Book of abstracts

PGMO DAYS 2024

November 19th and 20th, 2024

EDF Labs Paris-Saclay

Preface

This volume contains the extended abstracts of the talks presented at the conference PGMO-DAYS 2024 held on November 19th – 20th, 2024 at EDF Labs Paris-Saclay.

We especially acknowledge the support of EDF, FMJH and RT Optimisation. We thank CNRS, Institut polytechnique de Paris, Université Paris-Saclay and Inria. We also thank ROADEF, SMAI-MODE, and SMF. We are grateful to the plenary speakers and to the organizers of the invited sessions.

November 5, 2024 The organizers of the PGMO days
Palaiseau

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Didier Aussel: *New trends in bilevel optimization*
Matías R. Bender and Elias Tsigaridas: *Polynomials, Optimization, and Sampling*
Charles Bertucci: *Mean-Field Games*
Luce Brotcorne: *Bi-level problems in energy management*
Wesley Coelho and Dimitrios Papadimitriou: *Quantum Computing for Combinatorial Optimization*
Michel De Lara: *Theory and Algorithms in Sparse Optimization*
Sorin-Mihai Grad: *Fast algorithms in convex optimization*
Ziad Kobeissi: *Differential equations in machine learning*
Luca Nenna, Paul Pegon and Guillaume Carlier: *New trends in Optimal Transport and Applications*
Juan Peypouquet: *Dynamical systems and algorithms with applications in optimization, sampling and variational analysis*
Andrea Simonetto and Eric Bourreau: *Quantum algorithms: graphs, groups, and walks*
Cheng Wan and Pierre Gruet: *Auctions: algorithms and application in power and telecommunication markets*
Cheng Wan and Quentin Jacquet: *Optimization and heuristics in battery management for ancillary services*
Adrien Taylor: *Algorithmic optimization and online decision-making with application to robotics.*

PGMO 2024 PhD Prizes

The Gaspard Monge Program for Optimization, Operations Research, and their Interactions with Data Sciences, awards every year, under the scientific patronage of ROADEF and SMAI-MODE, two PhD prizes (exaequo). All the fields in Optimization, and Operations Research, including their Interfaces, are eligible. The applicants must have defended their PhD in France, during the previous civil year.

The two 2024 PGMO PhD prizes were awarded to

- *Tâm Lê*, for his PhD at Toulouse School of Economics, under the supervision of Jérôme Bolte et Edouard Pauwels, on "Nonsmooth calculus and optimization for machine learning: first-order sampling and implicit differentiation".
- *Antoine Oustry*, for his PhD at LIX, École polytechnique, under the supervision of Claudia D'Ambrosio and Leo Liberti, on "Global optimization of nonlinear semi-infinite programming problems: Applications in power systems and control".

The 2024 PhD prize committee was chaired by Luce Brotcorne (INRIA). It was composed of the following researchers:

- Members appointed by the PGMO Scientific Council
  - Guillaume Carlier (CEREMADE, Dauphine)
  - Johanne Cohen (LISN, Paris Sud)
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- Members appointed by the ROADEF
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  - Dominique Feillet (LIMOS, Mines de St Etienne)
  - Rosa Figueiredo (LIA Univ. d'Avignon)
- Members appointed by the group MODE of SMAI
  - Alexandre d'Aspremont (INRIA, ENS)
  - Luce Brotcorne (INRIA, Lille)
  - Yannick Privat (IECL, Nancy)

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Traffic Flow Models for Current and Future Mobility Challenges

Paola Goatin[1]

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Keywords: Macroscopic traffic flow models, multi-population models, multi-scale models, finite volume schemes, optimal control.

The mobility paradigms could undergo a significant transformation in the near future, as new technologies enable extended data collection and Vehicle-to-Vehicle (V2V) or Vehicle-to-Infrastructure (V2I) communication. This will offer novel means to control and optimize traffic flow. Within this context, mathematical models play an important role, allowing for the design and evaluation of new management approaches. In this talk, I will present applications to road traffic regulation using Connected Automated Vehicles (CAVs) [1] or dynamic routing [2]. Our results are based on the development of specific macroscopic models accounting for the interacting dynamics of the different classes of users. Numerical experiments show that controlling a small fraction of users is, in general, sufficient to consistently improve the global system performance.

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Moshi: a real-time spoken dialogue model

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In this talk, I will present Moshi, a joint speech-text foundation model and full-duplex spoken dialogue system.

Current systems for spoken dialogue rely on pipelines of independent components, namely voice activity detection, speech recognition, textual dialogue and text-to-speech. Such frameworks cannot emulate the experience of real conversations. First, their complexity induces a latency of several seconds between interactions. Second, text being the intermediate modality for dialogue, non-linguistic information that modifies meaning – such as emotion or non-speech sounds – is lost in the interaction. Finally, they rely on a segmentation into speaker turns, which does not take into account overlapping speech, interruptions and interjections.

Moshi solves these independent issues altogether by casting spoken dialogue as speech-to-speech generation. Starting from a text language model, Moshi generates speech as tokens from the quantizer of a neural audio codec, and separately models its own speech and that of the user into parallel streams. This allows for the removal of explicit speaker turns, and the modeling of arbitrary conversational dynamics. We extend the hierarchical semantic-to-acoustic token generation of previous work, by predicting time-aligned text tokens as a prefix to audio tokens. Our resulting model is the first real-time full-duplex spoken large language model, with a latency of around 200 ms in practice.

Incorporating Discrete Choice Models into Revenue Management Decisions

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Keywords: Discrete choice models, assortment optimization, multinomial logit model, revenue management

Over the last couple of decades, there has been enormous progress in using discrete choice models to understand how customers choose and substitute among products and incorporating this understanding into operational models to decide which assortment of products to offer to customers or what prices to charge. We owe some of this progress to increase in the computational power so that we can build and solve more detailed operational models, but perhaps, most of this progress is due to the fact that online sales channels started providing fine-grained data on how customers browse the products. In this talk, we will go over fundamental discrete choice models that have been used in building operational assortment optimization and pricing models, overview the main algorithmic approaches that have been developed to solve the operational models, and identify research prospects. The focus will be on both static models that make one-shot assortment optimization or pricing decisions, as well as dynamic models that explicitly capture the evolution of demand and inventories over time.

Control and Machine Learning

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Abstract

Systems control, or cybernetics, a term first coined by Ampère and later popularized by Norbert Wiener, refers to the science of control and communication in animals and machines. The pursuit of this field dates back to antiquity, driven by the desire to create machines that autonomously perform human tasks, thereby enhancing freedom and efficiency.

The objectives of control systems closely parallel those of modern Artificial Intelligence (AI), illustrating both the profound unity within Mathematics and its extraordinary capacity to describe natural phenomena and drive technological innovation.

In this lecture, we will explore the connections between these mathematical disciplines and their broader implications. We will also discuss our recent work addressing two fundamental questions: Why does Machine Learning perform so effectively? And how can data-driven insights be integrated into the classical applied mathematics framework, particularly in the context of Partial Differential Equations (PDE) and numerical methods?

Keywords: control theory, machine learning, supervised learning

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Global optimization of nonlinear semi-infinite programming problems: Applications in power systems and control.

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Keywords: Global optimization, semi-infinite programming, AC optimal power flow.

Computing global optima of nonlinear semi-infinite programming problems is difficult due to the infinite number of constraints and their (potential) nonconvexity. This thesis makes theoretical and practical contributions to address this challenge, focusing on applications to the optimization of electrical grids and the control of dynamical systems.

The first part is devoted to convex semi-infinite programming [1, 2]. First, we exhibit a convergence rate for the cutting-plane algorithm when the semi-infinite constraints are linear and the objective function is strongly convex. Second, we address a limitation of the cutting-plane algorithm: feasibility is achieved only asymptotically. Focusing on semi-infinite programs with a quadratic separation problem, we propose an iterative inner-outer approximation algorithm that generates a globally convergent sequence of feasible points. The third chapter presents an application of convex semi-infinite programming to minimal-time control.

The second part deals with the global optimization of finite and semi-infinite nonconvex optimization problems related to the dispatch of electricity. We tackle the standard AC optimal power flow problem, a quadratically constrained quadratic programming problem with a finite number of constraints. Our global optimization algorithm, based on a strengthened semidefinite programming relaxation and piecewise linear approximations, achieves state-of-the-art performance on a reference benchmark [3]. We address the numerical issues raised by this semidefinite programming relaxation for instances at a larger scale, by solving an unconstrained dual formulation with a bundle method to obtain certified lower bounds. Our numerical experiments on large-scale instances show significant improvement of the dual bounds [4]. Finally, we consider some uncertainties in the AC power flow model and solve the resulting semi-infinite programming problem with an adaptive discretization algorithm with global convergence guarantees.

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Nonsmooth calculus and optimization for machine learning: first-order sampling and implicit differentiation

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Keywords: Deep learning, Automatic differentiation, First-order methods, Nonsmooth nonconvex optimization, Implicit differentiation, Stochastic optimization

Machine learning problems are often formulated as the minimization of a nonsmooth and nonconvex expectation. Gradient-based methods are favored in this context due to their flexibility and scalability. To implement these methods, we observe a common practice of replacing classical derivatives with automatic differentiation, but without rigorous theoretical justification in the nonsmooth setting.

A notion of gradient called "conservative gradient" [4] provides a justification for such a practice by extending simple calculus rules to nonsmooth functions, such as the chain rule. In this thesis, we propose two extensions of the conservative calculus. A first result is a rule to differentiate nonsmooth functions under the integral operation [1] which justifies gradient sampling on nonsmooth functions. A second result is an implicit differentiation formula [2] motivated by bi-level problems such as hyperparameter selection or implicit layers.

Thanks to these new calculus rules, we propose a theoretical framework for analyzing algorithms that is furthermore compatible with practical implementations. A descent along curves property allows us to obtain the convergence of two popular optimization algorithms, the stochastic subgradient method and its heavy ball variant [1, 3]. Our approach involves a notion of stationarity that may depend on the use of calculus rules in practical implementations and generate absurd limit points. For both algorithms we finally show that these artifacts are generically avoided.

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Comparative Analysis of Quantum Annealing and Quantum-Classical Hybrid Solver on Cut Problems

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Keywords: Combinatorial optimization, MILP, Quantum Annealing, QUBO, Hybrid Solvers, Cut Problems, D-Wave.

This work addresses a cybersecurity optimization problem [1], reduced to various cut problems [2], which were tackled using Quantum Annealing (QA) from D-Wave [3]. Classical approaches such as Mixed Integer Linear Programming (MILP) were not considered, as they outperform quantum methods for the considered instances. Instead, the focus is on analyzing the scalability and performance of different Quadratic Unconstrained Binary Optimization (QUBO) formulations [4], and how their embedding on a quantum device impacts solution quality.

The study compares the performance of Simulated Annealing, Quantum Annealing alone and the Quantum-Classical Hybrid Solver which leverages QPU and a metaheuristic [5] on these problems, examining the effect of QUBO structure [6], embedding strategies [7], and solver selection. Special attention is given to how the interaction between variables influences the physical qubit requirements and how different workflows within the Hybrid Solver perform as the problem size increases.

