Locomotion in granular media.

Keywords: Mathematical Modelling, Control, Optimal control, Granular media, Soils.
Location: Centre Borelli, École Normale Supérieure (ENS) Paris-Saclay, Gif-Sur-Yvette
Contacts: François Alouges (email, webpage) and Aline Lefebvre-Lepot (email, webpage)

Context: Faced with the challenges of global warming and environmental changes (landslides, coastal erosion, floods), it is crucial to predict the dynamics of soils to better anticipate and prevent risks. These soils are heterogeneous materials composed of inert particles (sand grains, clay) and living organisms (bacteria, fungi). The organisms living in the soil significantly shape its mechanical and hydraulic properties [Voigtländer24]. However, our understanding of how they affect the soil's overall structure and efficiently propel themselves through granular material remains limited.

Objectives: The aim of this project is to better understand the locomotion strategies of living beings in soils. One would like to study the swimmers' ability to move (control problem) and their optimal strategies (optimal control problem). From a modeling perspective, this leads to granular medium / active structure interaction, alongside classical fluid / active structure interactions. While control and optimal control problems for active structures have often been studied in (linear) fluids governed by Stokes equations, granular models are typically nonlinear. This presents challenges and will require developing new mathematical methods, as those used for swimmers in Stokes fluids are specific to linear problems.

Tasks: The student will first examine simplified academic models, such as solids moving on a plane with frictional contact. While in [Tanaka12] the interaction is modeled by a viscous frictional force, we aim to use a solid friction model, leading to differential inclusion control problems [Stewart10]. Another direction will be to consider swimmers composed of several rigid links using an RFT-type interaction model (Resistive Force Theory). Again, compared to Stokes fluids [Alouges19, NGallardo21, Marchello22], the interaction here is non-linear and necessitates the development of new techniques. Numerical tests could also be carried out, either on a microscopic scale using the SCoPI granular code developed by Aline Lefebvre-Lepot [SCoPI] for the simulation of granular materials, or by implementing a numerical scheme for the RFT model.

Collaborations: The post-doctoral student will collaborate with François Alouges and Aline Lefebvre-Lepot, both being reseachers at Centre Borelli at ENS Paris-Saclay. Along with other collaborators they studied swimming in fluids governed by Stokes equations and developed methods from geometric control theory that are now classical in the field [Alouges08, Alouges13]. The research of Aline Lefebvre-Lepot focuses on modelling and simulation of granular media and suspensions. Some interactions with physicists in the FAST laboratory (Paris-Saclay University), specialists in the study of suspensions and granular media, are also expected.

Profile: PhD thesis in Mathematics/Numerical Analysis with a knowledge of mechanics.

Bibliography:

[Alouges08] Alouges, F., DeSimone, A., & Lefebvre, A. (2008). Optimal strokes for low Reynolds number swimmers: an example. Journal of Nonlinear Science, 18, 277-302.

[Alouges13] Alouges F., DeSimone A., Heltai H., Lefebvre-Lepot A. & Merlet B. (2013) Optimally swimming Stokesian robots, DCDS-B, Vol. 18, no. 5

[Alouges19] Alouges, F., DeSimone, A., Giraldi, L., Or, Y., & Wiezel, O. (2019). Energy-optimal strokes for multi-link microswimmers: Purcell's loops and Taylor's waves reconciled. New Journal of Physics, 21(4), 043050.

[Alouges22] Alouges, F., Lefebvre-Lepot, A., & Weder, P. (2022). Optimal strokes for the 4-sphere swimmer at low Reynolds number in the regime of small deformations. MathematicS In Action, 11(1), 167-192.

[Lefebvre09] Lefebvre-Lepot, A., & Merlet, B. (2009). A stokesian submarine. In ESAIM: Proceedings (Vol. 28, pp. 150-161). EDP Sciences.

[Marchello22] Marchello, R., Morandotti, M., Shum, H., & Zoppello, M. (2022). The N-link swimmer in three dimensions: Controllability and optimality results. Acta Applicandae Mathematicae, 178(1), 6.

[NGallardo21] Nuevo-Gallardo, C., Traver, J. E., Tejado, I., & Vinagre, B. M. (2021). Purcell's three-link swimmer: assessment of geometry and gaits for optimal displacement and efficiency. Mathematics, 9(10), 1088.

[SCoPI] https://lefebvre-lepot.perso.math.cnrs.fr/SCoPI/

[Stewart10] Stewart, D. E., & Anitescu, M. (2010). Optimal control of systems with discontinuous differential equations. Numerische Mathematik, 114, 653-695.

[Tanaka12] Tanaka, Y., Ito, K., Nakagaki, T., & Kobayashi, R. (2012). Mechanics of peristaltic locomotion and role of anchoring. Journal of the Royal Society Interface, 9(67), 222-233.

[Voigtländer24] Voigtländer, A., Houssais, M., Bacik, K. A., Bourg, I. C., Burton, J. C., Daniels, K. E., ... & Yang, J. Q. (2024). Soft matter physics of the ground beneath our feet. Soft Matter, 20(30), 5859-5888.