies are suitable to analyze, not only the ℓ_0 pseudonorm, but provide convex lower bounds for 0-homogeneous functions. We will also point out how to tackle the rank matrix function.

Finally, we discuss how the theory could open the way for possible algorithms in sparse optimization problems.

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Progressive decoupling of linkages in a class of optimization problems with non-elicitable convexity

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Keywords: Nonconvex optimization, Decomposition, Splitting, Stochastic programming

Commonly, decomposition and splitting techniques for optimization problems strongly depend on convexity. Implementable splitting methods for nonconvex and nonsmooth optimization problems are scarse and often lack convergence guarantees. One exception is the Progressive Decoupling Algorithm (PDA) [1], which has local convergence should convexity be elicitable. In this work we furnish the PDA with a descent-test and extend the method to accommodate a broad class of nonsmooth optimization problems with non-elicitable convexity. More precisely, we focus on the problem of minimizing the difference of convex and weakly convex functions over a linear subspace. This framework allows covering, in particular, a family of stochastic programs with nonconvex recourse and statistical estimation problems for supervised learning.

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A Fairness Framework for Coflow Scheduling

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Keywords: data transfer, coflow scheduling, fairness, progress, primal-dual scheduler

A coflow identifies a group of datacenter network flows sharing a common goal [1]. In data-intensive computing frameworks it represents the data transfer phase acting as synchronization barrier between successive computation phases occurring in different physical nodes. The Coflow Completion Time (CCT) is the time by which the last flow of a coflow is completed. Minimizing the average CCT of a coflow batch is *NP*-hard and inapproximable below a factor 2, with best to date deterministic approximation ratio of 4 [2,3]. While minimizing the average CCT increases the rate of computing jobs dispatched in a datacenter, it entails potential unfairness among different flows due to starvation. Fairness has thus been studied in the context of coflow scheduling: in this context, the efficiency/fairness tradeoff balances the system performance, i.e., the average CCT, versus the individual coflow performance. The standard notion of coflow fairness is defined by using the notion of *coflow progress*, i.e., by guaranteeing a minimum resource allocation to coflows scheduled concurrently [4]. Any further performance optimization, e.g., minimization of average CCT or maximization of port utilization, is usually pursued on top of this baseline constant non-pre-emptive rate allocation. In this work we introduce a fairness framework where the notion of progress corresponds to the average rate granted to a coflow while in service. It is controlled via the inverse metric, e.g., the *slowdown*, which measures the additional delay on the data transfer time of a coflow compared to isolation, i.e., when a coflow is scheduled alone in the datacenter fabric. A *slowdown* constraint provides a single parameter control able to strike the trade-off between average CCT and the individual coflow slowdown. The first contribution we discuss is a fully polynomial time algorithm, namely MPS (Minimum Primal Slowdown): it provides an accurate estimation of the minimum feasible slowdown for a batch of coflows. Second, we introduce the CoFair scheduler to extend the family of the primal-dual algorithms [3,5]: by using the output of MPS, it produces a *primal-feasible* σ-order. Finally, a simple order-preserving work-conserving transmission policy realizes the rate allocation for the actual coflow scheduling. CoFair is tested against state of the art coflow schedulers demonstrating more effective in attaining a flexible performance-fairness tradeoff [6].

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Optimal control of a bioeconomic model applied to the recovery of household waste

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Keywords: Optimal control, Pontryagin's principle, Bioeconomic models, Waste recovery

A mathematical model describing the process of generating energy from household waste treatment is analyzed. This is in fact an upgraded version of the earlier model introduced in [1]. It is a three-dimensional nonlinear system that depicts the concept of transforming household waste stored in a landfill into energy that flows to a user's network. More precisely, the state of the system describes at a broad scale a process of generating energy E by treating a quota of a waste stock x through K-valorization units (that may also consume a part of the produced energy for their functioning). Our main objective is to maximize the energy produced and transmitted to the user's network. In particular, we investigate the issue of determining an optimal investing strategy that monitors the deployment of treatment plants. Using Pontryagin's maximum principle (PMP, based on the *current value-Hamiltonian* framework [2]), we characterize, over a fixed time-window $[0, T]$, the optimal investment that maximizes the produced energy while limiting the overall costs. In addition, the efficiency of the suggested optimal control strategy is illustrated throughout this talk using a direct optimization method (implemented in `Bocop`: optimal control solver).

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New Runtime Guarantees for the Non-Dominated Sorting Genetic Algorithm II (NSGA-II)

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Keywords: Multi-objective optimization, evolutionary algorithms, runtime analysis.

The *non-dominated sorting genetic algorithm II* (NSGA-II) [1] is the most intensively used multi-objective evolutionary algorithm (MOEA) in real-world applications. Only very recently, the first mathematical runtime analysis for this algorithm was conducted [6]. In this talk, we report on the follow-up work conducted in the second author's M2 internship. In [3], we conducted the first runtime analysis on a bi-objective benchmark having multimodal objectives. It shows that the NSGA-II admits very similar performance guarantees as those shown earlier [5] for the simple GSEMO algorithm. Our experimental investigation finds that with crossover, the NSGA-II becomes much faster than the GSEMO on this benchmark. We give a mathematical proof for this speed-up in [2]. Building on a careful analysis of the population dynamics, in [4], we prove the first lower bounds on the runtime of the NSGA-II. They show that the previous analysis of the mutation-only version of the NSGA-II on the OneMinMax and OneJumpZeroJump benchmarks are asymptotically tight.

This work is part of the PGMO-funded project *Understanding and Developing Evolutionary Algorithms via Mathematical Runtime Analyses* (PI: Benjamin Doerr). The M2 internship of the second author was funded by this PGMO project.

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Parking duration optimization for electric vehicles in a loss queueing system

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Keywords: Electric vehicles, charging station, stochastic model

With the rise of Electric Vehicles (EVs), the demand for parking spots equipped with plugging devices in the charging stations (CSs) is tremendously increasing. To face this demand, a charging point operator responsible for one CS deals with limited resources, both in terms of the number of parking spaces equipped with a Charging Point (CP) as well as the available power. This motivates us to study how to improve the quality of charging service at a given CS. Each CP within the CS can deliver a maximum power, independently of the time. Moreover, there is a total limited maximum power available at the CS level, also independently of the time. Therefore, at each instant, the current charging EVs have to share the total power available at the CS according to a modified processor sharing rule, i.e EVs charge at the maximum power per CP when the number of charging EVs (CPs in use) is sufficiently low, otherwise the total power is uniformly shared. In our model, the stochastic nature of arrivals and departures at the CS is modeled by a queueing system [1]. EV users arrive at the CS according to a Poisson process and with a random amount of energy needed to fully charge their battery. An EV can occupy a CP without consuming power: each EV has a random parking duration and leaves the parking spot only when its parking time expires. The total number of EVs and the number of charging EVs at the CS at each instant follows a two-dimensional Markov process. In our work, and based on a study of the Markov process [1], a closed form approximation of the optimal average parking duration in terms of the expected energy received per EV is provided. The model is numerically simulated using a Python discrete-event simulation framework *Simpy*. The theoretical approximation is then compared with the optimal average parking duration obtained from the simulations.

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Robust MILP formulations for the two-stage p-Center Problem

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Keywords: p-Center problem, robust MILP formulations, *branch-and-cut* algorithm

The deterministic p-center problem (PCP) is one of the most studied facility location problems in the literature of location science [2]. It consists in installing p centers out of m available sites, assigning n clients to these p centers in order to minimize the *radius* which corresponds to the maximum distance or travel time between a client and its closest open site. The incorporation of uncertainty in the (PCP) has important applications in emergency logistics problems. Two approaches can be considered depending on whether the client allocations to the centers are made before or after the uncertainty is revealed. The first case corresponds to single-stage problems while the second case leads to two-stage problems in which the client allocations are recourse variables. In this context, most of the articles have focused on heuristic approaches.

We study the exact solution of the two-stage robust p-center problem $(RPCP_2)$ in which the uncertainty on the node demands and the travel times are modeled by box uncertainty sets. The $(RPCP_2)$ aims to find a set of p sites for which its regret of the worst-case scenario is minimal. Since a box uncertainty set contains an infinite number of scenarios, the evaluation of the regret is a major challenge when solving the $(RPCP_2)$. To make it tractable, we prove that only a finite subset of scenarios can be considered without losing optimality. We use this result to propose three robust formulations based on different deterministic MILP formulations of the (PCP) from the literature. To solve these reformulations, we introduce a *column-and-constraint* generation algorithm and a *branch-and-cut* algorithm. Moreover, we highlight how they can be adapted to optimally solve the single-stage problem $(RPCP_1)$ for which no exact resolution method has been previously introduced. We present a numerical study to compare the performances of our algorithms on randomly generated instances and a case study introduced in [1] with up to 52 demand nodes. We solve 80 instances for several values of p and number of sites.

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Design and dimensioning of natural gas pipelines with hydrogen injection

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Keywords: Hydrogen networks, gas pipelines, MINLP

The problem of designing and sizing a hydrogen injection pipeline network is to find both a network topology and the diameter dimensions of each pipe section for hydrogen distribution. Formally, given a digraph $G = (N, A)$, where N is the set of hydrogen supply and consumption nodes and A is the set of potential links through the hydrogen pipelines, we define a pipeline network topology as the connected partial graph $G' = (N, A')$ induced by a set of arcs $A' \subset A$. Each node has a demand for hydrogen or can supply a quantity. The aim is to minimize the cost of building pipelines and distributing hydrogen, so that the proposed network can meet customer demand. The flow of hydrogen in the network is governed by the physical quantities related by the equation of fluid mechanics:

$$(\pi_1 - \pi_2)D^5 = k'Q^2L \tag{1}$$

where π_1 and π_2 represent the square of the pressure at the inlet and outlet of a pipe, D and L are the diameter and length of the pipe, and Q is the quantity of gaz flowing in the pipe. Finally, k' is a constant.

Due to the form of equation (1), the mathematical models representing this problem are complex because they include polynomial terms up to degree 6, as well as some integer variables. In this study, we consider that the diameters of the pipes can only take a few possible values. This is a realistic assumption whose first effect is to increase the number of integer variables in the models. But it also allows us to reformulate the degree-6 polynomial model into a quadratic one. The resulting model is thus a quadratically constrained quadratic problem with binary variables. We also show that its continuous relaxation amounts to a convex optimization problem. Finally, fixing the topology network in this model leads to a mixed-integer linear problem and gives rise to fast heuristics. We implement the global solution method based on our discrete-diameter models, together with heuristics. We then compare our results with existing solution methods on randomly generated instances and on a case study based on realistic data from [1].

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Global Solution of Quadratic Programs by Interval Methods and Convex Reformulations

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Keywords: MINLP, QP, Interval methods, Semidefinite programming

We consider the quadratic optimization problem (QP) that consists in minimizing a quadratic function of mixed-integer variables under quadratic inequalities. Our objective is to make enumeration methods based on interval arithmetic and quadratic convex reformulation methods cooperate in order to better solve (QP), globally and rigorously. Our work leads to a new quadratic solver called `QIBEX` that is a quadratic variant of `IbexOpt`, a constrained nonlinear optimization tool using rigorous interval algorithmic operators [1, 4]. Interval methods provide `IbexOpt` with two main advantages: first, the guarantee of the solution obtained despite rounding problems on floating numbers; second, the possibility of defining the constraints and the objective function based on a wide variety of mathematical operators.

Our first contribution is an interval B&B algorithm that can solve globally (QP). At each node, the hybrid solver `QIBEX` uses a quadratic convex relaxation (PC^*) [2, 3] that is calculated in a preprocessing phase from the solution of a semidefinite optimization problem, together with a bisection heuristic dedicated to quadratic optimization. The interval features can then efficiently propagate this information for contracting all variable domains. Our experiments show significant speedups on integer quadratic instances.

The solvers that compute the optimal solution to (PC^*) are not rigorous because they are subject to round-off errors: they may output an objective function value that does not bound the optimal real cost by below. To overcome this, we correct the cost by using properties proven in [5]. Our experiments show that the rigorous version of `QIBEX` still remains efficient.

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Quadratic Error Bound of the Smoothed Gap and the Restarted Averaged Primal-Dual Hybrid Gradient

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Keywords: linear convergence, primal-dual algorithm, error bound, restart

We study the linear convergence of the primal-dual hybrid gradient method, also know as Vu-Condat method. After a review of current analyses, we show that they do not explain properly the behavior of the algorithm, even on the most simple problems. We thus introduce the quadratic error bound of the smoothed gap, a new regularity assumption that holds for a wide class of optimization problems including strongly convex-concave saddle point problems and linear programs. Equipped with this tool, we manage to prove tighter convergence rates.

Then, we show that averaging and restarting the primal-dual hybrid gradient allows us to leverage better the regularity constant. We also propose an adaptive way of setting the restarting instants. Numerical experiments on linear and quadratic programs, ridge regression and image denoising illustrate the findings of this work. In particular, we show that on some problems, the rate of convergence of the new method can be faster by orders of magnitude.

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A Boosted DC Algorithm for Non-Differentiable DC Components with Non-Monotone Line Search

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Keywords: DC function, boosted difference of convex functions algorithm, DC algorithm, non-monotone line search, Kurdyka-Łojasiewicz property.

In [1], we introduce a new approach to apply the boosted difference of convex functions algorithm (BDCA) for solving non-convex and non-differentiable problems involving difference of two convex functions (DC functions). Supposing the first DC component differentiable and the second one possibly non-differentiable, the main idea of BDCA is to use the point computed by the DC algorithm (DCA) to define a descent direction and perform a monotone line search to improve the decreasing the objective function accelerating the convergence of the DCA. However, if the first DC component is non-differentiable, then the direction computed by BDCA can be an ascent direction and a monotone line search cannot be performed. Our approach uses a non-monotone line search in the BDCA (nmBDCA) to enable a possible growth in the objective function values controlled by a parameter. Under suitable assumptions, we show that any cluster point of the sequence generated by the nmBDCA is a critical point of the problem under consideration and provide some iteration-complexity bounds. Furthermore, if the first DC component is differentiable, we present different iteration-complexity bounds and prove the full convergence of the sequence under the Kurdyka-Łojasiewicz property of the objective function. Some numerical experiments show that the nmBDCA outperforms the DCA such as its monotone version.

