Research Proposal: Stable Matching and its Generalization

The model of two-sided matching markets [Gale and Shapley, 1962] has a wide range of applications including labor employment, marriage market, and college admission. A key requirement of any solution to the matching problem is stability, which ensures the market participants have no incentive to abandon the current partner. Since the seminal work of [Gale and Shapley, 1962], stable matching has been an active area of research. In this research project, we aim to focus on the model of stable matching, incorporating practical considerations, and develop new results that bring insights into theory and practice.

1 Stable Matching with Indifferences

In the classical stable matching model, the preference profiles of both sides are strict and complete. The lattice structure of stable matching provides a significant property to this model that one side could be optimally matched in a single integral matching. However, when preference profiles admit ties, this property no longer holds. [Lin et al., 2024] studied this problem from an approximation perspective. For every participant in the market, a ratio between the individual optimal stable matching utility and the expected utility output from a given algorithm is defined. They derived a polynomial time algorithm to minimize this ratio. A matched lower bound shows that this algorithm is asymptotically optimal. There are several possible follow-up generalizations based on this model.

Many-to-one and Many-to-many Matching Markets [Lin et al., 2024] focuses on the case of one-to-one matching markets, while in reality, more complex cases may appear. In college admission [Roth, 1985], which is a many-to-one matching market, a college can admit many students, while a student could only be admitted to one college. Moreover, a labor market for freelancers could be viewed as a many-to-many matching market [Echenique and Oviedo, 2004], where companies may hire multiple freelancers for different projects, and freelancers may work for multiple clients at the same time. In these cases, whether a similar approximation ratio holds as in the one-to-one setting, and how should we develop algorithms to achieve this ratio is interesting and worth exploring.

Decentralized Algorithm for Stable Matching with Indifferences Decentralized matching is a pressing challenge in real-world online marketplaces where players must coordinate their actions indirectly [Niederle and Yariv, 2007, Liu et al., 2021]. In the previous work, [Lin et al., 2024] provides a centralized approximation algorithm which needs to be implemented by a platform since it needs to randomly pick a matching from a distribution. This algorithm does not generalize to the decentralized case if we allow the market participants to make their own decisions. Therefore, it would be interesting to study whether we could achieve the same approximation ratio in a decentralized setting. And whether the system could converge to an equilibrium.

Beyond Worst Case Analysis The algorithm proposed in [Lin et al., 2024] is asymptotically optimal, where the lower bound is proved based on the worst-case analysis. However, the lower and upper bounds differ in lower order terms, which opens the possibility for us to improve it in the "beyond worst case analysis" [Gimbert et al., 2020], i.e., if the preferences for each market participant are i.i.d. and possibly with ties, what would be the lower bound of the approximation ratio? Is there a better algorithm which could provide a tighter upper bound?

2 Online Stable Matching

Classical stable matching problems often consider a fixed market, where two sides of the market and their corresponding preferences are known before the game starts. This could be generalized to an online fashion, where one side of the market is offline and the other side of the market arrived one by one. The preference profiles of the offline side is known beforehand, while for the online side, the preference list for each agent is revealed upon arrival. We need to make an irrevocable matching for each online agent when she arrives. An agent is said to be satisfied if she is matched and does not participate in a blocking pair. The objective is to maximize the number of satisfied online agents, the number of satisfied offline agents, and the number of matched pairs.

When the offline side has only one agent and the online side has n agents, this reduces to the classical secretary problem [Ferguson, 1989]. [Babichenko et al., 2019] considered a similar setting, while focusing on a simplified serial dictatorship preference structure, i.e., both sides have global ranking over the other side. They corresponding upper and lower bounds for the quantities that we are interested in. Whether similar results hold in a market with general preference structure remains open and challenging.

3 Stable Matching and Optimal Transport

The canonical problem of optimal transport corresponds to finding a matching that maximizes utilitarian welfare, i.e., an aggregate of agents' utilities, while stable matching aims to output a matching without blocking pairs. [Echenique et al., 2024] studies the connection between these two fields, showing that by utilizing a certain convex transformation of agents' utilities as the cost function, the solution to the optimal transport problem is approximately stable. They only focus on matching markets with aligned preferences, which simplifies the preference structure. It would be interesting to explore the connections under general preference profiles or even the preferences admit ties, which may provide new insights into the structural properties of stable matchings.

References

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