Our findings suggest that the formulation of the QUBO, including the balance between linear and quadratic terms, plays a crucial role in achieving optimal results. While Quantum Annealing alone shows promise, the Hybrid Solver consistently performs better for larger instances, providing a more scalable and efficient solution for industrial cut problems.

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Equilibrium in Functional Stochastic Games with Mean-Field Interaction

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Keywords: mean-field games, Nash equilibrium, Volterra stochastic control, optimal portfolio liquidation, systemic risk, price impact

We consider a general class of finite-player stochastic games with mean-field interaction, in which the linear-quadratic cost functional includes linear operators acting on controls in L2. We propose a novel approach for deriving the Nash equilibrium of the game explicitly in terms of operator resolvents, by reducing the associated first order conditions to a system of stochastic Fredholm equations of the second kind and deriving their closed form solution.

Furthermore, by proving stability results for the system of stochastic Fredholm equations we derive the convergence of the equilibrium of the N-player game to the corresponding mean-field equilibrium. As a by-product we also derive an epsilon-Nash equilibrium for the mean-field game, which is valuable in this setting as we show that the conditions for existence of an equilibrium in the mean-field limit are less restrictive than in the finite-player game. Finally we apply our general framework to solve various examples, such as stochastic Volterra linear-quadratic games, models of systemic risk and advertising with delay, and optimal liquidation games with transient price impact.

Extensions of $\mathcal{KL}$ class and Lyapunov Functions for the Maximization of Discrete-time Dynamical System States

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Keywords: Discrete-time Systems, Trajectories Maximization, Lyapunov Functions, $\mathcal{KL}$ Stability Theory

In this presentation, we consider a particular maximization problem for which the constraints set is the reachable values set of a discrete-time dynamical system (in a finite dimensional state-space). This type of maximization problems arises in the analyze of the robustness or the performance of the system; for inverse problems or even in formal verification of systems. The optimal value in this context can be viewed as the value which penalizes the most the system with respect to some criteria.

More formally, the data of the problem are the initial conditions set X^{in}, the dynamics of the system i.e. a function T which maps a state to a preceding state and a real-valued objective function φ. Hence, we are interesting in maximizing $\varphi(T^k(x))$ under the constraints $x \in X^{\text{in}}$ and k is a natural integer. Thus, an optimal solution is a couple (x, k). When k is fixed, we are faced to a standard optimization problem. Our main difficulty (the novelty) is to find the optimal k. Under relative mild assumptions, the set of optimal integers is nonempty and has a maximal element K. We then propose a method to produce an upper bound of K. The first approach is to use geometric sequence homeomorphisms which are upper bounds of $\varphi(T^k(x))$. We can prove that this approach is optimal in the sense that it furnishes, in theory, K but associated homeomorphisms are not computable. In consequence, we adapt tools from dynamical systems classically used to prove system stability to compute interesting homeomorphisms. First, we consider discontinuous and unsigned extensions of $\mathcal{KL}$ functions for which we can exhibit an homeomorphism. Second, we extend the concept of Lyapunov functions adapted to our maximization problem. Moreover, we obtain both direct and converse theorems meaning that interesting homeomorphisms exist if and only such Lyapunov functions exist.

Optimal Control for Linear Systems with L^1-norm cost

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Keywords: Optimal Control, L^1-minimization, Linear Systems

In this paper, we study optimal control problems for finite dimensional linear systems, where control parameters take values in the Euclidean unit ball and the cost is the L^1-norm of the control function. We study both finite and infinite horizon problems. Main results concern the existence, uniqueness and structure of optimal solutions, as well as continuous dependence of the optimal cost on the data.

We also study some examples of two-dimensional linear systems. We describe optimal syntheses in infinite time horizon for the case of a free particle and for the harmonic oscillator. Finally we study all classes of two-dimensional hyperbolic systems.

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Escape Rate Games and Competitive Spectral Radii

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Keywords: Mean-Payoff Games, Joint Spectral Radius, Non-expansive Mappings, Growth Optimization

We introduce a new class of perfect information repeated zero-sum games in which the payoff of one player is the escape rate of a dynamical system which evolves according to a non-expansive nonlinear operator depending on the actions of both players. Formally, we fix two compact action spaces $\mathcal{A}$ and $\mathcal{B}$, a space state X equipped with a *hemi-metric* d (assymetric metric taking possibly negative values), an initial state $x_0 \in X$ and a family of non-expansive operators $(T_{ab})_{(a,b)\in\mathcal{A}\times\mathcal{B}}$, meaning that $d(T_{ab}(x), T_{ab}(y)) \leq d(x, y)$ for all $x, y \in X$. Two players, called *Min* and *Max*, alternatively choose actions in the sets $\mathcal{A}$ and $\mathcal{B}$. An infinite sequence of actions $(a_n, b_n)_{n\in\mathbb{N}^*}$ determines the following *escape rate*:

$$\limsup_{k\to\infty} \frac{d(T_{a_k b_k} \circ \cdots \circ T_{a_1 b_1}(x_0), x_0)}{k} .$$

Min wants to minize it whereas *Max* wants to maximize it. Taking X as the interior of a closed convex cone in $\mathbb{R}^n$ equipped with the Funk metric ([1]), we obtain subclasses of Matrix Multiplication Games [2], a 2-player extension of the joint spectral radius.

Under some continuity assumptions, we prove the existence of a *uniform value* ρ for our game, which we characterize by a two-player generalization of Mañe's lemma:

$$\rho = \max_v \inf_{x\in X} (Sv(x) - v(x))$$

where the maximum is taken over the set of Lipschitz functions v of constant 1, and S is the self-map of the space of Lipschitz functions given by $Sv(x) := \inf_{a\in\mathcal{A}} \sup_{b\in\mathcal{B}} v(T_{ab}(x))$. This extends the maximin characterization of[3],and allows us to show the existence of optimal strategies for both players. Additionnaly, having S act on the set of *distance-like* functions (f is *distance-like* if $\exists(\alpha, x^*) \in \mathbb{R} \times X$ such that $f(\cdot) \geq \alpha + d(\cdot, x^*)$), we provide a dual minimax characterization of the value, when d is the Funk metric, or when $(T_{ab})_{(a,b)\in\mathcal{A}\times\mathcal{B}}$ are *almost isometries*.

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APPLICATION OF QUANTUM COMPUTING TO SETTLE THE LARGE NUMBER OF FINANCIAL TRANSACTIONS

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Keywords: Discrete optimization, quantum computing, finance

A securities transaction is an exchange of securities versus payment in currency between two financial actors. The transaction is settled, if, inter alia, there are sufficient amounts of resources on the corresponding cash and securities accounts. Each night more than four hundred thousand transactions over a billion euro are settled in batches. Banque de France is in charge of Mathematical Optimization Module (MOM) [1], which is a component of a large European transactions settlement platform named T2S [3]. Given a batch and the limited time, MOM looks for a subset of transactions to settle whose size is as large as possible respecting all business constraints and taking advantage of some financial features reducing the number of failed transactions. It is composed of preparatory, construction and improvement solution phases. Since the batch size is too large, no algorithm can find neither optimal nor approximate solution for the entire batch on a conventional computer. Thus, MOM's preparatory phase splits a batch of transactions into the subsets to address smaller problems. The current splitting procedure has long runtime and its quality affects the final number of settled transactions. In this work, we present an approach that would improve the splitting results and the MOM's outcome. It is based on a Quadratic Unconstrained Binary Optimization problem solved by a Quantum Approximate Optimization Algorithm and implemented on the available quantum devices via Qiskit platform [2, 3].

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Optimal Operation of a Battery Energy Storage System Considering Cyclic Degradation and Revamping Strategies: a comparison of two formulations

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Keywords: battery storage system, optimal scheduling, cyclic degradation, capacity revamping, convex optimization

Battery Energy Storage Systems (BESS) are subject to inevitable capacity degradation phenomena due to frequent charging/discharging actions, whose dynamics are typically non-linear and difficult to be integrated into optimization problems. Nevertheless, neglecting this aspect may lead to an overestimation of the net revenues and to high-cost penalties for the inability of the BESS to provide the contracted services.

Within this context, we propose two deterministic scheduling models that co-optimize the *short-term* arbitrage operation (hourly) in the Wholesale Energy Market (WEM) and the *long-term* revamping actions (months) to restore the degraded capacity and comply with the requirements from the contracted services.

Both the models consider the objective function in (1) where the first term accounts for the revenues from selling and buying power [MW] (P_t^d, P_t^c respectively) at the λ_t market price [€/MWh] while the second term accounts for the revamping cost for buying new energy capacity R_t [MWh] at the market price θ_t [€/MWh].

$$\max_{P_t^c, P_t^d, S_t} [f(\lambda_t, P_t^c, P_t^d) - g(\theta_t, R_t)] \tag{1}$$

The dynamic evolution over time of the energy capacity E_t [MWh] is expressed by (2)

$$E_{t+1} = E_t - \gamma_t + R_t \tag{2}$$

where the energy capacity degradation γ_t [MWh] pursuant to cyclic actions is defined as in (3).

$$\gamma_t = \frac{\hat{E}}{2} \cdot \left| \frac{1}{\overline{N}(d_t)} - \frac{1}{\overline{N}(d_{t+1})} \right| \tag{3}$$

Here $\overline{N}(z)$ indicates the maximum number of full cycles which a BESS can perform at a certain Depth of Discharge (DoD) z before the capacity drops below 20% of the designed energy capacity $\hat{E}$ [MWh].

Even though both the models consider the same objective function (1), they rely on different mathematical frameworks and consider different re-formulation techniques to account for the above-mentioned constraints.

The first model is the Greedy Optimal Revamping Algorithm (GORA) which is based on a Dynamic Programming (DP) algorithm. This methodology allows to directly integrate the non-convex behaviour of cyclic degradation (3) and therefore consider different cycle-life curves. The feasibility of the contracted service is ensured by the *instantaneous* replacement of the degraded cells with brand new ones. The second model is the Optimal Capacity Revamping Algorithm (OCRA) which is formulated as a Mixed Integer Linear Programming (MILP) algorithm. Here, the cyclic degradation (3) is re-formulated with ad-hoc reformulation techniques, allowing for the integration of only specific families of cycle-life curves. Nevertheless, differently from the GORA, additional constraints have been added to allow for more flexible revamping policy where the system may *periodically* (e.g. every six months) purchase some extra (racks of) cells to increase its capacity.

Both the models have been developed using the Julia 1.7.3 programming environment and the simulations have been performed on an Intel® Core™ i5-52000 processor with 8.0 GB RAM. Concerning the OCRA, the optimization problem has been solved by means of the Gurobi 10.0.2 solver with a MipGap of 1%.

In both the cases, results have evidenced that considering the cyclic degradation while optimizing the system's operation lead to an increase in the net revenues (around 15%) at the end of the planning horizon. As a matter of fact, the models acknowledge the capacity degradation and therefore limit the cyclic action of the BESS during arbitrage operation in the WEM. This strategy, although decreasing the revenues from arbitrage operation, allows to abate the overall costs for the revamping actions.