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Periodical Infinite Horizon Optimization for Multistage Problems

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Keywords: Infinite horizon, multistage problems, convex optimization

The long-term operation of hydro and thermal power stations can be cast as a multistage stochastic optimization program—as in the case of the Brazilian interconnected power system. This illustrative problem setting commonly features seasonal periodicity arising from climate and/or energy consumption patterns. The traditional approach is to consider a large but finite horizon, which breaks this property. We propose an infinite horizon approach, allowing for further exploitation of the periodicity in the solution process. Additionally, this setting avoids terminal boundary conditions and introduces questions regarding the limit behavior of solutions over time. We present solution algorithms for such problems and showcase numerical examples, including simple deterministic examples that nonetheless exhibit complex behavior.

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Non-uniform Observability for Moving Horizon Estimation and stability with respect to additive perturbation

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Keywords: Moving Horizon Estimation, Nonlinear Observability, Time Varying Optimisation, Stability of solutions, Implicit Function Theorem

In tackling nonlinear dynamical estimation problems using the machinery of mathematical optimisation, a well-known computationally tractable method is to define a cost on past inputs and outputs on a moving horizon and to estimate the associated state trajectory by minimising that cost over state trajectories. The estimator is then built from the resulting optimal solution. This leads to Moving Horizon Estimation (MHE). In the classical litterature on robust stability of MHE methods, observability assumption are typically stated independently of any control input and a global solution of a possibly nonconvex program is assumed to be available, see chapter 4 of [1]. The former assumption is very strong and ignores necessary refinements coming from the notion of persistent inputs in nonlinear observability theory [2]. The latter assumption is impractical [3] as only local solutions can be usually obtained numerically. It allows one to weaken the notion of observability considered. Thus, this paper formalises the concepts of weakly and weakly regularly persistent input trajectory as well as their link to the Observability Grammian and the existence and local uniqueness of local solutions of MHE problems. Additionally, thanks to a new time-uniform Implicit Function Theorem, these notions are proved to imply the stability of MHE solutions with respect to small additive perturbation in the measurements, both uniformly and non-uniformly in time. Finally, examples of weakly regularly persistent input trajectories are given in the case of 2D bearing-only navigation [4].

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Coupled production management and energy supply planning of an industrial microgrid

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Mots clés : Stochastic Optimization, Renewable Energies, Production Planning

We consider the problem of jointly optimizing the daily production planning and energy supply management of an industrial complex, with manufacturing processes, renewable energies and energy storage system. It is naturally formulated as a mixed-integer multistage stochastic problem. This problem is challenging for three main reasons: there is a large number of time steps (typically 24), the renewable energy are uncertain and uncontrollable, and we need binary variables modeling hard constraints. We discuss various solution strategies, in particular Model Predictive Control, Dynamic Programming, and heuristics based on the Stochastic Dual Dynamic Programming algorithm.

We compare these strategies on two variants of the problem: with or without day-ahead energy purchases. Without day-ahead purchases, although it is clear taking uncertainties into consideration is a necessity, MPC has the best results, beating stochastic heuristics. However, with day-ahead purchases and therefore decisions at $t = 0$ impacting the whole horizon, stochastic heuristics perform better than MPC.

Monotonic Ensemble Classifiers for Biomedical Applications

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Keywords: Classification, Biomedicine, Monotonic Regression, Feature Pair Selection

The advent and use of RNA sequencing has led to the significant development of computational methods for the diagnosis and classification of cancers and other serious diseases. One of the challenges is the interpretability of the models. Many common models perform well, but are difficult to analyze in terms of biological reality.

Feature pairs monotonic models [1] can help to address this issue. Our approach originally consisted in testing all the pairs of genes and selecting the best ones to make an ensemble model [2].

We introduce a preselection scheme that drastically reduces the number of feature pairs that need to be considered, with provable guarantee to return the same result as the original approach run on the full set of feature pairs.

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Crossover for Cardinality Constrained Optimization

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Keywords: Evolutionary algorithms, runtime analysis, crossover, balanced operators

In order to understand better how and why crossover can be beneficial in evolutionary optimization, we consider the BoundMax benchmark. This generalization of the well-known OneMax function was introduced by Friedrich et al. [1] and has increasing fitness only as long as the number of 1-bits does not exceed a given parameter B and is negative, otherwise. The unique optimum is the string of B 1-bits followed by $n-B$ 0-bits. The literature gives a runtime bound of $\Theta(n^2)$ for the (1+1) EA on this benchmark [1]. Part of the difficulty when optimizing this problem lies in having to improve individuals that already have the correct number of 1s and 0s but not at the correct positions. Then, to progress, two bits must flip in the same iteration. The experimental literature proposes balanced operators, preserving the number of 1s, as a remedy. We analyze how balanced mutation and balanced crossover operators in different settings optimize the problem. With the best combination of operators, we achieve an optimization time in $\mathcal{O}(n \log n)$. As an additional contribution we analyze and discuss different balanced crossover operators from the literature.

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One-to-Many Matching Games

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Keywords: Stable Matching, Substitutability, Feasibility, Matching with contracts

The seminal work of Hatfield and Milgrom [4] extended many models of stable matching [1, 2, 3, 5, 6] to the setting in which doctors and hospitals are matched and, simultaneously, choose a contract from a finite set of options. The authors proved the existence of stable allocations under *substitute contracts* using a *cumulative offer mechanism* (COM).

Matching games add a new dimension to this problem by allowing the agents within a coalition to play strategic games and receive payments as outputs. They naturally faces two stability notions: *external stability*, the generalization of Hatfield and Milgrom's stability, and *internal stability*, a novel notion concerning the agents' deviations in actions within each game.

Efficient algorithms are designed to compute externally and internally stable allocations under classical game theory assumptions and *substitute strategies*. First, we propose a generalization of the classical *deferred-acceptance* algorithm of Gale and Shapley to compute externally stable allocations, in particularly proving that simpler algorithms than COM are enough to achieve the task. Second, a novel *strategy profiles modification* algorithm is designed to compute an internally stable allocation whenever the strategic games played satisfy a *feasibility* condition and the algorithm converges.

Feasibility in games is a novel property. We characterize it using *constrained Nash equilibria* and prove that many well-known games from the literature of game theory are feasible.

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State constraints in stochastic optimization

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Keywords: Optimization in Banach spaces, optimality conditions, regularization, convex stochastic optimization in Banach spaces, duality.

In this talk, which is based on the papers [1, 2], we present necessary and sufficient optimality conditions for convex stochastic optimization problems subject to almost sure equality and conical constraints. We refine classical results by Rockafellar and Wets from two-stage stochastic optimization to include states belonging to the Bochner space of essentially bounded random variables with images in a reflexive and separable Banach space. Under certain conditions, the optimality conditions given are necessary and sufficient. Lagrange multipliers exhibit different regularity depending on whether or not the assumption of relatively complete recourse is satisfied. We propose a Moreau–Yosida regularization for such problems and show consistency of the optimality conditions for the regularization problem as the regularization parameter is taken to infinity. Algorithmic approaches using stochastic approximation are discussed as well as possible applications in PDE-constrained optimization under uncertainty.

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A Mean Field Game model in Economics with spatial interactions in the human capital

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Keywords: Mean Field Games, Economics, spatial interactions, non monotone cost, non separable Hamiltonian

We study a Mean Field Game (MFG) model arising in Economics where each agent chooses its position in space and its level of human capital- i.e. the skills aimed at use in production possessed by each individual- and interacts with the other agents through the human capital. The main goal consists in studying the effect of spatial interaction terms (called in Economics spatial spillovers) on the abilities of each individual, due to the proximity, in terms of spatial distance, of other individuals. The resulting MFG system has a highly non standard structure, that is, a non-separable Hamiltonian and a non-monotone cost. In the talk we describe the economic model, we prove existence to the associated MFG system by a fixed point argument, and we provide some numerical experience.
This is a joint work with F. Gozzi (LUISS, Italy) C. Ricci (Pisa, Italy), and G. Zanco (LUISS, Italy). The economic model has been developed through contributions by G. Fabbri (CNRS Grenoble), D. Fiaschi (Pisa, Italy), F. Gozzi (LUISS, Italy).

Relaxed-inertial Proximal Point Algorithms for Problems Involving Strongly Quasiconvex Functions

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Keywords: proximal point algorithm, strongly quasiconvex functions, nonconvex optimization, generalized convexity)

Introduced in the 1970's by Martinet for minimizing convex functions and extended shortly afterward by Rockafellar towards monotone inclusion problems, the proximal point algorithm turned out to be a viable computational method for solving various classes of optimization problems, in particular with nonconvex objective functions.

We propose first a relaxed-inertial proximal point type algorithm for solving optimization problems consisting in minimizing strongly quasiconvex functions whose variables lie in finitely dimensional linear subspaces (see [2]). The method is then extended to equilibrium problems where the involved bifunction is strongly quasiconvex in the second variable (see [1]).

Possible modifications of the hypotheses that would allow the algorithms to solve similar problems involving quasiconvex functions are discussed, too. Numerical experiments confirming the theoretical results, in particular that the relaxed-inertial algorithms outperform their "pure" proximal point counterparts [3, 4], are provided, too.

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Newton Method for Stochastic Control Problems: Application to Decentralized Control of Energy Storage Systems

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Keywords: Stochastic control, stochastic Pontryagin principle, Forward-Backward Stochastic Differential Equations, Newton method, Energy Management

The Newton method is an iterative method to solve smooth non-linear optimization problems or equations. For unconstrained optimization problems, under some convexy and regularity conditions, local quadratic convergence can be established. Global convergence can be further obtained by using a line search procedure.

In this work [1], we generalize this method to convex stochastic control problems, where the decision variable is a stochastic process. We show that the Newton step can be crucially interpreted as the solution of an auxiliary Linear-Quadratic stochastic control problem which approximates the original problem around the current iterate, and which can be explicitly solved, by introducing a Riccati Backward Stochastic Differential Equation. This gives a practical and efficient way to compute the Newton step, by solving only (Backward) Stochastic Differential Equations, using for instance Monte-Carlo regression techniques.

Regarding the theoretical convergence properties, we show that, even under strong regularity assumptions, Lispchitz-continuity of the second order derivative does not hold in general, which prevents us from establishing local quadratic convergence of the method in the space of square integrable processes. To overcome this issue, we show that under some boundedness assumptions, the problem can be posed in an alternative vector space in which the second-order derivative becomes Lipschitz continuous: the space of essentially bounded stochastic processes. Using an adaptation of the line-search procedure, we design a globally convergent algorithm in the space of bounded processes, with quadratic convergence in a neighbourhood of the optimal solution, and with arithmetic convergence outside. The algorithm is used to solve a stochastic control problem aiming at controlling a large population of batteries in order to help balancing the electricity grid, using a methodology proposed in [2].

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Smart Charging under Uncertainty on Arrival and Departure Dates: a Robust Optimization Approach

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Keywords: Electrical Vehicles, Robust Optimization, Recourse policy, Binary Quadratic Optimization, Convex relaxations

The rising share of electrical vehicles leads to challenging optimization problems, like for instance, the operational management of a charging station. The charging station manager has for instance to choose how to allocate power to the various vehicles plugged at each time, while accounting for several potentially contradictory objectives: cost, satisfaction of mobility needs... In a deterministic setting, this problem can be modelled as a linear programming problem. In an uncertain context, it becomes much more involved to model and solve. One of the most impactful and difficult source of uncertainty for operational management of a charging station is the randomness of arrival and departure dates for electrical vehicles. In our linear programming models, this amounts to consider uncertainty for binary-valued parameters $(u_{v,t})_{v,t}$, where $u_{v,t}$ is equal to 1 when vehicle v is plugged at time t, and 0 else.

In this work, we aim at designing a stochastic optimization model for the management of a charging station, under uncertainty on arrival and departure dates, with several objectives. First, we want to account for real-time recourse. Second, we want to enforce non-anticipacivity of our decisions. Third, we wish to design an optimization model which can be solved in reasonable time, even for instances of moderate sizes (few tens of vehicles and time steps), to be consistent with the industrial objectives.

We argue that many classical methods do not satisfy these requirements due to curse of dimensionality with respect to state dimension (dynamic programming), number of time steps (multi-stage stochastic programming) or noise (scenario based models). Moreover, some stochastic optimization paradigms require to probabilize the uncertainty, which could be costly if statistical models have to be adapted to every single charging station, and dynamically updated.

Robust optimization approach is a more suitable candidate for which uncertainty is easier to model through the specification of an uncertainty set. In order to ensure real-time recourse to avoid over-conservatism, we assume the recourse decisions (power allocation to each plugged vehicle) are in parametric form (feedback), and show that their simplest consistent form is quadratic with respect to the uncertain parameters $(u_{v,t})_{v,t}$, and non-anticipativity can be ensured by properly constraining our feedback function parameters. We thus obtain a min-max problem, where the lower level problems are non-convex quadratic programming problems with binary variables. Nontheless, we show that by using linear relaxations of the lower level problems (i.e., by enlarging the uncertainty set), and duality-based reformulation techniques, we can recover a tractable single level linear programming problem (with additional dual variables), which solution is feasible and hopefully performant for the original robust optimization problem. Numerical results support the study.

On the Computational Complexity of the Moment-SOS Hierarchy for Polynomial Optimization

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Keywords: Polynomial optimization, semidefinite programming, computational complexity

The moment/sum-of-squares (moment-sos) hierarchy is one of the most celebrated and widely applied methods for approximating the minimum of an n-variate polynomial over a feasible region defined by polynomial (in)equalities. A key feature of the hierarchy is that it can be formulated as a semidefinite program of size polynomial in the number of variables n. Although this suggests that it may therefore be computed in polynomial time, this is not necessarily the case. Indeed, as O'Donnell and later Raghavendra & Weitz show, there exist examples where the sos-representations used in the hierarchy have exponential bit-complexity. We study the computational complexity of the sos-hierarchy, complementing and expanding upon earlier work of Raghavendra & Weitz. In particular, we establish algebraic and geometric conditions under which polynomial-time computation is possible. As this work is still ongoing, our results should be treated as preliminary.

Regularized Rényi Divergence Minimization through Bregman Proximal Gradient Algorithms

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Keywords: Variational Inference, Bregman Proximal Gradient Algorithms, Rényi Divergences, Exponential Families

Variational inference methods aim at approximating complex target probability distributions by a simpler distribution through the minimization of a divergence. In this work, we propose to minimize a regularized Rényi divergence over an exponential family. To solve this problem, we propose a relaxed moment-matching algorithm, which includes a proximal-like step. We also introduce a sampling-based implementation for non-conjugate models. Using results from information geometry [2], our algorithm is shown to be equivalent to a Bregman proximal gradient algorithm [4]. This novel point of view allows us to exploit the geometry of our approximate model [1] while using stochastic black-box updates [3].

