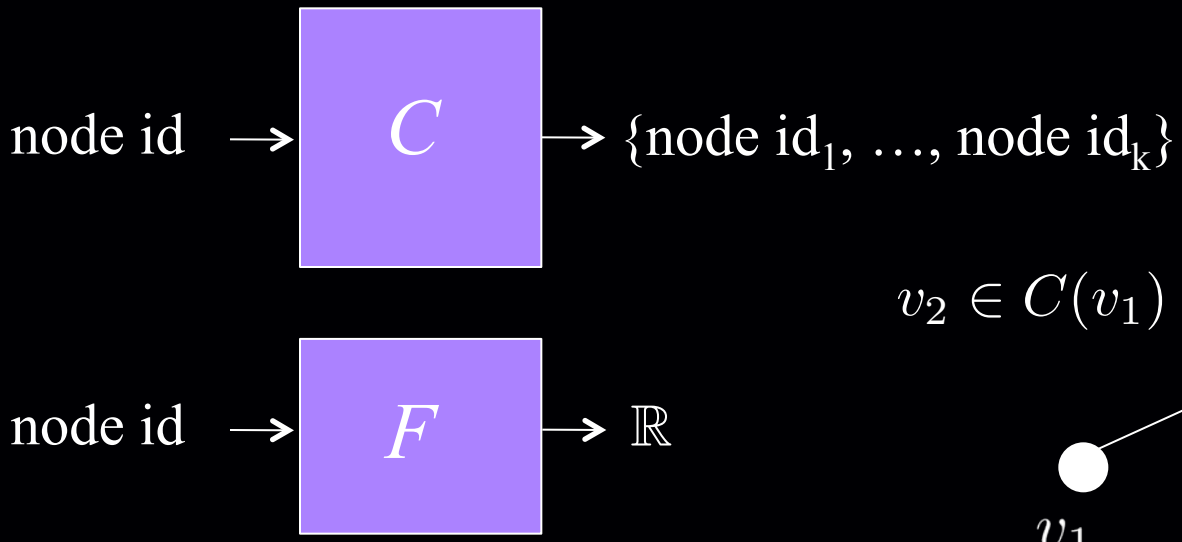
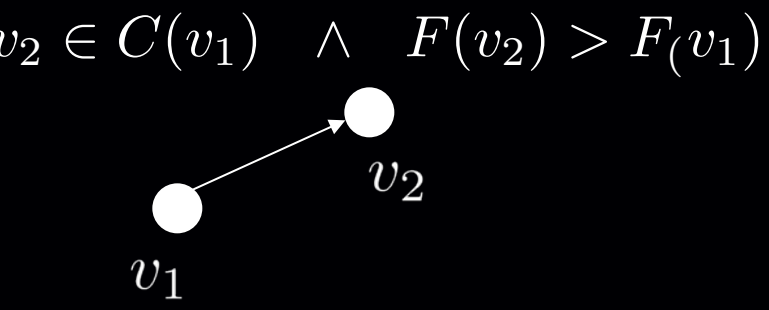
The Class PLS [JPY '89]

"Every DAG has a sink."

Suppose that a DAG with vertex set $\{0,1\}^n$ is defined by two circuits:

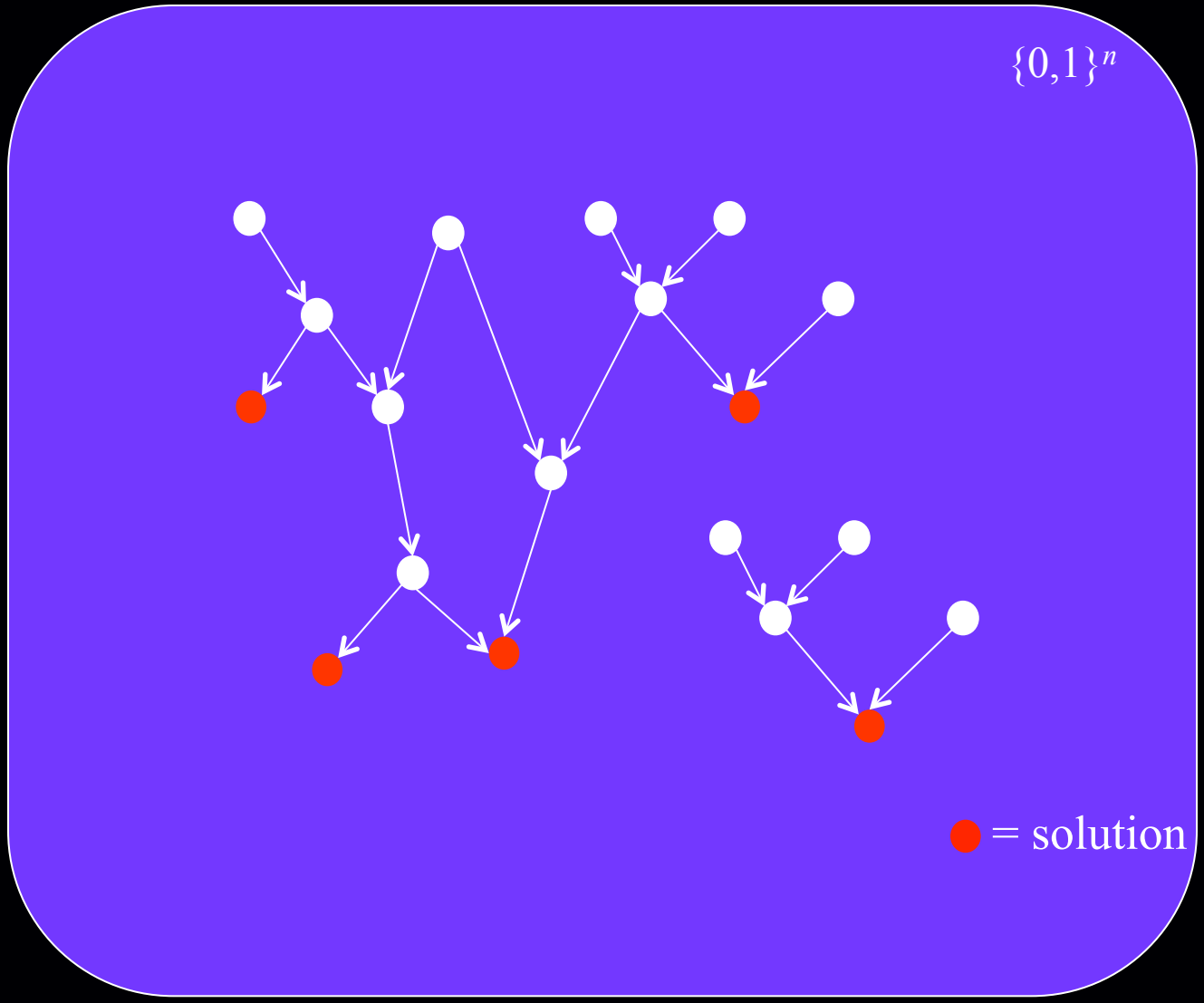




FINDSINK: Given C, F: Find x s.t. $F(x) \geq F(y)$, for all $y \in C(x)$.

PLS = { *Search problems in* FNP *reducible to* FINDSINK }

FINDSINK



LocalMaxCut is PLS-complete

LocalMaxCut: Given weighted graph $G=(V, E, w)$, find a partition $V=V_1 \cup V_2$ that is locally optimal (i.e. can't move any single vertex to the other side to increase the cut size.

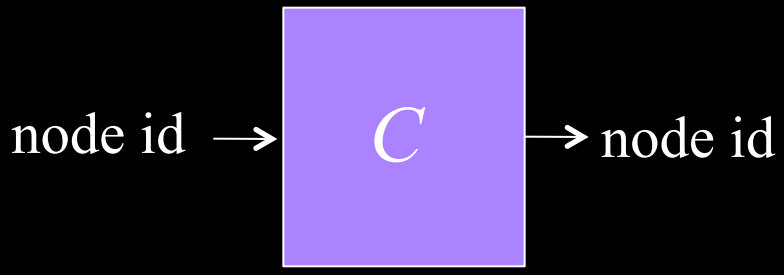
[Schaffer-Yannakakis'91]: LocalMaxCut is PLS-complete.

[Fabrikant-Papadimitriou-Talwar'04]: Pure Nash equilibria in potential games are PLS-complete.

The Class PPP [Papadimitriou '94]

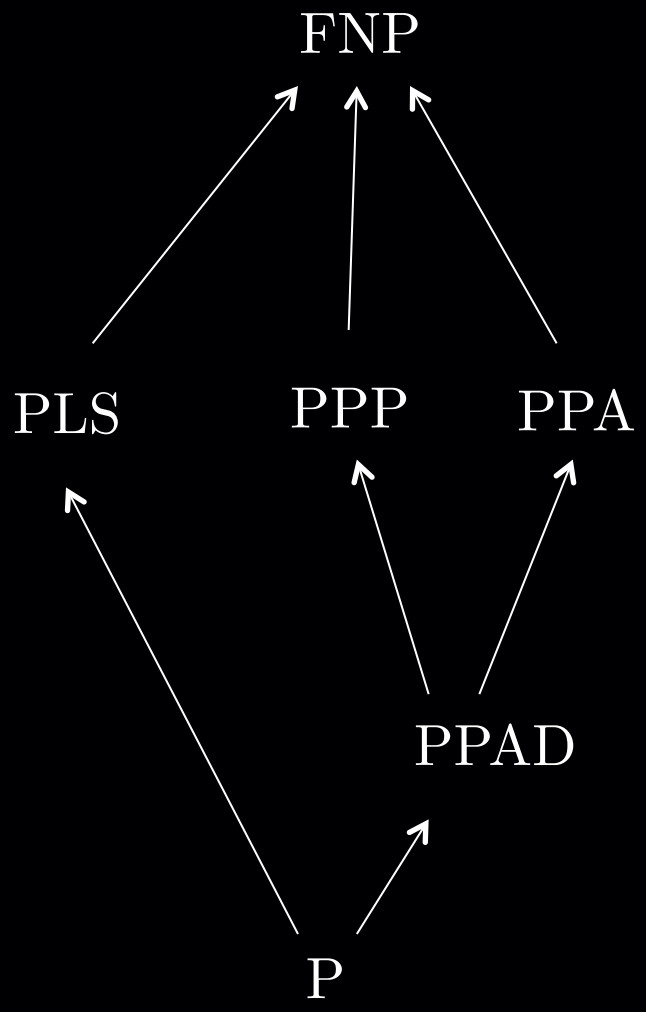
"If a function maps n elements to n-1 elements, then there is a collision."

Suppose that an exponentially large graph with vertex set $\{0,1\}^n$ is defined by one circuit:



COLLISION: Given C: Find x s.t. $C(x) = 0^n$; or find $x \neq y$ s.t. $C(x)=C(y)$.

PPP = { *Search problems in FNP reducible to* COLLISION }


FNP
PLS
PPP
PPA
PPAD
P

Thanks for your attention
Questions?