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Stationary Regimes of Piecewise Linear Dynamical Systems with Priorities

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Keywords: Piecewise-linear dynamics, emergency organizations, Markov Decision Processes, Blackwell optimality

Dynamical systems governed by priority rules appear in the modelling of emergency organizations [1] or of road traffic [2]. These systems can be described by piecewise-linear discrete time-delay systems of the form

$$x_i(t) = \min_{a\in\mathcal{A}_i}\Big(r_i^a + \sum_{\tau\in\mathcal{T}}\sum_{j\in[n]}(P_\tau^a)_{ij}x_j(t-\tau)\Big), \quad i\in[n],\ t\geq 0\,.$$

Here, $x_i(t)$ represents the numbers of occurrences of an event i up to time t, the sets $\mathcal{A}_i$ are finite, $\mathcal{T}$ is a finite set of integer delays, and the r_i^a as well as the $(P_\tau^a)_{ij}$ are real numbers. In the above applications, the r_i^a are determined by the resource allocation, and the (possibly negative) $(P_\tau^a)_{ij}$ are determined by input flows, routing policies and priority rules.

A basic question is to show the existence of stationary solutions $x(t) = u+\rho t$, with $u, \rho \in \mathbb{R}^n$. Then, the vector ρ determines the throughput. Moreover, by computing ρ as a function of the parameters r_i^a, we obtain a phase diagram showing the possible congestion phases, as a function of the resource allocation. The existence of stationary solutions has been observed in concrete applications, but establishing the existence in a more general setting has remained open.

We show that stationary regimes do exist under conditions involving the linear parts of the piecewise-linear dynamics. These conditions hold in particular when the spectra of these linear parts are included in the unit disk with 1 a semisimple eigenvalue (or not an eigenvalue). This extends a theorem of Kohlberg [3] on the existence of invariant half-lines of nonexpansive piecewise-linear dynamical systems to a class of dynamics that are not necessarily nonexpansive. The proof relies on topological degree theory, and it builds on the idea of "Blackwell optimality" from the theory of Markov decision processes.

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A Search-Free $O(1/k^{3/2})$ Homotopy Inexact Proximal-Newton Extragradient Algorithm for Monotone Variational Inequalities

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Keywords: Variational Inequalities, Monotone Inclusions, Proximal-Newton Algorithm, Iteration-Complexity

We present and study the iteration-complexity of a relative-error inexact proximal-Newton extragradient algorithm for solving smooth monotone variational inequality problems in real Hilbert spaces. We removed a search procedure from Monteiro and Svaiter (2012) by introducing a novel approach based on homotopy, which requires the resolution (at each iteration) of a single strongly monotone linear variational inequality. For a given tolerance $\rho > 0$, our main algorithm exhibits pointwise $O\left(\frac{1}{\rho}\right)$ and ergodic $O\left(\frac{1}{\rho^{2/3}}\right)$ iteration-complexities.

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Safe treatment of infeasible convex optimization problems via the augmented Lagrangian

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Keywords: Augmented Lagrangian, Convex Optimization, Infeasibility

This work focuses on constrained convex optimisation problems i.e. the form

$$\min_{x \in \mathcal{X}} f(x) \tag{1}$$
$$\text{s.t. } C(x) \in \mathcal{D} \tag{2}$$

where f, $\mathcal{X}$, and the feasible set $\{x \ , \ C(x) \in \mathcal{D}\}$ are convex. Various popular algorithmic strategies were designed to tackle such problems by solving sequences of unconstrained optimisation problems. That is, for instance, the case of interior point methods and augmented Lagrangian algorithms. The augmented Lagrangian [1] in particular is practically efficient and has strong theoretical guarantees under minimal assumptions when the feasible set associated to the constraints (2) is non-empty.

The setting where the constrained optimization problem is infeasible, however, only recently started to attract attention. This situation is particularly relevant for optimal control (e.g., Model Predictive Control [4]) and machine learning (e.g., neural networks using convex optimization layers [5]) where infeasible problems are commonly encountered. Hence, algorithms under consideration should remain stable also when dealing with such problems.

Recent literature studies this problem under various sets of assumption [2, 3, 1]. In this work, we analyze the general case with convexity as the sole assumption. Our approach is based on the classical relationship between the augmented Lagrangian algorithm and the dual proximal point algorithm.

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Evolutionary Algorithms Are Robust to Noise out of the Box

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Randomized search heuristics (RHSs) are generally believed to be robust to noise. However, almost all mathematical analyses on how RSHs cope with a noisy access to the objective function assume that each solution is re-evaluated whenever it is compared to others. This is unfortunate, both because it wastes computational resources and because it requires the user to foresee that noise is present (as in a noise-free setting, one would never re-evaluate solutions).

In this work, we show the need for re-evaluations could be overestimated, and in fact, detrimental. For the classic benchmark problem of how the $(1+1)$ evolutionary algorithm optimizes the LeadingOnes benchmark, we show that without re-evaluations up to constant noise rates can be tolerated, much more than the $O(n^{-2} \log n)$ noise rates that can be tolerated when re-evaluating solutions.

This first runtime analysis of an evolutionary algorithm solving a single-objective noisy problem without re-evaluations could indicate that such algorithms cope with noise much better than previously thought, and without the need to foresee the presence of noise.

Eddy current imaging of defects inside a steam generator with an inverse method

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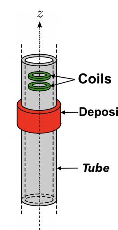
Keywords: PDE model, inverse problem, Eddy current imaging, shape optimization, gradient descent algorithm, level-set method

Eddy current probes are used to inspect the steam generator tubes inside a nuclear plant, identifying defects like deposits, wear, or cracks. Our study focuses on the inverse problem of reconstructing these defect shapes from probe measurements, starting with simulating eddy current data via a PDE model:

$$\begin{aligned} \nabla \times \mathbf{E} - i\omega\mu\mathbf{H} = 0 \text{ in } \Omega \\ \nabla \times \mathbf{H} - i\omega\sigma\mathbf{E} = \mathbf{S} \text{ in } \Omega \end{aligned} \tag{1}$$



where ω is the angular frequency, μ is the magnetic permeability, and σ is the conductivity.

Next, we optimize a defect shape by minimizing the residual between simulated and measured data, using a gradient descent algorithm with a level-set method for shape representation. We address deposits with both conductivity and magnetic permeability changes, which cause boundary instabilities, unlike prior work [1, 2] that focused on conductivity changes only. We then extend this approach to reconstruct cracks.

We explore two different methods for evaluating the derivative with respect to a shape:

- A surface-based shape gradient, the classic shape optimization approach from the Hadamard structure theorem. This shape gradient relies on the defect boundary and responds only to normal directional changes of the shape.
- A domain-based shape gradient which provides robustness to boundary instabilities and enables tangential adjustments of the shape. This is crucial for crack reconstruction where sensitivity to crack size and tip extension or contraction is essential.

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SDP Relaxations for polynomial Single-Leader-Multi-Follower Games

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A single-leader-multi-follower game (SLMFG) is an extension of a bilevel optimization problem, where the interaction between only one leader and one follower is considered. In this work we show that the powerful reformulation technique of describing the SLMFGs as equivalent so-called Mathematical Programs with complementarity con- straints (MPCC) extends more generally to SLMFGs, where the constraint functions are neither convex nor even quasiconvex, and present an equivalent first-order reformulation in terms of concatenated Karush-Kuhn-Tucker systems as well an MPCC reformulation. We provide simple examples illustrating the significance of our assumptions and reformulations. In the case where the functions involved in the games are polynomials, we establish that the global optimal value of SLMGs is the limiting optimal value of a sequence of semi-definite program relaxations under suitable conditions. This is obtained by equivalently transforming the follower games into systems of polynomials constraints and employing the celebrated Putinar's Positivstellensatz to derive semi-definite program relaxations. We also show by examples how polynomial SLMFGs can be solved via semi-definite programming methods.

Comparison between Robust Optimization, Stochastic Programming and Distributionally Robust Optimization for Unit Commitment under uncertainty

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Keywords: Optimization under uncertainty, Unit Commitment, Distributionally Robust Optimization, Robust Optimization, Stochastic Programming

The Unit Commitment (UC) problem, a key challenge for energy producers and network operators, involves determining the optimal production schedule for electricity generation units, such as thermal and hydro units, while accounting for both technical and economic constraints [1]. A significant source of complexity in UC arises from uncertainty, particularly in forecasting energy demand and wind power generation. In this study, we focus on demand uncertainty and adopt a two-stage approach, with commitment decisions made in the first stage. Traditional optimization techniques, including robust optimization and stochastic programming, have been widely applied to address uncertainty in UC. However, a newer approach, Distributionally Robust Optimization (DRO), has gained attention, particularly DRO methods based on ϕ-divergences [2] and the Wasserstein distance [3, 4]. Among these, the L^2 Wasserstein distance is unexplored in the UC context.

In this talk, we first provide new algorithms to address this new DRO case. We then examine and compare stochastic, risk-averse, robust and DRO approaches for UC under uncertainty, evaluating their potential advantages in term of cost, robustness and reliability of solutions.

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A Stationary Mean-Field Equilibrium Model of Irreversible Investment in a Two-Regime Economy

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Keywords: mean-field stationary equilibrium; irreversible investment; regime-switching; market concentration; value of economic stability.

We consider a mean-field model of firms competing *à la Cournot* on a commodity market, where the commodity price is given in terms of a power inverse demand function of the industry-aggregate production. Investment is irreversible and production capacity depreciates at a constant rate. Production is subject to Gaussian productivity shocks, while large non-anticipated macroeconomic events driven by a two-state continuous-time Markov chain can change the volatility of the shocks, as well as the price function. Firms wish to maximize expected discounted revenues of production, net of investment and operational costs. Investment decisions are based on the long-run stationary price of the commodity. We prove existence, uniqueness and characterization of the stationary mean-field equilibrium of the model. The equilibrium investment strategy is of barrier-type and it is triggered by a couple of endogenously determined investment thresholds, one per state of the economy. We provide a quasi-closed form expression of the stationary density of the state and we show that our model can produce Pareto distribution of firms' size. This is a feature that is consistent both with observations at the aggregate level of industries and at the level of a particular industry. We provide evidence that persistent periods of economic downturn increase market concentration. We demonstrate that firms with slowly depreciating production capacities are better off in a stable, average economy, whereas firms with quickly depreciating assets can benefit from sequences of boom and bust.

A Column Generation Approach for the Routing of Electricity Technicians

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Keywords: Column Generation, Dynamic Programming, Vehicle Routing

The maintenance of an electricity distribution network involves numerous daily technical interventions. In this problem, we are given a set of interventions each with associated time windows, location, necessary skills and duration, as well as a set of teams of technicians with associated set of skills. We need to find feasible routes on the interventions for each team, considering the time windows and skills, and ensure that each team returns to its departure depot before the end of the day. The primary objective is to maximize the total duration of completed interventions and as a secondary objective, we aim to minimize the overall routing cost. This problem can be formulated as a capacitated vehicle routing problem with time windows. Due to the large number of teams and interventions, this results in a large scale optimization problem, and its operational nature limits the time available for exact solving. Here, we propose a column generation approach where one subproblem per vehicle has to be solved and each potential route of a vehicle is considered as a new column in the master problem. To generate these routes, we rely on dynamic programming. Real world instances from EDF (Electricité de France) of historical technicians' interventions will be used to evaluate the effectiveness of the proposed methods.