Moreover, depending on the parameter α of the considered Rényi divergence, we are able to establish the relative smoothness of the objective, its relative strong convexity, or both properties. We leverage these new results to prove strong convergence guarantees for our deterministic method including monotonic decrease of the objective, convergence to a stationary point or to the minimizer, and convergence rates. We also show through a simple counter-example how the conditions for the convergence of an equivalent Euclidean scheme may fail. These new theoretical insights lead to a versatile, robust, and competitive method, as illustrated by numerical experiments comparing the Euclidean method of [3] with our algorithm.

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Discrete Optimization Methods for Problems with the L0-Pseudonorm

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Keywords: L0-Pseudonorm, Mixed Integer Programs, SOS constraints, Branch and Bound, Screening tests.

Finding a sparse representation is a fundamental problem in the field of statistics, machine learning and inverse problems, among many others [1]. This task can be addressed by leveraging an ℓ_0-norm penalization, which leads to optimization problems of the form

$$\min_{\mathbf{x}} \; f(\mathbf{x}) + \lambda\|\mathbf{x}\|_0 \tag{$\mathcal{P}$}$$

On the one hand, the function $f(\cdot)$ encodes the model to fit and on the other hand, the ℓ_0-norm enforces sparsity in the optimizers of $(\mathcal{P})$ by counting the number of non-zeros in its argument. The hyperparameter $\lambda > 0$ allows to balance these two terms. Problem $(\mathcal{P})$ is unfortunately NP-hard in the general case. Nonetheless, recent contributions proposed strategies to solve $(\mathcal{P})$ exactly and efficiently. Most leverage discrete optimization tools and methods.

We propose a survey of advances made the past decade to solve $(\mathcal{P})$ exactly. First, we review some key applications driving the interest in addressing $(\mathcal{P})$, despite its NP-hardness. Then, we detail how $(\mathcal{P})$ can be reformulated as a Mixed Integer Program. Such standard reformulation allows to rely on a broad class of off-the-shelf solvers. In particular, we focus on formulations employing Special Ordering Set constraints that model a logical dependency between variables [2]. We also present how tailored Branch-and-Bound algorithms can be constructed in order to solve $(\mathcal{P})$ more efficiently, at the cost of a slight perturbation of the problem [3]. Finally, we present how acceleration strategies deriving from ℓ_1-penalized problems can be adapted to accelerate the convergence of the different solution methods presented [4]. We enhance our presentation with illustrations and numerical results.

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HiGHS: The missing link for fully open-source energy system modelling

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Keywords: Open-source software, Linear programming, Interior point method, Energy system modelling

HiGHS [1] has emerged in the past couple of years as the world's best open-source linear optimization software. Its interior point (IPM) solver for linear programming (LP) problems was written by Lukas Schork [2]. Recently, this was identified by the PyPSA power system analysis team as being vastly more efficient than other open-source solvers for their energy system modelling problems. Indeed, for many problems, the performance of the HiGHS IPM solver was comparable with that of Gurobi. The absence of satisfactory open-source LP solvers has been a serious limitation on the development of open-source energy modelling systems that are crucial for stakeholders who cannot afford commercial software, such as non-governmental organizations, early-stage companies, and government ministries in lower income countries.

As well as introducing HiGHS, this talk will discuss why its IPM solver performs so much better than other open-source solvers for energy system modelling problems, why the HiGHS IPM solver is wholly uncompetitive with Gurobi for some instances, and the plans for addressing this.

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Image Reconstruction: Superiorization versus Regularization

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Keywords:

Image reconstruction plays an important role in a number of modern activities, such as experimental verification of radiotherapy. Mathematically, image reconstruction can be viewed as a least squares problem, but with the caveat that the 'optimal' solution is not necessarily the cleanest image. Several approaches have been suggested to deal with this issue. In this talk, we review some of these approaches and present the results of comprehensive numerical testing comparing different approaches.

A Turnpike Property for an Optimal Control Problem with Chance Constraints

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Keywords: turnpike phenomenon, probabilistic constraints, optimal control

The turnpike phenomenon is well-known in mathematical economics or in optimal control with PDE constraints (see, e.g., [1]). It roughly states that for a large time interval, close to its middle, the dynamic optimal state/control is close to the static optimal state/control (obtained when setting time-derivatives equal to zero and leaving initial and terminal conditions free in the optimization problem). In this talk we are going to consider a simple time-discrete linear optimal control problem but with the peculiarity of adding a joint probabilistic (or chance) constraint to the system. The model may be understood in the context of reservoir managment (water, gas) with uncertain inflow and controlled release. More precisely, an initial filling level of the reservoir has to be steered in expectation to a given terminal level while staying all the time with high probability in a desired region. The objective function consists of a term of tracking type for the expected values and a control cost. We provide sufficient conditions that imply the optimal expected trajectories of the filling level to remain close to a trajectory that can be characterized as the solution of an optimal control problem without prescribed initial and terminal condition. In this way we contribute to the study of the turnpike phenomenon from a new perspective.

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Entropic Lower Bound of Cardinality for Sparse Optimization

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Keywords: Sparse Optimization, Cardinality, Lower Bound, Entropy.

The cardinality of a vector is the number of its non-zero elements. In numerous fields such as machine learning, finance or energy, it is proved very useful to control the cardinality of decision vector in related sparse optimization problems. Several approaches have been proposed to deal with such very challenging nonconvex problems. Replacing cardinality by the convex approximation based on L_1 norm leads to tractable problems but solutions obtained are far from the expected one. In addition, this classical approach fails over the probability simplex.

We propose an novel approach to cardinality for dealing with sparse optimization problems [1] based on nonconvex continuous approximations of cardinality guarantying a more precise sparsity level than convex relaxation. Defining a family of ratios of norms, we prove that its limit is a lower bound of cardinality involving Shannon entropy. The result can also be stated by using Jensen's inequality. The related optimization problems can be solved by using nonlinear solvers like IPOPT. Some numerical experiments on the Finance Index Tracking problem illustrate the efficiency of the proposed approach (Figure 1).

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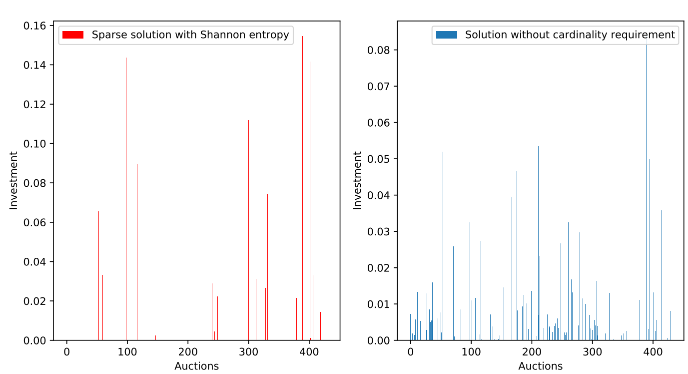

Figure 1: Sparse solution of Index Tracking Problem using Shannon entropy.

Optimal Voltage References for DC Microgrids using an Economic MPC Approach

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Keywords: optimization, model predictive control, DC Microgrids

Modern power systems will be characterized by a low inertia and fast voltage dynamics due to the increase of sources interfaced with power electronics and the disconnection of large traditional thermal generators [1]. Power electronics are commonly equipped with fast controllers that are able to set a desired voltage within few seconds [2]. When many power electronic devices are present in a power system, the question on how to set the voltage level on each power electronic device arises. The voltage levels on different nodes in a power system determine the power flows over power lines and thus have a high influence on the losses in a power system.

In this paper, we present an approach using economic Model Predictive Control (MPC) (c.f. [3]) to compute optimal voltage references for the power electronic devices in order to minimize the losses in a DC Microgrid. In particular, we show that economic MPC outperforms the traditional control hierarchy for computing the optimal references, which comprises a steady state optimization and a tracking MPC [4].

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Two-Dimensional Drift Analysis:
Optimizing Two Functions Simultaneously Can Be Hard

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Keywords: Black-Box Optimization, Evolutionary Algorithm, Drift Analysis

The performance of Evolutionary Algorithms (EAs) in dynamic environments, that is, environments in which the fitness function changes over time, has recently been studied (e.g. [2]). In this talk, we develop and analyze a minimal example TwoLin [1] of a dynamic environment that can be hard for EAs. The environment consists of two linear functions f_1 and f_2 with positive weights 1 and n, and in each generation selection is based on one of them at random. They only differ in the set of positions that have weight 1 and n. We show that the $(1+1)$-EA with mutation rate χ/n is efficient for small χ on TwoLin, but does not find the shared optimum in polynomial time for large χ.

To obtain the above result, we need to apply drift analysis in two dimensions. Drift analysis is one of the most powerful techniques to analyse the performance and the behaviour of EAs. Previously, it has only been applied in one dimension. Here, we are in the case of two random variables X_1, X_2, and the drift is approximatively given by $A \cdot (X_1, X_2)^T$ for a matrix A. The non-trivial case is that X_1 and X_2 impede each other's progress, and we give a full characterisation of this case.

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Per-Run Algorithm Selection (and Beyond)

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Keywords: dynamic algorithm selection, black-box optimization, exploratory landscape analysis

Optimization problems are omnipresent in different scientific disciplines, as well as in numerous real world scenarios. Very often, these problems have to be treated as a black box (e.g., too complex to be solved analytically, unavailable derivatives, etc). The only information we have access to are specific solution candidates and their qualities, i.e., the pairs of $(x, f(x))$ that we can use to guide the search towards a good estimate of the optimal solution.

Given a wide range of black-box optimization algorithms (most notably, but not limited to, population-based iterative heuristics) designed for different problems in mind, a meta-optimization problem arises: how does one select the best suited algorithm for a specific problem? This is well known as the *algorithm selection (AS) problem* [1]. At first, AS has relied on inherently biased human expert knowledge; automating this process has thus turned out to be a crucial step in gaining efficiency and performance. However, only knowing what the problem at hand is can often be insufficient, as different approaches might be more suitable depending on different *instances* of the same problem; this has paved a way towards *per-instance algorithm selection* [2].

To automatically recommend a good algorithm for a given problem instance, we first need to characterize the instance, which is typically done by extracting *landscape features* using *exploratory landscape analysis* tools, again only based on what we have access to in a black-box setting: solution candidates and their qualities. Naturally, this induces a computational overhead cost that we cannot afford in most real world applications, where each function evaluation requires expensive simulations or physical experiments.

We therefore propose a different perspective in which we take into account how instance landscapes look from the point of view of the optimization algorithm itself, and we build an AS pipeline on top of those locally observed features. To this end, we coin the term *per-run algorithm selection* for a process in which we extract landscape features from the samples observed by a default algorithm, and use that knowledge to recommend the best suited algorithm for the remainder of the search (until the total budget is exhausted).

We present first results in this direction for a portfolio of different black-box optimization heuristics, but more importantly, we show that this approach can be generalized to other scenarios, such as selection of acquisition functions in Bayesian optimization, in which *on-the-fly choices* lead to performance gains.

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Viscosity solutions of first order Hamilton-Jacobi equations in locally compact Hadamard spaces

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Keywords: Hamilton-Jacobi equations, discontinuous Hamiltonians, Networks, Hadamard spaces, DC functions.

We propose a novel approach to study first order Hamilton-Jacobi equations in a certain class of metric spaces called locally compact Hadamard spaces.

A metric space (X, d) is said to be a *Hadamard space* if, roughly speaking, it is a complete geodesic space and its geodesic triangles are "thinner" than the triangles of the Euclidean plane $\mathbb{R}^2$. They can be seen as a generalization of Hilbert spaces or Hadamard manifolds. Typical examples of Hadamard spaces include Hilbert spaces, metric trees and networks obtained by gluing a finite number of half-spaces along their common boundary.

Although Hadamard spaces are not manifolds in general, they carry a solid first order calculus resembling that of a Hilbert space. For example, a notion of tangent cone is well defined at each point of X. The tangent cone is the metric counterpart of the tangent space in Riemannian geometry or the Bouligand tangent cone in convex analysis. Furthermore, a notion of differential is well defined for any real-valued function $u : X \to \mathbb{R}$ that is Lipschitz and can be represented as a difference of two semiconvex functions (Lipschitz and DC functions in short). We propose to exploit all this additional structure that Hadamard spaces enjoy to study stationary and time-dependent first order Hamilton-Jacobi equation in them. In particular, we want to recover the main features of viscosity theory: *the comparison principle and Perron's method*.

In this talk, we give the main hypotheses we require on the Hamiltonian in this setting. Furthermore, we define the notion of viscosity using test functions that are Lipschitz and DC. Moreover, we show that we obtain the comparison principle using the variable doubling technique. Finally, we derive existence of the solution from the comparison principle using Perron's method in a similar manner as in the classical case of $X = \mathbb{R}^N$.

The main results are the following. First, we show that this new notion of viscosity coincides with the classical one in $\mathbb{R}^N$ by studying the examples of Hamilton-Jacobi-Bellman and Hamilton-Jacobi-Isaacs' equations. Furthermore, we prove existence and uniqueness of the solution of Eikonal-type equations posed in networks that can result from gluing half-spaces of different Hausdorff dimension.

Lyapunov Stability of the Subgradient Method with Constant Step Size

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Keywords: Differential inclusions, Lyapunov stability, semi-algebraic geometry

We consider the subgradient method with constant step size for minimizing locally Lipschitz semi-algebraic functions. In order to analyze the behavior of its iterates in the vicinity of a local minimum, we introduce a notion of discrete Lyapunov stability [2] and propose necessary and sufficient conditions for stability.

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A Two-Phase Sequential Algorithm for Global Optimization of the Standard Quadratic Programming Problem

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Keywords: Quadratic Programming, Complementarity Problems, Global Optimization

The Standard Quadratic Programming (StQP) problem [1] consists of finding a (global) minimizer of a quadratic form over the standard simplex. Hence, it can be written in the following form:

$$\textbf{StQP}: \begin{cases} \min f(x) = x^T Q x \\ \text{ s.t. } e^T x = 1, x \geq 0, \end{cases} \tag{1}$$

where $Q \in \mathbb{R}^{n \times n}$ is a symmetric matrix and e being the n-dimensional vector of ones.

Finding a global optimal solution to StQP (1) when Q is an indefinite matrix, or even giving a certificate of global optimality for a locally computed solution, is known to be NP-hard [2].

We introduce a new sequential algorithm for the StQP, which exploits a formulation of StQP as a Linear Program with Linear Complementarity Constraints (LPLCC). The algorithm is finite and guarantees at least in theory a δ-approximate global minimum for an arbitrary small δ, which is a global minimum in practice. The sequential algorithm has two phases. In Phase 1, Stationary Points (SP) with strictly decreasing objective function values are computed. Phase 2 is designed for giving a certificate of global optimality for the last SP computed in Phase 1. Two different Nonlinear Programming Formulations for LPLCC are proposed for each one of these phases, which are solved by efficient enumerative methods. Computational experiments with a number of test problems from known sources indicate that the two-phase sequential algorithm is, in general, efficient in practice. Furthermore, the algorithm seems to be an efficient way to study the copositivity of a matrix by exploiting an StQP with this matrix.

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Evolving Directed Acyclic Graphs for Automated Deep Learning: application to short-term load forecasting

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Keywords: AutoML, Directed Acyclic Graphs, Genetic Algorithm, Evolving Search Space

With the recent successes of Deep Learning on a wide range of applications, automatic design of neural networks has gained attention [1]. AutoDL decreases the need for human expertise by automatically designing an optimal architecture and choose the right hyper-parameters for a given task. This optimization problem is characterized by an expensive, black-box and noisy loss function, associated with a mixed-variable search space.

In this work, we propose a graph-based search space for the design of neural networks. We defined each network as a Directed Acyclic Graph (DAG) where the nodes represent the operations (the layers). Our search space is flexible compared to those from the literature [2, 4]. Each node can take the value of various neural network operations like convolutional, recurrent or normalization layers. Besides, we added state-of-the-art layers such as the self-attention from the Transformer model [3]. Moreover, our search space includes most of the layers' hyper-parameters and we added no constraints on the connections between layers.

Based on this search space, we implemented a genetic algorithm with specific evolutionary operators able to alternatively and/or hierarchically search for the best architecture and hyper-parameters. To speed up the optimization, we took advantage of the HPC clusters owned by Electricite De France (EDF). We applied our optimization scheme to several short-term load forecasting use cases from EDF, with different timelines, load signals and frequencies. In every use case, we found in less than 72 hours of computation, neural networks able to challenge or even outperform the handcrafted models designed by EDF experts.

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Parameter-Dependent Performance Bounds for Evolutionary Algorithms

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Keywords: evolutionary algorithms, benchmarking, Q-learning

It has long been observed that the performance of evolutionary algorithms and other randomized search heuristics can benefit from a non-static choice of the parameters that steer their optimization behavior. Mechanisms that identify suitable configurations on the fly ("parameter control") or via a dedicated training process ("dynamic algorithm configuration") are thus an important component of modern evolutionary computation frameworks. Several approaches to address the dynamic parameter setting problem exist, but we barely understand which ones to prefer for which applications. As in classical benchmarking, problem collections with a known ground truth can offer very meaningful insights in this context. Unfortunately, settings with well-understood control policies are very rare.

One of the few exceptions for which we know which parameter settings minimize the expected runtime is the LeadingOnes problem. We extend this benchmark by analyzing optimal control policies that can select the parameters only from a given portfolio of possible values. This also allows us to compute optimal parameter portfolios of a given size. We demonstrate the usefulness of our benchmarks by analyzing the behavior of the DDQN reinforcement learning approach for dynamic algorithm configuration.

Background: This talk is based on a paper presented at this year's genetic and evolutionary computation conference [1]. There, it won the best-paper award in the track for general evolutionary computation and hybrids (GECH).

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Feature Projection for Optimal Transport

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Keywords: Optimal transport, Entropic regularization, Signal tracking

Optimal transport is now a standard tool for solving many problems in statistics and machine learning. The optimal "transport of probability measures" is also a recurring theme in stochastic control and distributed control, where in the latter application the probability measure corresponds to an empirical distribution associated with a large collection of distributed agents, subject to local and global control. The goal of this paper is to make precise these connections, which inspires new relaxations of optimal transport for application in new and traditional domains.

The proposed relaxation replaces a target measure with a "moment class": a set of probability measures defined by generalized moment constraints. This is motivated by applications to control, outlier detection, and to address computational complexity. The main conclusions are (i) A characterization of the solution is obtained, similar to Kantorovich duality, in which one of the dual functions in the classical theory is replaced by a linear combination of the features defining the generalized moments. Hence the dimension of the optimization problem coincides with the number of constraints, even with an uncountable state space; (ii) By introducing regularization in the form of relative entropy, the solution can be interpreted as replacing a maximum with a soft-max in the dual; (iii) In applications such as control for which it is not known a-priori if the moment class is non-empty, a relaxation is proposed whose solution admits a similar characterization; (iv) The gradient of the dual function can be expressed in terms of the expectation of the features under a tilted probability measure, which motivates Monte-Carlo techniques for computation.

We illustrate the approach on two applications: i) MNIST digit classification using distance from [1] and ii) signal tracking by controlling the consumption of a set of consumers in an electricity network as in [2].The long version of this work is available on ArXiv [3].

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Moment relaxations for large scale AC-OPF problems

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Keywords: Nonconvex Optimization, Moment Hierarchy, AC-OPF, Power Systems

The moment-SOS hierarchy has proved relevant to address AC-OPF (Alternative Current - Optimal Power Flow) problems [1]. However, obtaining the convergence of the hierarchy may require to go beyond the first step $d = 1$ of the involved sequence of SDP relaxations (e.g. up to $d = 3$ in [1]), and thus to solve semidefinite programs whose size grows exponentially with the value of d. Thus, the application of the hierarchy to large scale AC-OPF problems (with $n \geq 10.000$ variables) remains a challenging task.

In this talk, we present some prospective techniques to tackle further steps ($d \geq 2$) of the hierarchy for large scale problems. In particular, we consider exploiting the sparsity of the problem, and its possible combination with decomposition techniques.

The presented work is the output of an ongoing research collaboration between the POP team at LAAS CNRS and RTE.

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Evolutionary Game of Coordination for Formation Flying

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Abstract

Many applications such as Search And Rescue missions require autonomous drones to operate in a coordinated fashion. Distributed control techniques make it possible to fly swarm of drones autonomously without a leader agent. In order to maximize the use of drone swarms it is necessary to carry out specific types of formation (flying in a square, forming a line and so on). The type of formation chosen evolves dynamically during the scenario according to the evolution of the environment. In the absence of a leader agent, it is necessary to set up a coordination process between agents to ensure that selfish agents converge towards the same geometric formation. The different possible formations are seen as the pure strategies of an evolutionary game of coordination. The evolutionary coordination game aims to converge during an evolutionary process a large population of players towards the choice of an identical strategy. However, the evolutionary game of coordination has several equilibria, some equilibria are stable, others unstable, some equilibria are pure strategies, others are mixed strategies. In the context of formation flying, it is a question of ensuring that the evolutionary process and the communication between agents makes it possible to converge towards a desirable equilibrium from an operational point of view. The equilibrium has to be stable and of pure strategy type to be applied to a small swarm (even if optimized considering a large population of agents). It is therefore a question of setting up a population dynamic between the agents of the swarm in such a way as to favor certain equilibria. The convergence and stability of this modified population dynamics are then analysed.

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Sparse polynomial optimization: old and new

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Keywords: Polynomial optimization, Semidefinite programming, Correlative sparsity, Term sparsity, Ideal sparsity

Polynomial optimization methods often encompass many major scalability issues on the practical side. Fortunately, for many real-world problems, we can look at them in the eyes and exploit the inherent data structure arising from the input cost and constraints.

The first part of my talk will focus on the notion of “correlative sparsity”, occurring when there are few correlations between the variables of the input problem.

The second part will present a complementary framework, where we show how to exploit a distinct notion of sparsity, called “term sparsity”, occurring when there are a small number of terms involved in the input problem by comparison with the fully dense case.

At last but not least, I will present a very recently developed type of sparsity that we call “ideal-sparsity”, which exploits the presence of equality constraints.

Several illustrations will be provided on important applications arising from various fields, including robustness of deep networks, quantum entanglement, optimal power-flow, and matrix factorization ranks.

This work is based on the recent monograph [1] and the preprint [2].

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A multi-objective exposure-oriented sensor location problem for border security in 3D

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Keywords: MILP, Bilevel programming, Minimal exposure path, Border surveillance

In a recent work [1], Lessin et al. proposed a bilevel algorithm to optimize the placement of heterogeneous sensors to maximize the minimum exposure of an intruder's penetration path through a defended region. In [2], Lessin et al. expanded on the previous problem to consider the relocation of sensors after an initial disruptive action of the intruder, in order to minimize the maximum sensor relocation time, and the total number of sensors requiring relocation. The defended region is modelized by a hexagonal mesh in 2 dimensions. In this talk, we expand on Lessin's work by taking into account the altitude of the terrain by checking for lines of sight (LoS) between the intruder and the sensors. This upgrade is very expensive computationally and proper care was necessary to keep the computation time under reasonable bounds. We also propose a variation of the initial formulation in order to minimize the number of sensors required to reach a desired exposure of the intruder.

We present and discuss preliminary results showing the performance of these modifications.

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Interior Point Methods in Optimal Control Problems: Convergence results and primal-dual solving algorithm

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Keywords: State constrained optimal control, interior point methods, primal-dual solver

This presentation deals with Optimal Control Problems (OCPs) with pure state and input constraints. One of the available approaches to solve these problems consists of adapting Interior Point Methods (IPMs) to state and input constrained OCPs [1, 2, 3]. However, adapting IPMs to OCPs is not straightforward and has not been wholly performed. Indeed, to be complete, this adaptation requires proving two things. Firstly, the optimal trajectories of the penalized problem strictly satisfy the constraints. Secondly, it requires establishing the convergence of the method to a point satisfying the first-order optimality conditions. In other words, to prove the convergence of primal and dual variables in function space. In [1], the authors demonstrate the interiority of solutions and convergence in the case of control constraint only. In [2], a primal-dual IPM for OCPs is presented, and convergence is proven only for control-constrained problems. In [3], the authors exhibit sufficient conditions on the penalty functions guaranteeing the interiority of solutions, but convergence is only proved for primal variables. The contribution of this work is to fill this gap by proving both interiority of the penalized trajectories and convergence of control, state, adjoint state, and constraint multipliers. To do so, sufficient conditions on penalty functions guaranteeing the interiority of local optimal solutions are exhibited, and we prove that logarithmic functions satisfy these conditions. In addition, we prove that the derivative of the penalty functions associated with any local optimal solutions satisfies a uniform boundedness property. Using some standard compactness argument with these uniform boundedness properties allows to prove strong, weak, or weak $*$ convergence, depending on the case, of these derivatives to the constraints multipliers. Then, the convergence of the adjoint state stems from the convergence of the state, control, and constraint multipliers. Finally, this paper provides a primal-dual solving algorithm based on Two Point Boundary Value Problems solver. This contribution has been submitted to SIAM in may 2022 and is still under review.

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Robust multi-stage optimization for glass production

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Keywords: Robust optimization, finite adaptability, multi-stage optimization, production planning

In the glass industry, visual and thermal properties of the glass sheets are obtained via the deposit of very thin layers of different materials. A standard way to perform this step is the use of a "magnetron," in which the materials are transferred from cathodes to the sheets using a magnetic field. Different materials can achieve a given property of the glass. Since the cathodes are very expensive, their activation and replacement have to be carefully decided to keep the production costs and the waste of materials low. Due to the complexity of the physical process, the consumption of the cathodes is partly uncertain, which makes finding the best activation and replacement decisions a highly challenging task. In an accurate modeling, the decisions have to be sequentially taken, leading to a multi-stage robust optimization problem.

To tackle the over-conservativeness of the current industrial practice, we propose to solve this problem within the framework of finite adaptability, introduced by Bertsimas and Caramanis [1]. It consists in splitting the uncertainty set into finitely many parts and in assigning to each part a constant recourse decision. Recently, Subramanyam et al. [2] proposed an exact method able to solve efficiently finite adaptable two-stage robust problems.

First, we show how to reformulate the studied problem in its two-stage version within the setting of Subramanyam et al. Preliminary results clearly demonstrate that finite adaptability brings a significant improvement upon the non-adaptable robust solutions, which are already better than the industrial solutions. Second, to deal with the full problem, we propose a multi-stage extension of the method by Subramanyam et al. Its theoretical and experimental validation is currently under investigation. Bertsimas and Caramanis have given evidences that in many situations finite adaptability provides a good approximation to the "complete" adaptability. Our last contribution is a theorem in the line of their work, showing that finite adaptability approximates asymptotically complete adaptability under mild continuity conditions.

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Quantum Side Channel Attacks

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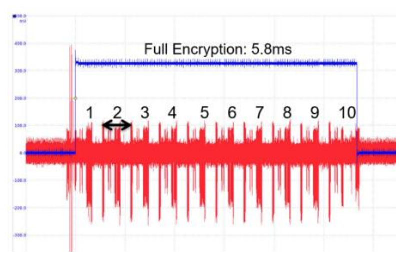
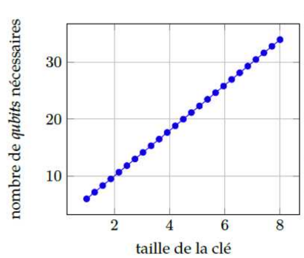
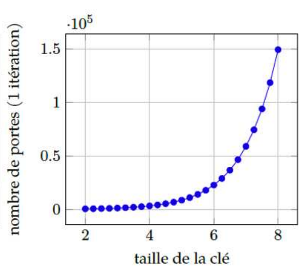
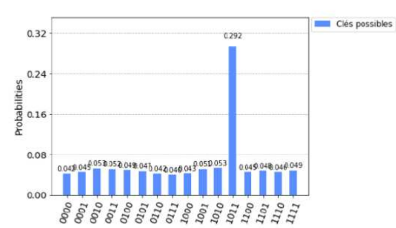
Keyword *: Cryptanalysis, Quantum Optimization.*

Quantum Computing is a new paradigm originated from Richard Feymann ideas. During 90's, Quantum Algorithm were discovered (*Polynomial-Time Algorithm for Prime Factorization on a Quantum Computer*, Shor 94, *A fast quantum mechanical algorithm for database search*, Grover 95). Recently, in the last 5 years, real Quantum Computing are available and first practical experiments validate expected results derived from the theory (*Quantum Supremacy using a programmable Quantum Computer*, Google 2019). We have investigated side channel attacks (*Differential Power Analysis*, Kocher 1999) from a Quantum point of view. By precisely decomposing the AES-128 algorithm (Advanced Encryption Standard), we proposed specific operators to extract hidden key from a sequence of plain text messages compared to encrypted results. To help us in this task, we exploit leaks from side channels (consumption power, EM emissions) by carefully spying the cryptoprocessor and microcontroller chips. Grover Adaptive Search was used with Qiskit python Libray from IBM to illustrate on small example efficiency of our attacks.