Optimizing the coalition gain in Online Auctions with Greedy Structured Bandits

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Keywords: Bandit, Auctions, Online learning

Motivated by online display advertising, this work considers repeated second-price auctions, where agents sample their value from an unknown distribution with cumulative distribution function F. In each auction t, a decision-maker bound by limited observations selects n_t agents from a coalition of N to compete for a prize with p other agents, aiming to maximize the cumulative reward of the coalition across all auctions. The problem is framed as an N-armed structured bandit, each number of player sent being an arm n, with expected reward $r(n)$ fully characterized by F and $p+n$. We present two algorithms, Local-Greedy (LG) and Greedy-Grid (GG), both achieving *constant* problem-dependent regret. This relies on three key ingredients: **1.** an estimator of $r(n)$ from feedback collected from any arm k, **2.** concentration bounds of these estimates for k within an estimation neighborhood of n and **3.** the unimodality property of r under standard assumptions on F. Additionally, GG exhibits problem-independent guarantees on top of best problem-dependent guarantees. However, by avoiding to rely on confidence intervals, LG practically outperforms GG, as well as standard unimodal bandit algorithms such as OSUB or multi-armed bandit algorithms.

Some Preliminary Insights on Project *GAALACTIC*: (Provably) *G*ood *A*pproximation *AL*gorithms for *A*dministrating *C*urtailments in the *T*elecommunications *I*ndustry *C*ontext

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Keywords: Approximation Algorithms, Optimal Use of Batteries, Load Curtailments

Our purpose here is to present a newly funded project, called *GAALACTIC*. The main topic of this project is to study, from a theoretical point of view, approximation algorithms for the problem of optimizing the use of a battery installed in a telecommunications network, provided that this battery can be used to perform so-called *load curtailments* (or simply *curtailments*).

Such a battery is typically installed in a telecommunications network as backup, in order to prevent customers from being impacted by potential power outages. However, other uses of batteries have also emerged in the past few years, in order to make them economically more appealing. As an example, a network operator may also wish to use the batteries of its own network to perform curtailments: in this case, in exchange for a given reward, this network operator agrees to avoid buying (and hence consuming) a given amount of electricity from the grid for a given amount of time. In order to do so, the network operator then has to use its own batteries, and at the same time to ensure some safety usage rules, as well as some market rules.

Recently, a series of papers have addressed this problem (or similar ones) with one or more batteries. In particular, a MILP formulation, as well as a graph-based heuristic, were proposed in [1] for the case with only one battery, and compared experimentally. However, while this heuristic can indeed be parameterized experimentally in order to provide solutions with improved quality, but also with a bigger running time, no theoretical study of its performance has been done yet. In this project, our main goal is thus to determine whether such a heuristic could be turned into a proper approximation algorithm that computes solutions with guaranteed quality.

This may involve determining optimal values for some of its parameters and/or slightly modifying some of its steps, but also writing down a formal analysis. We will review several preliminary results, namely some useful structural properties of optimal solutions, a first (but for now incomplete) rounding scheme having some nice characteristics, and some insights on which features the final algorithm should have, before concluding on what remains to be done.

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Mean Field Games in a Stackelberg problem with an informed major player

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Keywords: Mean Field Games, Stackelberg Games, Partial Information

We investigate a stochastic differential game in which a major player has a private information (the knowledge of a random variable), which she discloses through her control to a population of small players playing in a Nash Mean Field Game equilibrium. The major player's cost depends on the distribution of the population, while the cost of the population depends on the random variable known by the major player. Our main results are the following: first we rewrite the problem in a relaxed form and show that the major player has an optimal strategy for the game. This relaxed formulation involves the theory of MFG with a common noise (common noise being here the information disclosed by the major player) as developed in [1]. Second, we show that such an optimal strategy is still approximatively optimal in Stackelberg games with finite many small players. The fact that the optimal strategy in a mean field game provides an approximated Nash equilibria for the associated game with finitely many players is very classical: see [2, 3] for instance. The extension of such a property to Stackelberg equilibria in MFG with a large player has only been handled very recently in [4, Theorem 2.20] in a very general framework, but quite different from ours: indeed in [4], the major player can choose among the Nash equilibria of the $N-$player game, while in our set-up it is more natural that she has to handle any such Nash equilibrium. This difference leads us to show the approximate optimality of the control for the major player when the small players play *any* (approximate) Nash equilibria.

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Scheduling Autonomous Buses for On-Demand Transportation with Crossing Constraints

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Keywords: Autonomous vehicles, On-demand transportation, Bus scheduling

In order to reduce air pollution and improve the quality of life of their residents, many cities are choosing to develop public transportation, offering an alternative to the private cars. This is the case in Nantes, where the SNCF is replacing disused rails with an autonomous bus lane, in order to set up an autonomous bus system. During off-peak periods, these buses would provide transportation on demand, in order to meet user needs as effectively as possible with a small fleet of vehicles. In this abstract, we propose a method for managing bus timetabling.

The topology of the line is special, as the vehicles travel on a straight line where they can turn around at certain predefined points. In addition, certain sections of the track are too narrow for vehicles to cross, so an order of passage must be defined.

The problem studied includes a routing problem, since passengers have to be assigned to vehicles and the vehicle itinerary has to be determined, and a scheduling problem, since the timetable and order of passage on single-track sections of the track must be decided. This is a dynamic problem because requests may appear during the time horizon under consideration. This problem has several objectives, which are dealt with in the following order: (1) Minimize the number of used vehicles, (2) Minimize the distance covered by the vehicles, (3) Minimize lateness/earliness for passengers.

To address this problem, we propose an insertion algorithm. When a request is emitted, several feasible insertions are generated and ranked according to the routing objectives (objectives 1 and 2). Time windows are constructed based on the desired arrival time of each request, and then extended to each stage of the vehicle's route using forward and backward propagation as explained in [1], allowing feasibility to be checked. Each feasible *a priori* insertion is then added to a disjunctive graph modelling the vehicle scheduling problem. As similar problems are NP-hard ([2], [3]), we treat the scheduling problem with a linear programming model which is solved using a commercial solver, minimizing the scheduling objective (objective 3).

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Incremental Learning in Diagonal Linear Networks

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Keywords: theory of neural networks, linear networks, incremental learning

Diagonal linear networks (DLNs) are a toy simplification of artificial neural networks; they consist in a quadratic reparametrization of linear regression inducing a sparse implicit regularization. In this talk, I will describe the trajectory of the gradient flow of DLNs in the limit of small initialization. I will show that incremental learning is effectively performed in the limit: coordinates are successively activated, while the iterate is the minimizer of the loss constrained to have support on the active coordinates only. This shows that the sparse implicit regularization of DLNs decreases with time.

A Domain-Specific Modelling Language for a Multi-Energy Planning Tool

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Keywords: Domain-Specific Modelling Language, Information Structure, Multi-Energy Systems, Long Term Adequacy, Generation and Transmission Expansion Planning

Energy systems evolve rapidly to meet the ambitions of carbon neutrality, with an increasing share of renewable energy sources and increasing couplings between sectors. Capacity expansion planning and long-term adequacy studies are essential to assess the security of supply, and build economically consistent prospective mixes [1].

In an expansion planning problem, there are two levels of uncertainties: at the planning level (structural hypothesis on the mix, political decisions, ...) and at the operational level (demand, failures of generation assets, ...).

At the planning level, structural evolutions of the mix can be represented with a scenario tree. At the operational level, we may consider different modellings to capture the appropriate dynamics and uncertainties. As highlighted in [2], most long-term power system modelling tools use parallel Monte-Carlo simulations to take the uncertainty into account. This implies that for every given scenario, the realization of uncertainties such as the load or the renewable generation is known over all the optimization period. When there is little uncertainty on these quantities, this computation is relevant. However, as the variability in the system increases, this modelling may become inadequate. For instance, the decision to turn on a long-to-start thermal unit may be taken before knowing the exact value of wind production few hours later. Then the level of thermal generation can be adjusted once the wind production is known.

To improve the efficiency in conducting studies, modelling tools must allow to easily change the core structure of the model depending on the users' need: we want to easily configure a scenario tree or change the information structure of the operational level problem without coding in the tool.

To do so, we derive a Domain-Specific Modelling Language (DSML) from a generic formulation of long-term adequacy and planning problem. This DSML provides flexibility in the definition of the time structure, the assets dynamics and the information structure. We can also easily define scenario trees and couplings between their nodes. We present use cases to show how the DSML improves the efficiency in conducting studies by sharing modelling efforts.

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Sensitivity Analysis and Optimal Control for the Chemical Master Equation

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Keywords: Chemical Master Equation, continuous-time Markov chains, optimal control

We consider a chemical system composed of d species $\mathsf{X}_1, \dots, \mathsf{X}_d$ interacting via R reactions,

$$\text{for } 1 \leq r \leq R, \quad \mathsf{R}_r : \alpha_r \cdot \mathsf{X} \xrightarrow{a_r(t,x)} \beta_r \cdot \mathsf{X}, \quad \text{with } \mathsf{X} = (\mathsf{X}_1, \dots, \mathsf{X}_d).$$

Each reaction R_r is characterized by its stoichiometric coefficients $\alpha_r, \beta_r \in \mathbb{N}^d$ and its reaction rate $a_r(t,x)$. This reaction rate may depend on time $t \geq 0$ and on the state $x \in \mathbb{N}^d$ of the system. Being in state $x = (x_1, \dots, x_d)$ means that there are x_i molecules of species X_i, for all $1 \leq i \leq d$. Such a chemical system may be described by a continuous-time Markov chain $X(t)$, with law $p(t,x) = \mathbb{P}[X(t) = x]$, the probability to be in state x at time t. This law satisfies a Kolmogorov equation, called the Chemical Master Equation (CME) in this context,

$$\frac{\partial p}{\partial t}(t,x) = \sum_{r=1}^{R} \Big(a_r(t, x - \nu_r) p(t, x - \nu_r) - a_r(t,x) p(t,x) \Big), \quad \text{with } \nu_r = \beta_r - \alpha_r. \tag{CME}$$

This equation is used in biology (cf. [1]) to model chemical reactions taking place inside cells, in order to take into account the randomness of these reactions, in a context where the number of molecules involved is too low to make a continuous approximation of the state space $\mathcal{S} = \mathbb{N}^d$. Thus, the CME is a countable collection of ordinary differential equations.

In the simple case of bounded reaction rates a_r, the Cauchy-Lipschitz theorem ensures the existence and uniqueness of the solution, while the implicit function theorem allows to conduct a sensitivity analysis of this solution with respect to the data (initial condition, reaction rates, ...). However, practical applications in biology lead to consider reaction rates that are proportional to the number of reactants and therefore unbounded. In this framework, the uniqueness of the solution is no longer guaranteed. One may nevertheless use the appropriate (in a physical sense) notion of minimal solution (cf. [2]).