The Satellite Constellation Design Problem via MI(N)LP Boosted with a Genetic Algorithm

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Keywords: Satellite constellation design, Discontinuous coverage, MI(N)LP, Genetic algorithm

In a recent work [1], we introduced several Mixed-Integer (Non)Linear Programming (MI(N)LP) formulations for the satellite constellation design problem with discontinuous coverage and constrained revisit time. This problem represents an emerging application in aerospace engineering and consists in finding the minimal number of satellites and their orbital parameters such that the resulting constellation observes several targets on the Earth surface within regular time intervals (see, for instance, [2, 3, 4]).

In this talk, we keep on exploring the potentialities of operational research approaches for such an application by :

- describing new (more realistic) MI(N)LP formulations with a new definition for the satellite coverage area ;
- boosting our MI(N)LP formulations with a rather simple Genetic Algorithmic approach to provide better initial feasible (yet not necessarily optimal) solutions.

We present and discuss preliminary results, comparing the plain MI(N)LP formulations with their corresponding boosted versions, showing the computational benefits both in terms of CPU times and solution quality.

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MEAN FIELD ANALYSIS OF STOCHASTIC NETWORKS WITH RESERVATION

CÉDRIC BOURDAIS, CHRISTINE FRICKER, AND HANENE MOHAMED

The problem of reservation in a large distributed system is analyzed via a new mathematical model. A typical application is a real station-based car-sharing system with reservation that existed in Paris under the name Autolib' (2011-2018). Its fleet was composed, in August 2017, of 3.907 vehicles distributed in 1.100 stations. This system provided 303.006 rentals per month in August 2017. Its particularity was to offer both the possibility to reserve the car and the parking space at the destination. In this paper, we focus on a reservation policy called *double reservation* which is to reserve both the car and the parking space at the same time, a moment before picking up the car. Note that if the time between the reservation and the pick-up is zero, the policy is to reserve the parking space when the car is picked up. This policy is called *simple reservation* and was studied in a homogeneous framework by a mean-field approach in [1], using a large-scale analysis similar to that of bike-sharing systems in [2].

Intuitively, we expect to see a negative impact to the double reservation policy because the system will tend to reject more customers than if it offered only the car reservation. The goal of our work is to propose a model of such a car-sharing system, to analyze its asymptotic behavior and to study the impact, a priori negative, of the double reservation.

Such station-based car-sharing system can be described as a closed stochastic network where the nodes are the stations and the customers are the cars. In the paper, we study the evolution of the system when the reservation of parking spaces and cars is effective for all users. The asymptotic behavior of the underlying stochastic network is given when the number N of stations and the fleet size M_N increase at the same rate, in other words when N and M_N tend to infinity with $s_N = M_N/N$ tending to a constant s called *the average number of cars per station*. This sizing parameter s is a key parameter of the system. The analysis involves a Markov process on a state space with dimension of order N^2. It is quite remarkable that the state process describing the evolution of the stations, whose dimension is of order N, converges in distribution, although not Markov, to a non-homogeneous Markov process. We prove this mean-field convergence. We also prove, using combinatorial arguments, that the mean-field limit has a unique equilibrium measure when the time between reserving and picking up the car is sufficiently small. This result extends the case where only the parking space can be reserved. The proof is based on three main arguments. First, by applying queueing theory, the McKean–Vlasov process on the basic state space is identified with a tandem of four queues with an invariant measure of explicit product form. Thus, by simplifications, the problem of existence and uniqueness of the invariant measure of the non-homogeneous Markov process amounts to the same problem for a fixed point equation in dimension 2. Finally, the global inversion theorem and a monotonicity property allow us to conclude.

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Peak minimization for compartmental models

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Keywords: Epidemiology, optimal control, SIR model, maximum cost

In mathematical epidemiology, the transmission of a disease is evaluated in compartments which correspond to a partition of the population. This partition es determined by the different states of the disease. We work with optimal control problems which consist in minimizing the maximum over a time interval $[0, T]$ of a scalar function

$$\inf_{u(\cdot)} \max_{t \in [t_0, T]} I(t) \tag{1}$$

for compartmental dynamics. In this context I represents the compartment of infected individuals. This work has been motivated by the so-called overcrowding problem, which is a significant issue for hospitals. More precisely, the objective is to minimize the peak of infections in order to keep the health system below saturation levels.

In the particular case of the SIR dynamic, a control $u(t)$ affecting the transmission rate is introduced. We will present the optimal solution of (1) under a L^1 constraint over u, where in [1] we called NSN strategy by null-singular-null. We compare this strategy to which one presented in [2], where authors fix the duration taking the control in the same dynamical system.

In a more general case, we present numerical solution using reformulations proposed in [3] and Bocop solver. Taking advantage of the analytical solution of the SIR model, we evaluate the numerical performance of these reformulations for this kind of problems.

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Optimal Control of Tumor Immune Systems under Parametric Uncertainties

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Keywords: optimal control, cancer, parametric uncertainties, moments optimization, probabilistic certification

The last decades witnessed a considerable progress in experimental and clinical immunology as well as in modeling the immune system dynamics. The progress in cancer dynamics modeling motivated researchers to apply control approaches in order to schedule cancer treatments using optimal control strategies. Many works have been carried out in order to design cancer treatment protocols using mathematical models [2], [5]. One of the main complexities of such models is the presence of different types of uncertainties, which remains less considered in the literature.

This presentation will focus on the importance of parametric uncertainties considerations in the design of optimal cancer drugs protocoles. Therefore, it will firstly adress the use of the moment optimization framework for solving optimal control problems [1]. The latter being appropriate for the use of stochastic parametric uncertainties, it's efficiency will be investigated for cancer drugs schedules design [3]. Secondly, this presentation will adress a new probabilistic certification framework, that is based on chance-constrained optimization, in order to design optimal control profiles in the presence of parametric uncertainties [4].

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Constrained Smoothing and Out-of-range Prediction by Means of a Mathematical Optimization Approach

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Keywords: Data science, penalized splines, conic optimization

The complexity of the data generated at present in many diverse areas makes necessary the development of innovative methodologies which are able to incorporate human knowledge to, for instance, enhance interpretability and avoid misleading out-of-range predictions. In this work, we address the problem of estimating smooth functions in a regression setting, which are constrained to satisfy requirements about their sign and/or shape.

We assume that the smooth function to be estimated is defined through a reduced-rank basis ($B-$splines) and fitted via a penalized splines approach ($P-$splines [1]). When dealing with simple regression, necessary and sufficient conditions on the sign of the smooth curve are embedded into the fitting procedure as hard constraints, yielding a conic optimization model. This characterization can be adapted to require sign conditions on higher order derivatives of the curve, meaning that our methodology can deal with requirements concerning the monotonicity and the curvature of the curve. Our approach for multiple regression arises as a non-trivial extension of the one-dimensional shape constrained framework, imposing the constraints over a finite set of curves which belong to the hypersurface. Furthermore, previous results are generalized for the first time to out-of-sample prediction, either forward or backward.

We compare our approaches with other state-of-the-art algorithms on both synthetic and real-world data sets. Furthermore, an open source Python library is publicly available, `cpsplines`, which contains the implementations of all the methodologies proposed in this work [2].

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Risk management via optimal (partial) transport

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Keywords: Risk measures, optimal transport

In a variety of problems in operations research, a variable of interest $b = b(x_1, x_2, \ldots, x_d)$ depends on several underlying random variables, whose individual distributions are known but whose joint distribution is not. A natural example arises when the variables represent parameters in a physical system, whose individual distributions can be estimated empirically or through modeling (or a combination of both) but whose dependence structure cannot; an example of this flavour, originating in [1], in which the output b represent the simulated height of a river at risk of flooding, and the underlying variables include various design parameters and climate dependent factors. In political science, b might represent the outcome of an election and the x_i vote shares in different regions.

Metrics used in risk management depend on the distribution of the output variable b, and therefore, in turn, on the joint distribution of the x_i. A natural problem is therefore to determine bounds on these metrics; that is, to maximize the given metric over all possible joint distributions of the x_i with known marginal distributions. In this talk, we show that for a large class of metrics, the maximization can in fact be formulated as a traditional multi-marginal optimal transport problem with $d + 1$ marginals: the given marginals distributions of the x_i as well as another distribution arising from the particular form of the risk metric. In particular, in the special but important case of conditional value at risk, the problem further reduces to a multi-marginal partial transport problem on the d original distributions.

When the underlying variables x_i are all one dimensional, this allows us to derive a nearly explicit characterization of solutions in a substantial class of relevant problems, and facilitates the use of a very broad range of computational methods for optimal transport problems for others.

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Generalized Nash Fairness solutions for Bi-Objective Minimization Problems

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Keywords: Bi-Objective Optimization, Bi-Criteria Decision Making, Pareto optimal, Weighted Sum Method, Proportional Fairness, Bi-Objective Shortest Path, Bi-Objective TSP

In this paper, we consider a special case of Bi-Objective Optimization (BOO), called *Bi-Objective Minimization* (BOM), where two objective functions to be minimized take only positive values. As well as for BOO, most methods proposed in the literature for solving BOM focus on computing the Pareto-optimal solutions that represent different trade-offs between two objectives. However, it may be difficult for a central decision-maker to determine the preferred solutions due to a huge number of solutions in the Pareto set. We propose a novel criterion for selecting the preferred Pareto-optimal solutions by introducing the concept of *ρ-Nash Fairness* (ρ-NF) solutions inspired from the definition of proportional fairness. The ρ-NF solutions are the Pareto-optimal solutions achieving some proportional Nash equilibrium between the two objectives. The positive parameter ρ is introduced to reflect the relative importance of the first objective to the second one. For this work, we will discuss some existential and algorithmic questions about the ρ-NF solutions by first showing their existence for BOM. Furthermore, the set of ρ-NF solutions can be a strict subset of the Pareto set. As there are possibly many ρ-NF solutions, we focus on extreme ρ-NF solutions achieving the smallest values for one of the objectives. Then, we propose two Newton-based iterative algorithms for finding extreme ρ-NF solutions. Finally, we present computational results on some instances of the Bi-Objective Travelling Salesman Problem and the Bi-Objective Shortest Path Problem.

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Increasing electricity reliability in peri-urban areas of Sub-Saharan Africa with joint chance constraints

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Keywords: Probability functions; Stochastic programming; grid-connected mini-grids.

In this talk, we present a novel optimization dispatch of grid-connected mini-grids with joint chance constraints. In the context of sub-Saharan Africa, operators of grid-connected mini-grids face the following uncertainties: intermittent renewable energies supply, demand, frequency and duration of main-grids blackouts . These uncertainties pose new challenges to the classical power system's operation tasks [1]. We present an optimal dispatch with an islanding strategy, which makes sure that the operator uses planned reserves during the entire period of the main grid outage, such that customers enjoy a given p-reliability level of electricity supply. Results and comparison with deterministic approaches are presented. Also open problems and future work with probabilistic bilevel programs [2, 3] will be discussed.

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Spectral constraints in nonlinear programming

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Keywords: Nonlinear programming, Spectral optimization, Robust optimization, Optimal Power Flow

The robust solution of large-scale nonlinear programming (NLP) is still a challenging task today, notably when it comes to guarantee the iterates remains feasible with relation to a set of nonlinear constraints (potentially nonconvex). It is well-known that one can guarantee feasibility by using a subtle arrangement of line-search or trust-region techniques, but feasibility may be difficult to maintain if the problem is highly nonconvex. In this work, we propose to maintain feasibility by exploiting spectral information about the Jacobian of the nonlinear constraints. Using a smooth approximation of the min operator, one can constrain the minimum eigenvalue to be greater than a given threshold. As a result, we can force the algorithm to remain feasible with some controllable margin. We reuse the results of Lewis & Sendov [1] to evaluate the first- and second-order derivatives of the spectral constraints, allowing us to cast the model as a generic nonlinear problem. We present numerical results on the popular optimal power flow problem (OPF), where we use the newly introduced spectral constraints to guarantee we satisfy strictly the physical constraints of the problem. We show the method is numerically tractable.

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MadNLP: A Structure-Exploiting Interior-Point Package For HPC

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Keywords: Interior-point methods, Nonlinear programming (NLP), High Performance Computing (HPC)

Has nonlinear programming become boring? Indeed, most of the theoretical developments have been made during the 1980s and 1990s, culminating in the release of easy-to-use nonlinear optimization solvers (Ipopt, Knitro) now widely adopted by the community. There is a consensus that interior-point method (IPM) is among the most robust methods to solve nonlinear problem. IPM solves a nonlinear problem by solving its Karush-Kuhn-Tucker (KKT) equations using a homotopy method, leading to the resolution of one linear system at each iteration of the algorithm. Hence, in a large-scale setting, all the computational complexity is delegated to the sparse linear solver being employed under the hood. That component is the principal limiting factor when it comes to solve very large-scale nonlinear problems (with more than 1 millions variables and constraints), as commonly encountered in the optimization of energy systems. In this talk, we present a method to harness the structure of the nonlinear problem (degrees of freedom, potential arrowhead structure) to alleviate the limitation of interior-point in the very large-scale regime. The method has been implemented in MadNLP, a flexible interior-point solver developed entirely in Julia. We demonstrate the numerical capability of MadNLP in a HPC setting by bringing a practical way to accelerate IPM on Graphical Processing Units (GPU), paving the way to use IPM on future exascale supercomputers. We assess the performance of the method on various use-cases coming from the optimization of energy systems, notably (1) nonlinear problems with graph-structure (optimal powerflow) (2) nonlinear problems with a two-stage stochastic structure (stochastic optimal powerflow) (3) nonlinear problems with a dynamic structure (linear-quadratic MPC).

Scaling up Multi-Agent Reinforcement Learning with Mean Field Games and Vice-versa

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Keywords: Mean Field Games, Reinforcement Learning, Multi-agent Systems

In this talk, I will present how Mean Field Games allow to scale multi-agent systems to an infinite number of agents, and how Reinforcement Learning can scale Mean Field Games in terms of model complexity. I will first explain what are Mean Field Games and present different definitions and their corresponding settings [1]. I will then introduce two algorithms to find Nash equilibrium in Mean Field Games: Fictitious Play (FP) [2] and Online Mirror Descent (OMD) [3], that both converge to the Nash equilibrium under the monotonicity condition. Finally, I will present how these two algorithms can be adapted to model-free environments using Deep Reinforcement Learning [4]. This problem was particularly challenging because FP and OMD both require summing or averaging quantities, a difficult operation with non-linear functions such as Neural Networks.

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Projective Cutting Planes with multiple cuts per iteration for robust linear programming

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Keywords: Cutting-Planes, Robust Linear Programming, Ray Projection

Many research projects start from a problem and identify the most effective method to solve it. This work emerged the other way around. We considered an existing method [1] and searched for (new) problems it may solve efficiently. The new method aims to improve the widely-used `Cutting-Planes`, by upgrading its well-established separation sub-problem to a projection sub-problem: given some feasible $\mathbf{x} \geq \mathbf{0}$ in some polytope P and direction $\mathbf{d}$, what is the maximum t such that $\mathbf{x} + t \cdot d \in P$? This question is easy if one can list all constraints of P, but it is (much) harder in a `Cutting-Planes` context where P has prohibitively-many constraints.

This projection sub-problem is more difficult to implement than the standard separation. So why should we bother, why should we care? Can't say we should. But there may be some benefits down the road. Long story short, the new algorithm Projective Cutting-Planes generates interior points (feasible solutions) along the iterations. The classical `Cutting-Planes` proceeds only by removing infeasibility. The new one proceeds both by removing infeasibility and by producing feasibility. Suppose one has to choose one of the following constraints:

(1) $10x_1 + 20x_2 + 6x_3 \leq 30$

(2) $x_1 + 2x_2 + x_3 \leq 3$

When $\mathbf{x} \in \mathbb{R}^3_+$, constraint (2) implies (1). Yet, by applying the separation logic on $\mathbf{x} = [1\ 1\ 1]$, constraint (1) seems tighter because $10 + 20 + 6 - 30 > 1 + 2 + 1 - 3$, *i.e.,* the separation sub-problem returns (1) because it seems more violated by [1 1 1]. The projection sub-problem $[0\ 0\ 0] \to [1\ 1\ 1]$ leads to the pierce point $[\frac{3}{4}\ \frac{3}{4}\ \frac{3}{4}]$ and returns constraint (2) which is tight for this pierce point. Constraint (1) is not tight because it reduces to $36 \cdot \frac{3}{4} \leq 30$ or $27 \leq 30$.

When we first submitted [2], we initially solved robust linear programming by comparing the classical `Cutting-Planes` and the new `Projective Cutting-Planes`. We generated in both cases only one cut per iteration, because we only wanted to compare the two methods. Pointing out that the most effective `Cutting-Planes` for this problem uses multiple cuts per iteration, a referee asked as if we could also adapt `Projective Cutting Planes` for a multi-cut setting. The answer was yes, and – a bit to our surprise – the multi-cuts variant of `Projective Cutting-Planes` was really faster. For example, it solved in 8 seconds the instance `stocfor3`, an instance that required thousands of seconds for all other Cutting Planes mentioned above.

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Energy management in a hydrogen-based microgrid through constrained optimization

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Keywords: Energy management, Hydrogen-based Microgrid, Model Predictive Control

Abstract Systems for generating, storing and transporting energy in the form of *hydrogen*, are a reliable and non-polluting (green) alternative to classical energy systems. To produce hydrogen in renewable energy systems, *electrolysers* are used to split water molecules into hydrogen and oxygen with the first being stored for further use. Due to intermittency in generation and load it is important to manage efficiently the workload of the electrolysers as they have significant inertia (i.e., long start time and limited storage capacity).

We propose an energy management scheme where, as a first step, the various components of the system (sources, storage, loads, and converters) are characterised and modelled in a tractable way, using state of the art tools (Dymola, subsequent FMU export for further use in Matlab/Python).

We develop a reliable optimization-based control framework capable of handling the power flow within the hydrogen-based energy system under different scenarios. Simulations validate the proposed approach.

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Surrogate Component Approach for Self-Consumption Management

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Keywords: Power Self-Consumption, PhotoVoltaic, Scheduling, Bi-Level, Routing

The Problem: Multi-level decisional [1] models usually involve several players, tied together by some hierarchical or collaborative links. They aim at providing scenarii which would be the best in case all the players accept a common rule (centralized paradigm), or (collaborative paradigm) at searching for a compromise. Standard approaches involve decomposition schemes, hierarchical (Benders, Stackelberg,…) or transversal (Lagrangean), but a trend, boosted by the rise of machine learning technology [3], is to replace some levels by surrogate constraints or estimators. We apply it to the joint management of local photovoltaic energy ([2]) production and its consumption by a fleet of electric vehicles.

So we consider here a fleet of K small identical electric vehicles, required to perform VRP: *Vehicle Routing Problem* tours, that means to visit a set of *stations* $J = \{1,\ldots, M\}$ within a time horizon $[0, TMax]$. Moving from station j to station k requires $\Delta_{j,k}$ time units and an amount $E_{j,k}$ of energy. An *Elementary Trip* π is any VRP sub-tour that a vehicle may perform without recharging at the depot. It requires $T(\pi)$ time units and an amount $E(\pi)$ of energy. In order to implement a *self-consumption* policy, we are provided with a photo-voltaic facility *PV-Plant*, which rules a set B of identical batteries and which produces energy that it distributes between currently idle batteries or that it sells to the market. In case this energy is not enough, the *PV-Plant* can also buy energy to the market. Vehicles may switch their battery by coming back to *PV-Plant*. This *plug out/in* operation is instantaneous. It comes that while the vehicles are running with *active* batteries, *idle* batteries are recharged at *Depot* before being used again by the vehicles. The time space $[0, TMax]$ is divided into small periods $i = 1,\ldots, N$, all with same length p. We denote by C^R the *recharge per period capacity*. by R_i (A_i, B_i) the expected production (energy unit purchase price, energy unit sale price) of the *PV-Plant* at period i. So resulting *PV_Prod_VRP* decision problem comes as follows:

PV_Prod_VRP: {Simultaneously schedule the vehicles and the *PV-Plant*, in such a way that:

- Every station is visited at least once by the fleet;
- The global energy load of the batteries does not to decrease between the beginning and the end of the process.
- Some global cost is minimized, which combines VRP cost with the *PV-Plant* cost of energy self-consumption.

Handling *PV_Prod_VRP* with Surrogate Components: MILP formulation of *PV_PROD_VRP* is hardly practicable and does not fit uncertain or collaborative contexts. So we short-cut the *PV_Prod* level through the introduction of surrogate constraints and estimators and apply the following parametric *VRP_Surrogate* process:

***VRP_Surrogate* Parametric Algorithmic Scheme:**

While Not *Stop* do

1st step: Compute an elementary trip collection Π_0 which minimizes some parametric cost $\alpha.\Sigma_{\pi \in \Pi_0} T(\pi) + \gamma.\ \Sigma_{\pi \in \Pi_0} E(\pi)$. Do it while performing a *Branch and Cut* process, involving specific *Strong No Subtour constraint;*

2nd step: Assign any $\pi \in \Pi_0$ a period set $I(\pi)$ in such a way some surrogate cost $\Phi(\pi \to I(\pi))$ be minimized;

- A first estimator involves on a *pricing mechanism* which, at any period i, assign a price $Q_{i,n}$ of n batteries in recharge at period i. This price system Q is obtained through a learning process.
- Another one involves a *convolutional* neural network *N_Energy* with 421 synaptic coefficients, which estimates the quality of a scheduled trip collection Σ_0.

3rd step: Update the flexible parameters, solve low level sub-problem and update the best current solution.

Numerical Experiments: Experiments, performed while using libraries CPLEX12 and TensorFlow/Keras (for Machine Learning), make appear that this approach efficiently drives us towards solutions close to optimality.

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Degree Theory for the Linear Complementarity Problem and related Piecewise Linear Systems

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Keywords: Linear Complementarity Problem, Absolute Value Equation, Degree Theory

The *linear complementarity problem* $\mathrm{LCP}(q, M)$, where $q \in \mathbb{R}^n$, and $M \in \mathrm{M}_n(\mathbb{R})$, the space of $n \times n$ real matrices, is to determine $v, w \in \mathbb{R}^n_{\geq 0}$ with $v^T w = 0$ so that

$$v \;=\; Mw + q\,. \tag{1}$$

It provides a common framework for numerous optimization tasks in economics, engineering and computer science. An $\mathrm{LCP}(q, M)$ is uniquely solvable for arbitrary q if and only if M is a P-matrix, that is, a matrix whose principal minors are all positive. If $\mathrm{LCP}(q, M)$ is solvable—possibly non-uniquely—for arbitrary q, then M is called a *Q-matrix*. There exists no comprehensive characterization of Q-matrices. We study this question by investigating the following equivalent problem. Let $b \in \mathbb{R}^n$ and $A \in \mathrm{M}_n(\mathbb{R})$. Then the *absolute value equation* (AVE) poses the problem to find a vector $z \in \mathbb{R}^n$ so that

$$z - A|z| \;=\; b\,, \tag{2}$$

where $|\cdot|$ denotes the componentwise absolute value. We will study solvability of the AVE, but with our eyes on the Q-matrix problem. To this end we investigate the piecewise linear function

$$F_A : \mathbb{R}^n \to \mathbb{R}^n\,, \quad z \mapsto z - A|z| \tag{3}$$

associated to the AVE (2) and determine its degree in terms of the *aligned spectrum* of A:

$$\mathrm{Spec}^{\mathrm{a}}(A) := \{\lambda \geq 0 \mid \exists x \neq 0 \,:\, |Ax| = \lambda x\}. \tag{4}$$

We denote by $\mathrm{c}^{\mathrm{a}}(A)$ the *aligned count* of A, the number of aligned values of A which are larger than 1. Under a minor genericity assumption on A (the non-generic matrices are contained in a homogeneous hypersurface) we prove that

$$\deg F_A \;\equiv\; \mathrm{c}^{\mathrm{a}}(A) + 1 \mod 2\,.$$

We further prove an exact but slightly more technical formula for the degree of F_A. LCP-analogues of these concepts and results are derived. As a corollary we obtain that an LCP-coefficient matrix M with even (LCP-)aligned count is a Q-matrix.

This presentation is based on the preprint [1].

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How Principal Component Analysis can improve Bayesian Optimization at High Dimensionality

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Keywords: Bayesian optimization, principal component analysis, dimensionality reduction, global optimization, black-box optimization

Bayesian optimization (BO) is not well suited for high-dimensional problems. Several recent works aim to solve this problem [1], however they all have some limitations, such as the assumption of intrinsic lower dimensionality, the underestimation of correlation effects between variables, and the dependence on prior knowledge about a particular use case.

In this presentation, I introduce two algorithms that address the problem of applying BO to high-dimensional search spaces: PCA-BO [2] and KPCA-BO [3]. They aim to improve the scalability of BO by hybridizing it with principal component analysis (PCA), i.e., they perform an adaptive linear (PCA-BO) or a non-linear (KPCA-BO) feature mapping to a low-dimensional space in which they run the standard BO algorithm. The performance of (K)PCA-BO is compared with BO, CMA-ES, and other BO-based algorithms in terms of empirical convergence rate on the multimodal problems of the COCO/BBOB benchmark suite. I show that while both PCA-BO and KPCA-BO tend to stagnate as the number of evaluations increases, they perform very well on small budgets. Given the important role of BO in many applications, e.g., automated machine learning and mechanical design, I hope to discuss ideas for further improvements with PGMODAYS participants.

This presentation is based on joint work with Kirill Antonov and Hao Wang from Leiden University, The Netherlands, Maria Laura Santoni, from University of Camerino, Italy, and Carola Doerr from Sorbonne University, France.

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Stochastic blackbox optimization methods in the presence of dynamical constraints

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Keywords: Stochastic optimization, Dynamical constraints, Neural ODE.

In simulation-based optimization, one is often faced with constraints that stem from physical processes, and are expressed under the form of differential equations. Recent interest in machine learning architectures based on differential equations has generated renewed interest for this class of problems. Indeed, it gave rise to several complex optimization formulations where the dynamics play a prominent role and the objective function can be viewed as the result of an expensive procedure, typically not directly available to the optimizer.

In this talk, we investigate constrained optimization problems where the objective function is the result of a blackbox simulation, but the dynamics expressed in the constraints are available as a white box. We provide an algorithmic framework that is equipped with theoretical guarantees, even when the objective function cannot be accessed directly, and stochastic estimates are available instead. We also illustrate the performance of our algorithm on task involving neural architectures inspired by differential equations.

Simulation and optimal control of heating and cooling systems: A case study of a commercial building

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Keywords: optimal control, grey-box model, energy efficiency, black-box optimization, heating and cooling

This work proposes a novel energy conservation measure that optimizes the planning of heating and cooling systems for tertiary sector buildings. It consists of a model-based predictive control approach that employs a grey-box model built from the building and weather data that predicts the building heat load and indoor temperature. Different from classical optimization approaches where the discretized differential algebraic equations are integrated into the optimization formulation, our model is calibrated using black-box multiobjective optimization, which allows for decoupling the predictive model from the optimization problem, thus having more flexibility and reducing the total computational time. Moreover, rather than requiring the angle of solar radiation, solar orientation and solar masks to calculate the radiation data, our approach requires only a simple model of the solar irradiance. The calibrated model is then used by heating and cooling optimization strategies that aim at reducing the energy consumption of the building in the next day while satisfying the indoor thermal constraints. Results from a case study of a commercial building during heating and cooling seasons show that the proposed approach was able to yield up to 12% of energy savings.

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A Framework for Identifying Resistance-related Gene Signatures in scRNA-seq data using Non-Negative Matrix Factorization

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Keywords: Single-cell RNA-seq, Gene signatures, Non-negative matrix factorization, Ovarian cancer, Resistance

Although many high-grade serous ovarian cancer (HGSOC) patients initially respond to chemotherapy, eventually patients will develop recurrent disease and resistance to the treatment. On a cellular level, the resistance may be explained by a sub-population of cells within the highly heterogeneous tumor tissue that survives the effects of the drug and subsequently propagates. It is assumed that the survival of this cell sub-population is due to the activity or inhibition of specific cellular processes, which are characterized by the expression of a particular set of genes. Thanks to the recently established single-cell RNA-seq (scRNA-seq) technologies, the analysis of the inter- and intra-tumoral heterogeneity has become possible at the level of individual cells, enabling the identification and transcriptional characterization of different sub-populations of cells.