Using stability estimates for the minimal solution, we establish la differentiability of this solution with respect to the data. We then give an expression for the directional derivative involving an adjoint equation, for which we also define a concept of minimal solution. This allows us to derive optimality conditions (in the form of a Pontryagin's principle) for a class of optimization problems for which we control the reaction rates.

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Valuation of aFRR Activation in Germany for a Standalone Battery

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Keywords: Battery valuation, aFRR activation, Intraday market, Deterministic optimization

Our work aims to decompose the sources of revenue for a battery operating in various markets in Germany and to identify the most profitable ones. This involves backtesting an optimization strategy using historical data. Our approach simulates consecutive optimizations in the capacity market, the spot market, the intraday market, and the aFRR activation market within a deterministic framework. We demonstrate that, although the intraday and aFRR activation markets are significant sources of revenue during the period considered (mid-2022 to mid-2023), the additional cycling caused by aFRR activation should be managed with an appropriate bidding strategy.

Resolution of a 9-nodes graph coloring instance using a QAOA based approach color instances of graph coloring problem

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The graph coloring problem involves assigning a color to each vertex in a given graph $G(V,E)$ so that no two adjacent vertices share the same color, while minimizing the total number of colors used. Graph coloring is one of the oldest topics in graph theory. Such approach has been successfully applied to TSP [1] and Flow-Shop [2] using the subexceedant function [3] that gives a decomposition in the factorial base. Here, we introduce a rank representation that can be converted into a color assignment for the nodes, resulting in a valid graph coloring and qubit strings are used to model the rank.

By using a meta-heuristic-based approach, we derive a probability distribution that favors high-quality solutions, specifically those with very few colors, aiming for the optimal solution with the minimal number of colors. The effectiveness of this modeling is demonstrated on a 9-node graph, showing a superior probability distribution compared to Hadfield's driven Hamiltonian. Additionally, since the Hamiltonian is used to model rank, it requires a low number of gates, making IQAOA (Iterative Quantum Approximate Optimization Algorithm) suitable for simulations based on noisy environments. To conclude IQAOA is an alternative to the classical QAOA based approach [4] [5].

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Moderate Exponential-time Quantum Dynamic Programming Across the Subsets for Scheduling Problems

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Two key types of quantum algorithms are used to tackle combinatorial optimization problems: heuristic and exact. Heuristic methods, like Variational Quantum Algorithms [1] are hybrid quantum-classical algorithms (e.g., QAOA), tackle problems such as MAX-CUT, Traveling Salesman, or MAX-3-SAT, reformulated as QUBO models. However, with noisy quantum computers, no quantum advantage has been demonstrated yet due to small problem instances and the empirical nature of these heuristics. In contrast, exact quantum algorithms already offer theoretical speed-ups by providing optimal solutions with a high probability of success. Algorithms like Grover's Search [2] deliver quadratic speed-ups for specific search problems, forming the basis for numerous exact algorithms. For example, Durr and Hoyer [3] use Grover's Search in Quantum Minimum Finding (QMF), a hybrid quantum-classical algorithm to find minimum of an unsorted table. Later Ambainis et al [4] combine QMF with in dynamic programming to address NP-hard problems like the Traveling Salesman Problem and the Minimum Set Cover problem. The problems of interest must satisfy a property allowing them to be solved using classical dynamic programming in $O^*(c^n)$ where c is naturally not smaller than 2. Ambainis et al. introduced a hybrid algorithm that reduces the complexity to $O^*({c_{quant}}^n)$ for $c_{quant} < c$. For example, the classical Held and Karp dynamic programming algorithm solves the Traveling Salesman Problem (TSP) in $O^*(2^n)$, while the hybrid algorithm of Ambainis reduces this to $O^*(1.728^n)$.

This work aims to provide a generalized hybrid quantum-classical algorithm that adapts the seminal idea of Ambainis et al. to a broader class of problems, particularly those that can be tackled using Dynamic Programming Across Subsets (DPAS). The focus is on NP-hard scheduling problems, which involve finding optimal assignments of jobs to machines over time, subject to various constraints and objectives. The algorithm adapts the classical dynamic programming structure by incorporating quantum speed-up techniques reducing the exponential time complexity for several scheduling problems, including those with precedence constraints, release times, and weighted tardiness. New Results are described in the following table, for more references refer to [5].

Problem	Our hybrid algorithm	Best classical algorithm
$1\|\tilde{d}_j\|\sum w_j C_j$	$\mathcal{O}^*\left(\sum p_j \cdot 1.728^n\right)$	$\mathcal{O}^*(2^n)$, T'kindt et al. [32]
$1\|\|\sum w_j T_j$	$\mathcal{O}^*\left(\sum p_j \cdot 1.728^n\right)$	$\mathcal{O}^*(2^n)$, T'kindt et al. [32]
$1\|prec\|\sum w_j C_j$	$\mathcal{O}^*\left(1.728^n\right)$	$\mathcal{O}^*((2-\epsilon)^n)$, for small ϵ, Cygan et al. [6]
$1\|r_j\|\sum w_j U_j$	$\mathcal{O}^*\left((\sum w_j)^3 \cdot \sum p_j \cdot 1.728^n\right)$	$\mathcal{O}^*(\sum w_j \cdot \sum p_j \cdot 2^n)$, Ploton and T'kindt [25]
$1\|r_j\|\sum w_j C_j$	$\mathcal{O}^*\left((\sum w_j)^3 \cdot (\sum p_j)^4 \cdot 1.728^n\right)$	$\mathcal{O}^*(\sum w_j \cdot (\sum p_j)^2 \cdot 2^n)$, Ploton and T'kindt [25]
$F3\|\|C_{\max}$	$\mathcal{O}^*\left((\sum p_{ij})^4 \cdot 1.728^n\right)$	$\mathcal{O}^*(3^n)$, Shang et al. [28], Ploton and T'kindt [26]

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Witsenhausen Model for Leader-Follower Problems in Energy Management

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Keywords: leader-follower, demand response, energy

With the transition from centralized, controllable energy sources to decentralized, renewable options, managing energy consumption has become more complex. The unpredictability of renewable sources like solar and wind complicates balancing supply and demand. As a result, demand response has emerged as a key aspect of energy management [1], where the consumer (follower) adjusts his electricity consumption based on the prices set by the supplier (leader).

After briefly recalling the Witsenhausen intrinsic model (W-model) in its general form, we adapt it to analyze this leader-follower dynamic [3], where the follower makes decisions based on the actions of the leader, in the context of demand response. First, we focus on Nature, which includes both players' private knowledge, such as the supplier's production costs and the consumer's unwillingness to shift his consumption, as well as exogenous factors like weather conditions that impact heating demand and renewable energy production.

Next, we introduce W-games, also called Games in Product Forms, derived from the W-model, where each player has personal data, that is an objective function and a belief about Nature. We then apply game theory concepts, examining Nash and Stackelberg equilibria [2]. This analysis delves into the players' decision-making processes and links them to bilevel optimization [4], highlighting the often overlooked information structure. After that, we explore how the W-model framework can be adapted to address more complex problems, notably multi-leader-multi-follower cases. By reformulating such problems from the literature, we aim to offer a clearer and more systematic approach to demand response problems in energy management.

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Rochet-Choné Model, Bi-Level Optimization, and Quantization for Electricity Pricing

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Keywords: Pricing, Rochet-Choné Model, Bilevel Optimization, Quantization.

Pricing strategies in electricity markets are crucial for optimizing supply and demand dynamics, ensuring that both producers and clients make effective decisions. These strategies help manage grid stability, encourage efficient energy use, and promote competition among suppliers, ultimately benefiting clients through more competitive prices and better options.

In our presentation, we model the pricing problem using a Principal-Agent framework under adverse selection: the Rochet-Choné model [2]. This model addresses pricing in a monopolistic market under reservation constraints and captures the interaction between the provider and consumers. The goal is to determine tariffs that balance profitability with social welfare. To solve this problem, we discretize the client space and solve the corresponding bi-level problem using the value function approach.

Additionally, we introduce quantization methods aiming to clarify and improve the understanding of the overall supply offered by the electricity provider[1]. By utilizing a menu of pricing offers (contracts) derived from the Rochet-Choné model, we demonstrate how to identify the subset of contracts that best represents the initial menu. On one hand, we present this as a mixed-integer bi-level optimization problem, and on the other hand, we introduce greedy heuristics to compute the reduced menu, including the use of the Newton polygon and Fenchel conjugacy. To evaluate the methods' performance and practicality, we conducted experiments on instances generated by real life data provided by EDF. Our findings show that reducing the contract menu size from 100 to 10 results in only a 4% decrease in the objective value, while reducing it to 20 contracts leads to just a 1% decrease in the objective value with our best method in dimensions 2 and 3.

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Mean field optimal control with piecewise deterministic Markov processes

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Keywords: Optimal control, mean field, smart charging, generalized conditional gradient algorithm, piecewise deterministic Markov processes.

This presentation addresses the theoretical resolution and numerical approximation of a mean field optimal control problem for piecewise deterministic Markov processes, a class of non-diffusion stochastic models introduced in [1]. This problem models the optimal charging of a large fleet of electric vehicles.

Optimality conditions are obtained through a linearization procedure inspired from [2] and are studied via a system of coupled partial differential equations, similar to those encountered in mean field games.

The problem is discretized using an explicit finite difference scheme and solved numerically with the generalized conditional gradient algorithm. A linear convergence result, comparable to the one found in [3] for diffusion processes, shows that algorithm's convergence rate and constants are independent of the discretization parameters. Our proof technique leverages the strong convexity of part of the objective function on the set of measures/controls pairs satisfying the discrete continuity equation.

Finally, this problem is part of a broader class of problems studied both theoretically and numerically in [4] and motivated by their applications in smart charging. Specifically, it can be applied to curtailment problems.

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Particle Methods for High-dimensional Multi-marginal Optimal transport

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Keywords : multi-marginal transport, partial optimal transport, risk management

Optimal transport offers a convenient way to model some risk estimation problems [1], the goal of which is to find a joint probability measure which maximizes a certain cost that belongs to the class of spectral risk measure. These costs are (highly) nonlinear, since the quantity of interest is not the *mean* risk, but rather a weighted average giving greater weight to the highest dangers. We focus on a specific spectral risk measure called *expected shortfall*, or *conditional value at risk*, namely the average danger in the worst $q\%$ of cases, with $q \in (0, 1)$ a fixed proportion. For this cost, the problem reads as a partial multi-marginal optimal transport problem, which we study in the case of unidimensional, compactly supported, absolutely continuous marginals.

A key result in transport theory is that that under certain conditions, solutions are supported on a low-dimensional set (typically, the graph of a function). This greatly motivates the idea to numerically solve high-dimensional transport problems using *particle methods*. Roughly speaking, instead of looking for an approximate solution supported on a fixed grid, where the weights of the various points are to be optimized, we restrict ourselves to measures that are *uniform* linear combinations of a fixed number of Dirac masses, the positions of which we optimize. In the former method, most Dirac masses will get zero weight due to the low-dimensional support of the solution, hence considerably reducing the quality of the discretization for a given grid size. On the other hand, the Lagrangian approach of the latter method ensures that the vast majority of Dirac masses actively contribute to the approximate solution. The particle-method discretized problem can be shown to Gamma-converge to the original problem as the number of Dirac masses tends to infinity.