Our objective is to identify the mechanisms of chemotherapy resistance. To address this challenge, we have a unique scRNA-seq dataset of 57 samples from 42 patients before and after chemotherapy. The key challenges are that 1) we do not have resistance labels for the cells and 2) we do not know which genes may be implicated in the resistance. Therefore, our strategy is 1) to find general patterns of gene expression (gene signatures) across different sub-populations of cells (describing several cellular processes of the identity and state of the cells) and 2) from this set of gene signatures, identify those that underlie the resistance mechanisms.

For our first objective, we have selected non-negative matrix factorization (NMF, [1]) to decompose the gene expression of each cell into a linear combination over latent factors, each latent factor representing a gene signature. To circumvent the prominent problem of batch effects between samples, we applied NMF to each sample individually, and merged the samples only on the basis of the inferred gene signatures. Preliminary results on our data show several signatures that are unique to samples, as well as some signatures that are found across different samples. In our future work, we hope to be able to associate some of these signatures to chemotherapy resistance.

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A mean field control approach for smart charging

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Keywords: mean field control, mean field limit, smart charging

The increase in the number of electrical vehicles (EV for short) raises some challenges in the management of the equilibrium production/consumption of the electrical network. To avoid congestion effects, the charging of large fleets of EVs must be optimized. In this work we present a mathematical model to manage the consumption of such a fleet, using a mean field limit approximation. The charge of the fleet is modeled by the control of the distribution of the states m of the EV population, over a period $[0,T]$. The distribution satisfies the continuity equation on $[0,T]\times[0,1]\times I$:

$$\partial_t m_i + \partial_s(m_i b_i) = -\sum_{j\neq i}(\alpha_{i,j}m_i - \alpha_{j,i}m_j), \tag{1}$$

where b_i is the charging rate, $[0,1]$ the range of the state of charge of the EV battery, and I a finite set of modes of charging. The objective is to determine α minimizing a suitable cost $J(m,\alpha)$ under congestion constraints $\int_0^1 m_i(t,ds)\leq D_i$, where $D_i>0$ is given.

Our fist result [1] is the characterization of the optimal solutions as a system of two coupled PDEs and the regularity of the optimal control. We also provide numerical simulations [2].

In a second part [3], we compare the mean field control problem to an optimization problem with a finite number n of EVs. We obtain the convergence of the value and of the solution of the finite population problem as $n\to\infty$, thanks to the regularity of the optimal controls obtained in the first part, and thanks to a superposition principle adapted to this context

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A Reverse Stackelberg Game Model for Grid Usage Pricing with Local Energy Markets

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Keywords: reverse stackelberg games, game theory, bilevel optimization, quadratic inner problem, grid usage pricing, local energy markets.

In the context of the massive penetration of renewables and the shift of the energy system operation towards more consumer–centric approaches, local energy markets are seen as a promising solution for prosumers' empowerment. Various local market designs have been proposed that often ignore the laws of physics ruling the underlying distribution network's power flows. This may compromise the power system's security or lead to operational inefficiencies. Therefore, including the distribution network in clearing the local market arises as a challenge.

We propose using grid usage prices (GUPs) as an incentive mechanism to drive the system towards an economically and operationally efficient market equilibrium, subject to security constraints. Our approach requires expressing the incentive policies as affine functions of the prosumers' active and reactive power outputs. This setting falls into the category of reverse Stackelberg games, where we look for the optimal policy in the space of affine functions [1]. This approach takes advantage of controllability guarantees for the problem's unconstrained setting, which hopefully will enable the DSO to influence the output of the market towards an optimally determined target point. Market-related properties of the policy, such as economic efficiency, individual rationality, incentive compatibility, and fairness, will be rigorously studied.

Two alternative solution approaches are proposed: The first is based on the Gauss–Seidel algorithm, which capitalizes on the hierarchical structure of the reverse Stackelberg game. Meanwhile, the second is based on the KKT reformulation of the problem and then the application of exact penalty theory to the resulting non–convex smooth problem. Finally, extensive computational experiments will be carried out on different IEEE test feeders to assess the performance of the proposed approach statistically.

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An Equilibrium Analysis of Risk-Hedging Strategies in Decentralized Electricity Markets

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Keywords: Peer-to-peer market, risk hedging, generalized Nash equilibrium, Stackelberg game, Bilevel optimization

Currently, due to the large-scale integration of Distributed Energy Resources (DERs), electricity markets are starting to restructure - from centralized to decentralized local market designs. We investigate equilibrium problems arising in various decentralized designs of the electricity market involving risk-averse prosumers. The prosumers have the possibility to hedge their risks through financial contracts that they can trade with peers or purchase from an insurance company. We build several market designs of increasing complexity, from a one-stage market design with inter-agent financial contract trading to a Stackelberg game where an insurance company acts as a leader and prosumers are followers. We derive risk-hedging pricing scheme for each model and show that the Stackelberg game pessimistic formulation might have no solution. We propose an equivalent reformulation as a parametrizated generalized Nash equilibrium problem, and characterize the set of equilibria.

We prove that the insurance company can design price incentives that guarantee the existence of a solution of the pessimistic formulation, which is ε close to the optimistic one. We then derive economic properties of the Stackelberg equilibria such as fairness, equity, and efficiency. We also quantify the impact of the insurance company incomplete information on the prosumers' risk-aversion levels on its cost and social cost. Finally, we evaluate numerically the proposed risk-hedging market models, using residential data provided by Pecan Street.

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Efficient Quantum Encodings for Combinatorial Optimization Problems: The Sub-Graph Isomorphism Case

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Keywords: Combinatorial optimization, variational algorithms, quantum computing, sub-graph isomorphism

With the steady advances in quantum technology to develop reliable quantum computers and software interfaces, quantum computing is well on track to disrupt traditional workflows in solving hard problems. A natural question is then whether quantum-based approaches can help the resolution of combinatorial optimization problems, which are both widespread in real life and very hard to solve classically. Here, the answer is more nuanced.

A class of promising candidates to solve combinatorial optimization problems on near-term quantum computers is the class of variational algorithms, which alternate quantum evaluations and classical steps. Here, one big open question towards quantum advantage is how we encode the problem into the quantum hardware to be able to scale better (e.g., solve larger problems, solve the same problems faster) than on classical computers.

In this talk, I will present some work in tackling this open question by looking at a specific combinatorial problem. In particular, I will propose a novel variational method for solving the sub-graph isomorphism problem on a gate-based quantum computer. The method relies (1) on a new representation of the adjacency matrices of the underlying graphs, which requires a number of qubits that scales logarithmically with the number of vertices of the graphs; and (2) on a new Ansatz that can efficiently probe the permutation space. Simulations are then presented to showcase the approach on graphs up to 16 vertices, whereas, given the logarithmic scaling, the approach could be applied to realistic sub-graph isomorphism problem instances in the medium term.

The talk is based on [1].

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Regularized smoothing for solution mappings of convex problems, with applications to two-stage stochastic programming and some hierarchical problems

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Many modern optimization problems involve in the objective function solution mappings or optimal-value functions of other optimization problems. In most/many cases, those solution mappings and optimal-value functions are nonsmooth, and the optimal-value function is also possibly nonconvex (even if the defining data is smooth and convex). Moreover, stemming from solving optimization problems, those solution mappings and value-functions are usually not known explicitly, via any closed formulas.

We present an approach to regularize and approximate solution mappings of fully parametrized convex optimization problems that combines interior penalty (log-barrier) with Tikhonov regularization. Because the regularized solution mappings are single-valued and smooth under reasonable conditions, they can also be used to build a computationally practical smoothing for the associated optimal-value function and/or solution mapping.

Applications are presented to two-stage (possibly nonconvex) stochastic programming, and to a certain class of hierarchical decision problems that can be viewed as single-leader multi-follower games.

Continuous nonconvex relaxations of the ℓ_0 pseudonorm

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Keywords: ℓ_0 optimization, non-convex continuous penalties, exact relaxations

Sparse optimization problems are massively used in fields such as statistics, computer vision, signal/image processing, and machine learning. A typical formulation is given by

$$\hat{\mathbf{x}} \in \left\{ \arg \min_{x \in \mathbb{R}^N} \frac{1}{2} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\mathbf{x}\|_0 \right\}, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$ (with usually $M \ll N$), $\mathbf{y} \in \mathbb{R}^M$, and $\|\cdot\|_0$ is the ℓ_0 pseudo-norm that counts the number of non-zero entries of a vector. Finally, λ controls the trade-off between data fidelity and sparsity. Due to the combinatorial nature of the ℓ_0 function, Problem (1) is non-convex and belongs to the NP-hard class of complexity [1]. Yet, it has received considerable attention owing to its major importance in many fields.

In this talk, we will review relaxations of (1) where the ℓ_0 term is replaced by a continuous and separable approximation of it, such as those depicted in Fig. 1. This raises the question of which one to choose? There, we will distinguish two kinds of motivations: those based on *statistical* arguments and those based on *optimization* arguments.

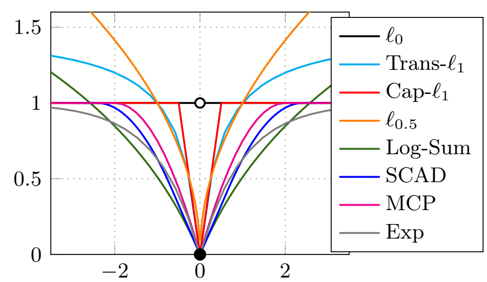

Figure 1: Popular non-convex sparsity promoting penalties in dimension one.

From a statistical perspective, we will follow [2, 3] arguing that a "good" penalty should lead to an estimator which is *unbiased* when the true solution is large, *a thresholding rule* that enforces sparsity, and *continuous with respect to the data*.

From an optimization point of view, we will follow [4, 5] and present relaxations that "reduce" the non-convexity of Problem (1) (e.g., less local minimizers, wider basins of attraction) while preserving its solution(s).

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Local minima and their basins of attractions for piecewise affine functions with polyhedral support

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Keywords : Piecewise affine functions, Polyhedral support, Hyperplane arrangements, Potential field descriptions, Neural Networks

Abstract Piecewise affine functions with a polyhedral support appear often in control/optimization applications (usually as simplifications of originally nonlinear functions but sometimes directly from the problem requirements). Whenever their support is generated by a hyperplane arrangement partitioning, interesting assertions may be made about their structure. Specifically we are interested in : i) reduced-complexity representations which provide equivalent descriptions ; and ii) the location of minima points and their associated basins of attraction.

We consider two applications of piecewise functions : i) we study how such constructs may be used in potential field descriptions ; and ii) how they relate to the neural networks with ReLU activation functions.

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Balancing Efficiency and Privacy in a Decision-Dependent Network Game

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Abstract

We consider a network game, where End Users (EUs) minimize their cost by computing their demand and generation while satisfying a set of local and coupling constraints. Their nominal demand constitutes sensitive information, that they might want to keep private. We prove that the network game admits a unique Variational Equilibrium, which depends on the private information of all the EUs. A data aggregator is introduced, which aims to learn the EUs' private information. The EUs might have incentives to report biased and noisy readings to preserve their privacy, which creates shifts in their strategies. Relying on performative prediction, we define a decision-dependent game $\mathcal{G}^{\text{stoch}}$ to couple the network game with a data market. Two variants of the Repeated Stochastic Gradient Method (RSGM) are proposed to compute the Performatively Stable Equilibrium solution of $\mathcal{G}^{\text{stoch}}$, that outperform RSGM with respect to efficiency gap minimization, privacy preservation, and convergence rates in numerical simulations.

Keywords: Performative Prediction, Equilibrium Learning, Gradient Method, Market

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Linear lexicographic optimization and preferential bidding system

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Keywords: linear lexicographic optimization; preferential bidding system; column generation)

Some airlines use the preferential bidding system to construct the schedules of their pilots. In this system, the pilots bid on the different activities and the schedules that lexicographically maximize the scores of the pilots according to their seniority are selected. A sequential approach to solve this maximization problem is natural: the problem is first solved with the bids of the most senior pilot; then it is solved with those of the second most senior without decreasing the score of the most senior, and so on. The literature admits that the structure of the problem somehow imposes such an approach.

The problem can be modeled as an integer linear lexicographic program. We propose a new exact method, which relies on column generation for solving its continuous relaxation. To design this column generation, we prove that bounded linear lexicographic programs admit "primal-dual" feasible bases, and we show how to compute such bases efficiently.

Another contribution on which our exact method relies consists in the extension of standard tools for resource-constrained longest path problems to their lexicographic versions. This is useful in our context since the generation of new columns is modeled as a lexicographic resource-constrained longest path problem.

Numerical experiments show that this new method is already able to solve industrial instances provided by Air France, with up to 100 pilots. By adding a last ingredient in the resolution of the longest path problems, which exploits the specificity of the preferential bidding system, the method achieves for these instances computational times that are compatible with operational constraints.

Asynchronous Bundle Methods

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Keywords: Bundle Methods, Convex nondifferentiable optimization, Asynchronous approach, Energy optimization

Efficiently optimizing complex nondifferentiable functions, composed by a sum of a large number of terms the computation of each of which may be costly, is a crucial task in many applications such as energy optimization. The use of HPC architectures may be required to obtain appropriate performances, but the existing parallelisation approaches, mostly based on the standard master-slave paradigm, do not scale well to a large number of cores. This has justified the recent surge of interest in asynchronous approaches that may allow a higher scalability. We propose a very general asynchronous approach that relies on the availability of multiple resources not only for the "oracles" (black-box) that compute the function components, but also of multiple master problems, ran in parallel, to provide a stream of potential iterates for the oracles. Each of the master problems is in principle a separate algorithm, typically one of the many variants of bundle methods with some specific setting for its (numerous) algorithmic parameters, which would typically converge towards an (approximate) optimal solution if ran in isolation. We add to the mix a principal entity managing their interaction, and we examine how different capabilities of the principal entity must be handled in order to ensure that the overall asynchronous approach maintains the good convergence properties of the underlying algorithms while allowing harmonious collaboration between them towards the goal of more efficiently solving the problem. We show how our approach can work with a very wide selection of the practical nondifferentiable optimization algorithms proposed in the literature.

Minimizing the Difference of Convex and Weakly Convex Functions via Bundle Method

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Keywords: Nonsmooth Optimization; DC Programming; Bundle Methods

We consider optimization problems with objective and constraints being the difference of convex and weakly convex functions. This framework allows covering a vast family of nonsmooth and nonconvex optimization problems, in particular, those involving Difference-of-Convex (DC) functions with known or unknown DC decomposition, functions whose gradient is Lipschitz continuous, as well as problems that comprise certain classes of composite and nonconvex value functions. We investigate several stationary conditions and present a proximal bundle method to compute critical points for problems of this class. Our algorithm, which employs an improvement function and an original rule to update the proximal parameter to ensure convergence, relies on cutting-plane approximations of the convex functions and linearizations of the weakly convex ones to construct a sequence of convex quadratic subproblems yielding new iterates. The practical performance of the method is illustrated by numerical experiments on some nonconvex stochastic problems.