We also address the issue of dealing with the constraints on the marginals within the framework of this particle method. As we have a partial transport problem, these are inequality constraints, and we take them into account by adding a penalization term to the objective function. The latter term consists in the sum of the squared 1-dimensional partial Wasserstein transport costs from each marginal to the (fixed) probability density by which it should be bounded in the original problem. We briefly explain why a 2-dimensional regularization seems to be necessary to numerically compute these 1-dimensional partial transport costs.

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The Geometry of Sparsity-Inducing Balls

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Keywords: sparsity, ℓ_0 pseudonorm, orthant-monotonic norm

One formulation in sparse optimization consists in seeking an optimal solution among vectors with at most k nonzero coordinates. This constraint is hard to handle, and a strategy to overcome that difficulty amounts to adding a norm penalty term to the objective function. The most widely used penalty is based on the ℓ_1-norm which is recognized as the archetype of sparsity-inducing norms [2].

In this talk, we present k-support norms, generated from a given source norm, and show how they contribute to induce sparsity via support identification. In case the source norms are the ℓ_1- and the ℓ_2-norms, we analyze the faces and normal cones of the unit balls for the associated k-support norms and their dual top-k norms.

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Asymptotic log-Sobolev constants and the Polyak-Łojasiewicz gradient domination condition

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Keywords: functional inequalities, Polyak-Łojasiewicz inequality, log-Sobolev, asymptotic, small temperature

The Polyak-Łojasiewicz (PL) constant for a given function exactly characterizes the rate of convergence of gradient flow uniformly over initializations, and has been of intense recent interest in optimization and machine learning because it is strictly weaker than strong convexity yet implies many of the same results. In the world of sampling, the log-Sobolev inequality plays a similar role, governing the convergence of Langevin dynamics from arbitrary initialization in Kullback-Leibler divergence. In this talk, we present a new connection between the PL constant and log-Sobolev inequalities by studying the latter in the small temperature limit. Our main result is that the two are equal under mild assumptions. In the course of our proof, we also establish some non-asymptotic estimates for the log-Sobolev and Poincaré constants of PL functions which may be of independent interest.

Random Cell Exploration and its applications to linear bilevel programming

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Keywords: Random Cell Exploration, Linear bilevel programming, Polyhedral geometry, Piecewise linearity

In this talk, we present a new heuristic to deal with piecewise linear problems, that we call the Random Cell Exploration method (RCEM). The method is based over the principle that the piecewise structure in general is hard to compute, but for (almost) any fixed point, the region containing the point where the problem is linear might be tractable to determine. This work is motivated by the previous contribution [3], where piecewise linearity has been applied to stochastic linear bilevel programming. A preliminary form of RCEM was developed, by regarding the stochastic linear bilevel problem as a particular case of the Bayesian approach of [4], first introduced by Mallozzi and Morgan in [2]. This work have been developed together with A. Svensson (Universidad de O'Higgins), G. Muñoz (Universidad de Chile), and Gonzalo Contador ()Universidad Técnica Federico Santa María).

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Global Optimization Algorithm through High-Resolution Sampling

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Keywords: Global Optimization, Accelerated Methods, Langevin Dynamics, Sampling Algorithms, Logarithmic Sobolev Inequality

We present an optimization algorithm that can identify a global minimum of a potentially nonconvex function with high probability, assuming the Gibbs measure of the function satisfies a logarithmic Sobolev inequality. Our global optimization method is built on a sampling algorithm, drawing inspiration from both overdamped and underdamped Langevin dynamics, as well as from the high-resolution differential equation known for its acceleration in deterministic settings. While the focus of the work is primarily theoretical, we demonstrate the effectiveness of our algorithm on the Rastrigin function, where it outperforms recent approaches. This talk is based on the paper [1].

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Is Maze-Solving Parallelizable?

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Keywords: metrical task systems, copllective exploration, online algorithms, advice complexity, decision making

Can you find a short path, without a map? Is maze-solving parallelizable? These questions can be cast formally using the toolbox of competitive analysis, a toolkit that was developed since the 1990s to study online decision-making. In this talk, I will present two problems that are known as "layered graph traversal" (introduced by the literature on online algorithms) and "collective tree exploration" (introduced by the literature on distributed algorithms). We will describe the connexions between these problems and use them as a case study of challenges arising in decision-making such as: collective strategies, advice complexity and switching costs. The techniques discussed in the talk are mainly based on online convex optimization.

This talk is based on joint works with Laurent Massoulié [1] and Laurent Viennot [2] at Inria and Xingjian Bai and Christian Coester at Oxford University [3] as well as on [4].

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Towards Effective Datasets for Training Data-driven Models for Smart Grid Security Assessment

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Keywords: Smart Grids, Data-driven Methods, Security Assessment, Training Data

Due to the decentralisation of energy resources and the electrification of the heat and transport sectors, it is becoming increasingly more difficult for network operators to supervise and manage smart grids. Research points at the development of decentralised energy management tools (e.g., based on artificial intelligence) to develop strategies for the individual decision-making of numerous agents (e.g., smart electric vehicle charging), that aggregated respect the physical constraints of the grid. To train these tools it is therefore necessary to foresee problematic operational states (e.g., under/overvoltage, exceeding the rated powers of line/transformer) which could damage equipment, trigger protections, and cause service disruptions. While this can be done through power flow simulations (i.e., solving a system of non-linear physical equations with a numerical solver [1]), their high computational cost hinders computing speed. An alternative is using a traditional power system simulator to generate labelled datasets in an off-line stage, and then training a data-driven model to act as a surrogate for a fraction of the computational cost of the conventional tools [2]. In this manner, the computing time associated with the training of the AI-based energy management tools is greatly reduced. We compared three state of the art data-driven approaches to identify if an operational point (i.e., electricity demand at each consumption point) is classified as "safe" or "unsafe" (i.e., absence/presence of congestions) [3]. To this end, we propose and evaluate novel data generation strategies, and compare them to the standard random generation approach in the literature, highlighting the inadequacy of the latter when applied to low voltage networks. This is tested on the IEEE European Low-Voltage Test Feeder, with reports on computational times for training and inference, as well as significant improvements in classification performance using alternative data generation strategies. This work is conducted as part of the ANR "EDEN4SG" project under grant agreement ANR-22-CE05-0023.

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Relative Value Iteration for Infinite-Horizon SDDP: Application to Hydroelectric Problem

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Keywords: Dynamic Programming, Stochastic Dual Dynamic Programming, Infinite Horizon, Long-Term Water Management

Motivated by long term hydroelectric water management problems, we study Dynamic Programming (DP) algorithms dedicated to solving sequential decision-making problems with continuous (but compact) state and control. For this setting the Stochastic Dual Dynamic Programming (SDDP), introduced by [1], which iteratively refine cuts approximating the so-called value function, has long been proven effective. However, its extension it to infinite-horizon settings (see [2]) poses both theoretical and computational challenges, especially when the discount factor β approaches 1. In contrast to the usual long-term Brazilian hydroelectric problem that limit the planning horizon to 10 years due to computational constraints, our model considers the infinite-horizon case to ensure better long-term policies.

One numerical burden for the infinite horizon setting with $\beta \approx 1$ is that the cut generated take a long time in getting close enough to the true value function to be relevant. To circumvent that we propose a variant of SDDP that rely on the relative value iteration approach as described in [3, Vol.II Sect.4]. In particular we additively shift the cut computed so that they are more relevant, *e.g.*, more cut are active after the same number of iterations. We discuss various normalization strategies, exact and heuristic, and their convergence.

Finally, numerical results show the interest of this approach.

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Machine Learning-Driven Inflow Forecasting for Optimizing Hydropower Maintenance Scheduling

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Keywords: Stochastic hydropower optimization, Inflow prediction, Machine Learning, Maintenance scheduling.

The aim of this study is to evaluate the impact of different Hydropower Production Function (HPF) formulations on maintenance scheduling under uncertainty. Three approximation methods are used: a convex hull approximation, piecewise linear approximation, and a polynomial function. Accurate scenario generation plays a crucial role in comparing these methods, as inflow uncertainty is one of the most important challenges for stochastic hydropower optimization. Therefore, in this study, three machine learning models : Long Short Term Memory (LSTM), Kernel Ridge Regression (KRR), and XGBoost are employed to forecast future inflows based on historical data. The predicted inflows from each model are then compared using different strategies to identify which model provides the most accurate results. The results from each model will be used then to generate scenarios for evaluating and comparing the three approximation methods, and to assess their impact on the solution.

Law-Smooth Update Scheme for the Cross-Entropy Optimization Algorithm and Application

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Keywords: Cross-Entropy method, Law-Smooth update, CMAES, Hyperspectral

It is well known that the Cross-Entropy (CE) algorithm [1], based on Gaussian distributions family, is significantly less efficient than successful methods such as CMAES [2] or even particle-based approaches such as particle swarms [3]. Nevertheless, the Gaussian-based CE approach implements similar ingredients as the CMAES approach, but it is significantly penalized by its law updating step after sample selection. Variants of the CE approach have been proposed, featuring a smoothing of the distribution parameters. We show why these approaches by smoothing law parameters may result in wrong convergence, and propose an approach based on smoothing laws [4]. We implement this CE updating approach and show that it gets close to the performance of CMAES on an applicative example. Our application concerns the robust optimization of band selection for anomaly detection in multispectral remote sensing images [5]. As it is important to take into account the variability in the scenes observed, as well as the diversity of objects likely to be encountered, we have drawn on robustness measures, such as quantile, to quantify the variations in anomaly detection criteria over the 100 hyperspectral images of the benchmark dataset, HAD100 [6]. Finally, we also explore a generalization of this CE update to laws of the exponential family.

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Stochastic Approximation with the Two-Norm Discrepancy

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Keywords: Stochastic gradient method, shape optimization, free boundary problem, two-norm discrepancy, optimization under uncertainty.

In this talk, we discuss the convergence of the projected stochastic gradient method in function spaces. We highlight applications and challenges in this setting and summarize existing results. A particular focus will be on a well-known phenomenon in optimal control called the two-norm discrepancy whereby coercivity of an objective can only be shown in a weaker norm than that in which it is differentiable. This was the topic of [1], where the convergence of the projected stochastic gradient method with respect to the appropriate norm was proven. As an application, we consider Bernoulli's exterior free boundary problem with a random interior boundary. We recast this problem into a shape optimization problem by means of the minimization of the expected Dirichlet energy. By restricting ourselves to the class of convex, sufficiently smooth domains of bounded curvature, the shape optimization problem becomes strongly convex with respect to the weaker norm. The theoretical findings are supported and validated by numerical experiments.