Stochastic Dual Dynamic Programming and Lagrangian decomposition for Seasonal Storage Valuation in SMS++

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Abstract: The increase in the share of renewable energy sources in the power system, especially intermittent ones like solar and wind, brings several challenges. As both energy production and demand vary throughout the year (in winter, for example, there is a reduction in the supply of solar energy while the demand for energy for heating increases), energy storage becomes a relevant factor to take advantage of excess energy produced in certain seasons of the year and respond to increased energy demand in others. An important system for seasonal storage is that formed by cascading hydroelectric reservoirs. The management of such systems is a challenging task which hinges on a good assessment of the future value of stored energy (water) in these systems. In order to assess the value of water, and thus be able to properly manage these reservoir systems, a large-scale multi-stage stochastic problem spanning a year must be solved, where each stage is a weekly unit-commitment problems. Since the unit-commitment problems are non-convex in nature, they make the seasonal storage valuation problem unsuitable for what would otherwise be the most natural approach to solve it, i.e., Stochastic Dual Dynamic Programming. In this work we exploit the natural convexification capabilities of Lagrangian relaxation to devise a Stochastic Dual Dynamic Programming approach to the seasonal storage valuation problem where the Lagrangian Dual of the single-stage subproblems is solved (using a Bundle-type approach), which corresponds to solving the convexified relaxation of the original problem. This is known to be at least as tight as, and typically strictly tighter than, the standard way to convexify a stochastic MINLP amenable to the Stochastic Dual Dynamic Programming approach, i.e., the continuous relaxation. We report on extensive experiments which show that these huge-scale stochastic problems can indeed be solved on HPC architectures. This is made possible by the use of the SMS++ software framework (https://smspp.gitlab.io) for large-scale problems with multiple nested forms of structure, and in particular by its native parallel capabilities that allow to exploit two different forms of parallelisation (between scenarios in the SDDP approach and within subproblems in the Lagrangian one) at the same time.

Keywords: Bundle Methods, Lagrangian Relaxation, Stochastic Dual Dynamic Programming, Seasonal Storage Valuation

A Maximum Principle for some Chance-Constrained Control Problems

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Keywords: Optimal control problems with uncertainties, Chance constraints , Pontryagin's principle

Many physical or engineering systems are usually described by complex models including inherent uncertainties related to the evolution of the system or to the environment in which this evolution takes place. This is the case for example in finance or in energy management where uncertainties about the price of commodities, demand or supply must be taken into account in the mathematical formulation. In this talk, we will consider a class of control problems with chance constraints.

The notion of constraints in probability (or chance constraints) constitutes a major axis that aims at analyzing and computing optimal solutions that are robust in the sense of probability with respect to the model's uncertainties. This notion is different from the "worst-case" approach by the fact that it aims to define a robust solution against the uncertainties for a reasonable level of probability, while the worst-case problem, which aims for a robust solution against all uncertainties, may admit no feasible solution (and even when an admissible strategy exists, it is generally too pessimistic).

In deterministic control problems, it is known that the optimality conditions can be described by the so-called Pontryagin maximum principle. We will show in this talk that a maximum principle can also be obtained for a large class of chance-constrained control problems governed by some differential equations.

AutoML - Benefits, Reality, Future

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Keywords: Algorithm configuration, algorithm selection, usability, meta-algorithms, automated machine learning.

With the increasing adoption of machine learning (ML) for many practical applications there is a need for techniques to make adoption more efficient. This is the gap that automated machine learning (AutoML), and more generally automated artificial intelligence (AutoAI), aims to fill. The key goal of AutoML is to automate (parts of) the machine learning pipeline. Some of the most prominent and well-developed components from the AutoML toolbox include meta-algorithms, such as, automated algorithm selection (AAS) and configuration (AAC). Both of these techniques have shown substantial benefits in many areas of computer science, including mixed-integer programming (MIP) and Boolean satisfiability (SAT).

One of the most highlighted benefits of AAS and AAC (and more generally AutoML and AutoAI) is the possible performance improvement that can be gained over hand-designed systems. Orthogonal to this, two other benefits can help democratise ML and AI. Automating ML (and other AI processes) reduces the required human effort and, by abstracting away the underlying ML components, decreases the required expertise to start using ML.

In practice, however, AutoML is not as widely adopted as one might expect based on the potential benefits [1]. Adoption seems to be inhibited by issues such as the usability of AutoML frameworks, insufficient expertise in AutoML, and the required computational costs.

As a first step towards increasing the adoption of AutoML, we aim to improve the access to- and understandability of meta-algorithms, such as AAS and AAC. To this end, we introduce the Sparkle platform [2]. Starting with computer scientist not actively involved in the development of meta-algorithms, Sparkle aims to make meta-algorithms accessible and understandable to progressively wider audiences. To achieve this, Sparkle implements a command line interface of self-explanatory commands to simplify correct use of AAS and AAC, and produces detailed reports including results, the experimental setup, and references. Beyond the scope of Sparkle, many other AutoML techniques support different parts of the ML pipeline. To truly democratise ML, these too, have to be advanced to make them broadly accessible and understandable.

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Some Optimal Control Problems Arising From Tumour Containment

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Keywords: Tumour containment, comparison principle, optimal control, drug resistance

Many cancer treatments are administered at a high dose, and are initially successful, but eventually fail as the tumor becomes resistant. When a cure is unlikely, an alternative is to try to simply stabilize the tumour, through a moderate, time-varying, patient specific dose. The hope is to diminish both treatment toxicity and selection for resistance to treatment, allowing us to control the tumor for a longer time. Some experimental results are promising [1].

This may be understood theoretically through mathematical models with treatment sensitive and treatment resistant cells. If these cells are in competition, killing too many sensitive cells may boost the growth of resistant cells, and lead to a shorter survival time. In simple models, if resistant cells are fully resistant, and death occurs when the tumor reaches a critical size, it may be shown that treatments aiming at tumor stabilization are superior to more aggressive treatments. This follows from simple comparison principles [2, 3]. However, more realistic variants of this problem would benefit from an optimal control approach. We will discuss some of these variants, with the hope of generating collaborations.

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Improving subtour constraints generation in Branch-and-Cut algorithms for TSP with Machine Learning

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Keywords: Traveling Salesman Problem, Subtour Elimination Constraints, Branch-and-Cut, Machine Learning

Branch-and-Cut (B&C) is a widely-used algorithm for solving integer programming (IP) problems. In recent years, applying Machine Learning (ML) to improve fundamental decision problems of B&C algorithms is an active research domain. While much of ML research focuses on variable selection [1], node selection [2], and cut selection [3], less attention has been paid to the question of how to design a cutting plane generation strategy in B&C algorithms. This question is crucial since generating cutting planes can be a computational burden when the instance's size increases. In this work, we focus on improving subtour elimination constraints (SEC) generation in B&C algorithms for Traveling Salesman Problem (TSP) by ML. SEC is a well-known constraint used to exactly solve TSP. In the IP formulations for TSP, SEC is usually dynamically generated as cutting planes in the course of B&C algorithms because of its exponential cardinality. Our approach is to take advantage of Machine Learning to handle two questions before launching the separation process to find violated SEC cuts on a node of the B&C search tree: 1) Does there exist violated SEC cuts? 2) If yes, is it worth generating them?. For the former, we use a Graph Neural Network to detect the existence of violated SEC cuts from fractional solutions. For the latter, we formulate the problem of sequentially deciding to generate SEC as a Markov decision problem and train a Reinforcement Learning agent to tackle it. Not only can our method leverage the geometric structure of optimal solutions but it also offers the flexibility of the instance's size. Numerical results show that our proposed approach can significantly accelerate the performance of B&C algorithms for TSP with SEC.

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Decomposition Methods for the Multi-Network Optimal Power Flow Problem

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Keywords: Optimal Power Flow, Decomposition, Mixed-Integer Non-Linear Programming, Mixed-Integer Linear Programming, Linear Relaxation

The optimal power flow (OPF) problem concerns the problem of determining the operational efficiency of power systems through the objective of minimising operating costs, subject to power flow constraints and demands given throughout a transmission network. OPF problems have a high level of practical importance, and a vast amount of research has studied them in the last 50 years (see e.g., [1, 2]).

OPF problems are typically modelled as mathematical programs with a large number of non-linear constraints, such as those involving bilinear terms, which are notoriously difficult to solve. Recently, a number of relaxation approaches have been presented for solving non-linear programs with bilinear terms, such as binary expansions and McCormick envelopes. These approaches allow solvers to find under- and over-estimators for the bilinear terms in the non-linear constraints.

The advantage of binary expansions is that the accuracy of the estimators can be set by the user. However, increasing accuracy leads to a significant increase in the number of complicating binary variables present in the expansion. Hence, efficiently solving more accurate relaxations is key to finding solutions to OPF problems effectively. Decomposition techniques which mitigate the effect of binary variables, such as combinatorial Benders decomposition (CBD) [3], have been shown to be effective for problems with similar structures.

We first compare different linear relaxations of OPF problems, showcasing how CBD can be used to find arbitrarily accurate solutions efficiently. Afterwards, we apply the relaxation approach to a multi-network OPF problem, where nodes in different networks can be connected. This leads to a tree-based model, where each node of the tree is solved using decomposition.

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On a Concept of a Generic Intersection Cut Callback

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Keywords: intersection cuts, polyhedron, nonconvexity, MINLPs

Intersection cut framework [1] can generate valid linear inequalities for a nonconvex set S. The framework requires a corner polyhedron relaxation of S, and a convex S-free set, which does not contain any point of S in its interior. In this talk, I will review the recent development of intersection cuts in MINLPs (e.g quadratic [2]/polynomial programming and signomial programming), featuring the construction of a variety of S-free sets. On the other hand, implementation of intersection cut requires much knowledge of a solver's data structure and numerical stability. Software engineering can help here, as the solver can encapsulate cut separation procedures and provides an intersection cut callback without need for symbolic representation of S. Then, the users only need to provide defining-variables, zero- and first-order oracle access to the S-free set. I will also discuss some limitations of intersection cuts and open problems.

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Stability analysis of controlled resource allocation in bacterial models

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Keywords: self-replicator model of bacteria, optimal resource allocation, stability analysis, cylindrical algebraic decomposition of semi-algebraic sets

The dynamical system analyzed in this work represents a simplified version of a bioprocess used in scientific research and in the chemical and pharmaceutical industries for the production of value-added chemical compounds [3]. The model is two-fold: on one side, it considers a self-replicator bacterial model representing the main cellular functions involved in growth and chemical production: metabolism and production of proteins. On the other hand, it models a continuous bioprocess, a production scheme occurring in a bioreactor that allows steady-state operation for long periods of time, avoiding shutdown for cleaning and maintenance. The study of the steady-state equilibrium, that is connected with the use of constant controls, is crucial and related to the turnpike phenomenon in optimal control (*e.g.*, when maximising dynamically the bacterial biomass). A typical situation is indeed as follows [1]: optimising the equilibrium w.r.t. the constant control provides a hyperbolic equilibrium of the underlying Hamiltonian singular dynamics described by Pontrjagin Maximum Principle. This is a strong motivation to study the stability properties of the main equilibrium of the system. This equilibrium of interest is parameterised by the control, and its existence is described by a series of strict polynomial inequalities in terms of the system parameters. Its local stability can be assessed by means of the well-known Routh-Hurwitz stability criterion that yields an additional inequality. By leveraging the degree pattern of the specific allocation resource, we significantly simplify the first steps of the so-called "Cylindrical Algebraic Decomposition" algorithm [2] and are able to prove local stability of the equilibria of interest for a dense subset of the parameters, including the control.

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Statistical Discovery of Transcriptomic Cancer Signatures using Multimodal Local Search

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Keywords: Transcriptomics, Multimodal Optimization, Local Search, Cancer, scRNA-seq.

Cancer exhibits *intra-tumor heterogeneity* of gene expression that can be profiled using *single-cell RNA sequencing* technology. 'Precision' treatment strategies attempt to exploit this for treatment recommendations tailored to patient-specific heterogeneity patterns in specific sets of genes (*signatures*). Identification of treatment-relevant signatures requires their observation across multiple patients. However, *inter-patient heterogeneity* makes direct integration of gene expression data across patients, and subsequent identification of signatures, unreliable.

For the case of glioblastoma, Neftel *et al.* [1] have circumvented this problem using an approach that first heuristically determines a candidate set of patient-specific signatures, and then identifies similar candidates across patients, to obtain biologically validated results.

We expand and improve upon this approach using: (i) a statistically well-founded approach to score general signatures in individual patients, (ii) robust ranks instead of normalized RNA expression levels, (iii) a straightforward extension of the patient-specific score to a global score across all patients, and (iv) a *gradient* structure of the global score function.

Since this binary partitioning problem is NP-complete, we use a randomized optimization method in a multimodal setup, across 10,000 runs. Our greedy algorithm identifies signatures by starting from a random gene set, then iteratively moves to the best signature in its neighborhood, using an efficient partial evaluation of the objective function, until a local optimum is found.

Out of the seven 50-genes signatures that we found in the *glioblastoma* data set of 7,167 genes and 6,855 cells, five had a high degree of similarity to all eight of the metaprograms from the original study, with some of our signatures including genes from two metaprograms simultaneously. Gene set enrichment analysis of one of the remaining two signatures identified a specific neuronal process that is biologically plausible within the biological context.

Our approach is free from ad-hoc thresholds, simple, transparent, robust, and can yield biologically plausible results. We believe that our approach allows for bypassing the need for a complicated process of generating individual signatures in every sample and their further integration, and hence represents a useful addition to the tool belt of methods tackling the signature search problem.

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Carbon offsetting: a game perspective

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This presentation will describe a project on decarbonization funded by the IMPT. Carbon offsetting schemes are developing strongly and remain at the center of existing emission reduction frameworks for states or certain sectors. This project focuses on the conditions under which the environmental integrity of such schemes can be guaranteed. Through the progressive development of a formalized representation, mobilizing game theory and motivated by a multi-disciplinary analysis of these schemes, it aims to refine our understanding of the conditions that can ensure the integrity of such schemes.

Author Index