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Stable Parameters of Mean-Field Neural ODEs

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Keywords: Neural ODEs, Mean-Field Approximation, Deep Neural Networks, Jacobi Principle, Gradient Descent, Polyak-Lojasiewicz Condition, Functionnal Inequalities

Performing regression tasks with deep neural networks can be modeled as an optimal control problem for an ordinary differential equation. We investigate a relaxation of this problem where controls are taken to be probability measures over the parameter space and the cost involves an additional entropy penalization. We are particularly interested in the stability of the optimal parameters – where stability is understood in terms of unique solvability of a certain linearized system of pdes. We show that, for a lot of initial data (in terms of the initial distribution of the features), there is actually a unique stable global minimizer in the control problem. Moreover we prove that stable minimizers satisfy some local Polyak-Lojasiewicz condition and that the (continuous analog of the) gradient descent with backpropagation converges exponentially fast when initialized nearby a stable minimizer.

An Introduction to Witsenhausen Model and Games in Product Form

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Keywords: Witsenhausen intrinsic model, games in product form, energy

This talk is the first one of a series of two (to be followed by Thomas Buchholtzer's talk).

Energy markets are becoming more complex, under the penetration of highly variable renewable energies (solar, wind) and of technology (telemeters). With uncertain load, producers expect consumers to be more flexible and to shift part of their demand when needed. As a correlate, the design of contracts between producers and consumers is made more complex because of the stochasticity induced by renewable energies and consumption, but also by the variety of producers and consumers profiles (more or less flexibility).

We propose to use the so-called Witsenhausen intrinsic model (WIM), also called W-model, to represent strategic interactions between leaders (producers) and followers (consumers). For this purpose, we present the WIM — with agents, Nature, a product set, a product sigma-field, information of an agent as a subfield of the product sigma-field, strategies [3, 4, 1] — and its extension into Games in Product Form (GPF), also called W-games — with players, payoff functions, beliefs, mixed strategies "à la Aumann" [2].

This talk paves the way for the following one, which will present examples of bilevel programs between different actors of the new energy markets, such as producers, consumers, aggregators.

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$\mathcal{H}_0$-Couplings and the ℓ_0 Pseudonorm
(part of the session
Theory and Algorithms in Sparse Optimization)

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Keywords: generalized convexity, ℓ_0 pseudonorm, orthant-strict monotonicity

In [3], we introduced the class of Capra-couplings, where Capra stands for Constant Along Primal RAys. Any Capra-coupling is defined by means of a norm. When this norm is orthant-strictly monotonic, a notion introduced in [4], we were able to prove in [2] that the ℓ_0 pseudonorm was Capra-convex, thus extending our original result established for the Euclidean norm in [1]. Little by little, we realized that our approach, initially designed for the ℓ_0 pseudonorm, could possibly be extended to 0-homogeneous functions, studied by couplings that were themselves 0-homogeneous in the first variable, and linear in the second one.

In this talk, we introduce a new class of (additive) $\mathcal{H}_0$-couplings, and study the induced conjugacy acting 0-homogeneous functions. We do the same for multiplicative $\mathcal{H}_0$-couplings and the induced polarity. With this, we are able to explore the best lower approximation of 0-homogeneous functions by closed convex and by positively 1-homogeneous closed convex. Finally, we establish new results of hidden convexity for the ℓ_0 pseudonorm.

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A Derivative-Free Method for Chance-Constrained Problems with Right-Hand Side Uncertainty

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Keywords: Stochastic programming, Global optimization, Mixed-integer optimization

This work addresses (mixed-integer) joint chance-constrained optimization problems in which the only uncertain parameter corresponds to the right-hand side coefficients in an inequality system. It exploits one-dimensional marginals to construct an upper model for the underlying joint probability function, which need not be differentiable nor satisfy generalized concavity properties. Based on this model, a global optimization derivative-free method is proposed. The approach iteratively approximates the probability function within an outer approximation algorithm that is shown to compute, under mild assumptions, an approximate global solution to the underlying nonconvex chance-constrained problem. When the problem's data is linear, the outer approximation algorithm requires solving (approximately) a mixed-integer linear programming problem per iteration. Numerical experiments on academic chance-constrained problems highlight the approach's potential and limitations.

Towards Graph Encoding for Quantum Computing with Neutral Atoms

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Keywords:unit-disk graph encoding, Rydberg atom arrays, maximum independent set, quantum computing.

Programmable quantum systems using Rydberg atom arrays are well-suited for solving quantum algorithms efficiently. The maximum independent set (MIS), a key NP-hard problem, can be natively encoded on these systems [2]. In the MIS problem, adjacent vertices in a graph cannot both be part of the solution. Rydberg atom arrays consist of neutral atoms placed in programmable positions in 2D. Each graph vertex corresponds to an atom, and nearby atoms, within a constant-radius disk, cannot both be excited. This represents the MIS constraint for graphs where adjacent vertices are at distance less than this radius in the Euclidean plane, known as Unit-Disk Graphs (UDG). Recognizing UDGs and finding valid vertex positions (UD embedding) are NP-hard, and ensuring these positions fit the physical grid introduces an additional challenge: grid embedding. This constraint can be addressed by adding gadgets and auxiliary qubits [1]. A key goal is to minimize the number of qubits needed to encode graphs with arbitrary connectivity, as the main limitation of neutral atom quantum machines is the number of available qubits. Additionally, Rydberg atom arrays must place atoms on a grid, rather than freely in the plane. The main result presented by QuEra [1] is the arrangement of the vertices in a lattice form using gadgets. These gadgets allow transforming any graph into a UDG, from which the MIS of the original graph can be recovered from the transformed graph. The proposed transformation encodes all graphs but requires a number of auxiliary qubits in $O(n^2)$, which remains too costly.

We propose a different approach, the Local UD encoding. The idea is to use gadgets similar to those of QuEra but to transform only the minimal non-UD structures of the graph. For example, the claw $K_{1,6}$, which is not UD, can be transformed with claws $K_{1,3}$. Transforming local structures uses fewer auxiliary qubits but does not provide a grid embedding. We therefore propose to first use local transformation then find a UD embedding with heuristics and finally apply search algorithms to find an embedding that fit the grid of the machine.

Finally, we also present a global approach to the problem. First, a preprocessing step is applied to reduce the size of the instances with recursive simplicial fixing [3], then the graph is partitioned to parallelize the quantum solving process. Finally we use the previously described Local UD encoding on each part of the graph.

Perspectives includes developing methods for selecting separators to partition the graph while respecting the constraints of parallelized computation, as well as further developing UD embedding heuristics.

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Block coordination of nonlinear networks and discrete optimization

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Keywords: nonconvex MINLP, bilevel optimization, nonlinear networks, augmented lagrangian

Nonlinear networks [1], also known as potential-based flows [3], serve as flow models for a variety of abstract and physical systems, including water or gas distribution, electric and thermodynamic circuits, road transportation. Nonlinear network flows present significant variational and dual properties [4].

Decision problems to operate or design these infrastructures often translate into discrete bilevel programs, as finding network configurations with associated flows of minimum joint costs. A priori, these bilevel programs are nonconvex MINLPs, some with additional synchronization constraints, appearing at any level, for example:

1. pump scheduling, for water network operation, gives a sequence of configurations synchronized through the storage states [2]

2. discrete network design, for traffic planning, gives one configuration for multiple synchronized commodities.

Synchronization makes these problems highly combinatorial and challenging. In this work, we are interested in developing separable algorithms based on the penalization of the synchronization constraints. We propose to exploit the strong duality property of the inner level nonlinear problems. We present applications to the two problems above and experiments with variants of the augmented lagrangian method, including ADMM.

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Demographic parity in regression and classification within the unawareness framework

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Keywords: Statistical fairness, demographic parity, optimal transport, unawareness framework

Statistical fairness seeks to ensure an equitable distribution of predictions or algorithmic decisions across different sensitive groups. Among the fairness criteria under consideration, demographic parity is arguably the most conceptually straightforward: it simply requires that the distribution of outcomes is identical across all sensitive groups.

We focus on the unawareness framework, where the prediction function cannot make direct use of the sensitive attribute (thus, individuals are not treated differently based on discriminating factors). In a regression setting, we solve the problem of finding the optimal unaware prediction function satisfying the demographic parity constraint. Interestingly enough, this problem boils down to solving a two-dimensional optimal transport problem for some degenerate cost. We study this optimal transport problem and exhibit relevant properties of the associated prediction function.

Runtime Analysis of the $(\mu + 1)$ GA: Provable Speed-Ups from Strong Drift towards Diverse Populations

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Keywords: genetic algorithms, crossover, Jump benchmark, run time analysis, speed-up

Most evolutionary algorithms used in practice heavily employ crossover. In contrast, the rigorous understanding of how crossover is beneficial is largely lagging behind. In this work, we make a considerable step forward by analyzing the population dynamics of the $(\mu + 1)$ genetic algorithm when optimizing the Jump benchmark. We observe (and prove via mathematical means) that once the population contains two different individuals on the local optimum, the diversity in the population increases in expectation. From this drift towards more diverse states, we show that a diversity suitable for crossover to be effective is reached quickly and, more importantly, then persists for a time that is at least exponential in the population size μ. This drastically improves over the previously best known guarantee, which is only quadratic in μ.

Our new understanding of the population dynamics easily gives stronger performance guarantees. In particular, we derive that population sizes logarithmic in the problem size n already suffice to gain an $\Omega(n)$-factor runtime improvement from crossover (previous works achieved comparable bounds only with $\mu = \Theta(n)$ or via a non-standard mutation rate).

This work is part of the PGMO-funded project *Mathematical Analysis of Complex Randomized Search Heuristics* (PI: Benjamin Doerr).

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Difficulties of the NSGA-II with the Many-Objective LeadingOnes Problem

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Keywords: Multi-objective optimization, evolutionary algorithms, runtime analysis.

Recent studies have shown that the widely used Non-dominated Sorting Genetic Algorithm II (NSGA-II) struggles with many-objective optimization, particularly when faced with problems involving more than two objectives. While NSGA-II performs well in bi-objective problems, its performance deteriorates as the number of objectives increases. These difficulties were further demonstrated in a runtime analysis of the m-OneMinMax problem [1], where NSGA-II was found to frequently miss parts of the Pareto front, especially when handling more than two objectives.

In our case, we consider the m-LeadingOnesTrailingZeros (mLOTZ) problem, a generalization of the classical bi-objective LOTZ problem. This problem is particularly interesting to explore, as its failure further emphasizes the inefficacy of the NSGA-II in many-objective scenarios. Unlike previous problems such as m-OneMinMax, where every solution lies on the Pareto front, in the case of mLOTZ, not every solution is Pareto optimal, adding another layer of complexity. Our analysis demonstrates that the NSGA-II frequently fails to cover a significant portion of the Pareto front. Specifically, with n as the problem size (i.e., the bitstring length) and m as the number of objectives, when the initial population size N is less than aM, where $a > 1$ and $M = (2n/m + 1)^{m/2}$ represents the size of the Pareto front, the algorithm struggles. We show that for any number of iterations T, there is at least a $1 - T\exp(-\Omega(\frac{m}{n}M))$ probability that the combined parent and offspring population fails to cover a constant fraction of the Pareto front.

In other words, to obtain full Pareto front coverage, the runtime required is exponential in $n^{(m-2)/2}$. Our theoretical results provide strong evidence of this inefficiency, both in expectation and with high probability. These findings reinforce that the NSGA-II struggles significantly with many-objective optimization problems, as its performance declines sharply when the number of objectives increases.

This work is part of the PGMO-funded project *Mathematical Analysis of State-of-the-Art Multi-Objective Evolutionary Algorithms* (PI: Benjamin Doerr)

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Dynamic Parking Pricing for Electric Vehicles at public Charging Stations

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Keywords: Charging station, electric vehicle, Markov chain, parking pricing

The growing adoption of Electric Vehicles (EVs) can lead to congestion at the Charging Stations (CSs) in terms of access to the charging points. Such congestion can be reduced through the implementation of parking pricing strategies.

Queueing theory has been widely used to model parking occupancy, where the service time corresponds to the parking duration [1]. However, existing studies on parking pricing using queues, such as [2], do not account for the variation in arrival rates throughout the day.

In this work, the parking fee is applied only to EV users exceeding a predefined parking time limit, and increases linearly with their parking durations. This policy encourages EV users with shorter parking duration to join the CS (and discourages those with longer stays), thereby freeing up charging points for other EV users. Both the time limit and the rate at which the parking fee increases are variables to be set by the CS operator. The occupancy level of the CS is modeled as an time-inhomogeneous Markov chain, where the transition rates – i.e., the frequency of EV users joining and leaving the CS, are controlled through the parking pricing strategy.

The design of an optimal parking pricing strategy is examined in terms of the expected profit of the CS operator generated from charging the EVs. This analysis considers various dynamic pricing approaches, based on (i) the instantaneous occupancy level, (ii) the time of day, or (iii) a combination of both. Our study extends the work of [3], which only considers a static parking policy. The different pricing strategies are also numerically compared in terms of various metrics such as their externalities on the EV welfare. The numerical results indicate that a time-based dynamic pricing strategy yields better performance.

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Comparative study of quantum methods in the resolution of track findings instances

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Track finding can be considered as a complex optimization problem initially introduced in particle physics involving the reconstruction of particle trajectories. A track is typically composed of several consecutive segments (track segments) that resembles a smooth curve without bifurcations. In this paper various modeling approaches are explored in order to assess both their impact and their effectiveness in solving them using quantum and classical methods. We present implementations of three classical models using CPLEX, two quantum models running on actual D-Wave quantum computers, and one quantum model on a D-Wave simulator. To facilitate a fair comparative study and encourage future research in this area, we introduce a new set of benchmark instances, categorized into small, medium, and large scales. Our evaluation of these methods on the benchmark instances indicates that D-Wave methods offer an excellent balance between computation time and result quality, outperforming CPLEX in numerous cases.

A single collision between two protons from the two beams of the Large Hadron Collider (Evans 2011) can generate thousands of new particles. The collision points can be identified with relatively precise coordinates using detectors with the innermost components built of multiple layers of silicon sensors arranged with cylindrical symmetry around the beam tube. This leads to a significant computational challenge in reconstructing the trajectories of charged particles starting from the small energy deposits in the detector, a process known as track finding.

In this paper, we propose a numerical experiment based on the Peterson's cost formula (Peterson 1989) with a Quadratic Constrained Binary Model (QCBM) and a Quadratic Unconstrained Binary Model (QUBM) solved using a quantum solution with D-Wave and a non-quantum approach with CPLEX, and a Binary Linear Program (BLP) solved only with CPLEX and a non-quantum approach. We have created a new dataset extracted from the TrackML Kaggle challenge (Calafiura 2018), containing data simulated using ACTS (Ai 2019), and we have defined a set of instances composed of 10 small-scale instances with between 70 and 700 hits, of 10 medium-scale instances with 875 and 2450 hits and of 10 large-scale instances with 2625 and 4200 hits. The instances and the numerical experiments are available at the following web page

https://perso.isima.fr/~lacomme/track_finding/data.html

aimed to stimulate fair future research on the topic. Our findings indicate that achieving high accuracy with non-quantum methods requires substantial resolution time. Although Simulated Annealing on classical computers and the Hybrid method for solving CQM offer significantly shorter resolution times, their accuracy is considerably lower. In contrast, solving the model using D_QUBM provides high accuracy within an acceptable resolution time. Using quantum computing will be a promising method to solve this problem in the future.

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Defining Lyapunov functions as the solution of a performance estimation saddle point problem

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Keywords: Performance estimation problem, Lyapunov analysis, randomized optimization algorithms

We reinterpret quadratic Lyapunov functions as solutions to a performance estimation saddle point problem. Like in [UBTG24], this allows us to automatically detect the existence of such a Lyapunov function and thus numerically check that a given algorithm converges. The novelty of this work is that we show how to define the saddle point problem using the PEPit software [GMG+24] and then solve it with DSP-CVXPY [SLB23].

This combination gives us a very strong modeling power because defining new points and their relations across iterates is very easy in PEPit. We can without effort define auxiliary points used for the sole purpose of designing more complex Lyapunov functions, define complex functional classes or study complex algorithms.

In a first numerical experiment, we numerically show that when solving a convex-concave saddle point problem whose smoothed duality gap has the quadratic error bound property [Fer23] with the Chambolle-Pock algorithm [CP11], step sizes satisfying $1 < \sigma\tau\|L\|^2 < 3/4$ may not lead to a faster rate of convergence that $\sigma\tau\|L\|^2 = 1$ even though they are larger.

Then, we conjecture and prove a convergence result for randomized coordinate descent which is slightly improved from [LX15].

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Efficient Sampling of Constraint Spaces in Practice

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Keywords: sampling, convex bodies, high dimensions, Monte Carlo algorithms, experimental analysis of algorithms

Sampling from a given probability distribution constrained by a convex space is a fundamental problem that arises across various fields, from optimization [1] to quantitative finance [2] and systems biology [3]. These problems are critical for modeling uncertainty, making decisions under constraints, and understanding complex systems. However, designing efficient sampling algorithms in high-dimensional spaces remains a significant challenge.

In this talk, we will introduce the relevant theoretical background, define the main problems, and delve into the core challenges associated with sampling in constrained spaces. We will explore how these challenges manifest in practical applications, briefly touching on real world examples from biology and finance. Furthermore, we will outline the current state-of-the-art methods available for some important constraint models, focusing on their computational efficiency, scalability, and accuracy.

From a practical viewpoint, we will demonstrate the implementation of these methods in open-source software, providing hands-on insights into their implementation and applicability. The discussed software is publicly available at `https://github.com/GeomScale`, enabling researchers and practitioners to integrate these advanced sampling techniques into their workflows.

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State Abstraction Discovery for Infinite Horizon Dynamic Programming

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Keywords: State Abstraction, Reinforcement Learning, Dynamic Programming

A Markov Decision Process (MDP) is a framework used to solve stochastic dynamic control problems where the environment evolves under the actions of an agent [2]. The high dimensionality of MDPs can be addressed using decomposition techniques such as state and action space abstractions in Hierarchical Reinforcement Learning [1]. Although state abstraction methods have been explored over the past decades, explicit methods to build useful abstractions of models are rarely provided.

We provide a practical criterion for constructing meaningful abstractions [3]. To achieve this, we estimate the error introduced by abstraction and evaluation of the abstract MDP through the following bound:

$$\|\tilde{V} - V^*\|_\infty \leq \frac{1}{1-\gamma}\left(\|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty + \operatorname{Span}_{S_k} \mathcal{T}^*\tilde{V}\right)$$

where $\tilde{V}$ is the value on the abstract MDP, V^* is the optimal value function, γ is the discount factor, $\mathcal{T}^*$ is the optimal Bellman operator, $\Pi\mathcal{T}^*$ is the approximate optimal Bellman operator and $\operatorname{Span}_S V := \max_{s\in S} V(s) - \min_{s\in S} V(s)$.

Finally, we propose a method to solve MDPs with controlled error by progressively refining the abstractions allowing us to discover explicit state abstractions. We prove the convergence of this method under both the discounted and total reward criteria. Recent implementations demonstrate that this approach reduces the solving time relatively to traditional methods and other aggregation algorithms for a variety of models. It also reveals the underlying dynamics of the MDPs without making assumptions about the problem's structure.

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Integrated crew management for rail freight

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Keywords: Rail freight, crew scheduling, crew rostering, column generation

The construction of yearly schedules for drivers is a problem met in multiple branches of transportation. Generally, the planning process is addressed sequentially: in a first step—the Crew Scheduling—, driving tasks (assumed to be the same every week) are combined together to form daily duties; in a second step—the Crew Rostering—, these duties are organised together in rosters, providing the yearly schedule. A common variation of the latter problem, the Cyclic Crew Rostering, is used for freight train drivers at the SNCF (the main French railway company), but also in other industries [1]. For this problem, the drivers are first split into teams. Then the cyclic roster of each team is obtained by cyclically rolling out a weekly schedule of an individual driver to provide the yearly schedule. Such rosters are easy to memorize and ensure fairness and a similar level of proficiency among the drivers.

The decomposition of the planning process into Crew Scheduling and Crew Rostering is justified by the numerous operational rules on duties and rosters, as well as the combinatorial complexity of the two problems. However, this decomposition results in sub-optimal solutions. Lin and Tsai [2] propose to solve the integrated version via column generation. The SNCF is faced with additional challenges making their approach irrelevant: some trains must be driven at night (due to priority of passenger over freight in France), leading to rosters with multiple duties on the same day, and the objective function is not linear in the duties.

We propose a column generation approach to the integrated Crew Scheduling and Cyclic Crew Rostering problems, in which columns are not rosters but "long shifts," representing the working period between two days off, making the objective function linear. A non-trivial contribution is the modeling of the pricing sub-problem as a Resource Constrained Shortest Path Problem, within the setting of Parmentier [3]. Our approach has already been able to provide a solution to an instance of 265 trains, with a cost 10% lower that the one achievable with the current sequential approach.

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Generalized Kantorovich-Rubinstein Duality and Applications to Dictionary Learning

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Keywords: Optimal Transport, Inverse Problems, Duality

In Dictionary Learning, ones finds a decomposition of a signal into elementary dictionary entries. We study the case where the dictionary is a function, the decomposition is a measure and the signal results from integration of the dictionary by the measure. This setting encompasses inverse problems, where the imaging function plays the role of the dictionary. The learning process is a sparse optimization over the space of measures, generalizing the classical lasso problem.

We offer a new perspective on this problem using a generalized Kantorovich-Rubinstein duality. Measures are put in duality with c-concave functions using Minkowski gauges. The resulting couple of dual Banach spaces provides a good setting for the study of the optimization problem by Fenchel duality. This new spaces also shortcircuits some of the technical difficulties found in the Continuous functions / Radon measures setting.

The introduction of transportation costs allows to study the "sliding" steps — iterations where the support of the measure is moved, as Wasserstein gradients. Our developments are done when the cost is the Bregman Divergence of an entropy h. In this setting the transport map is the gradient of a convex function, generalizing Brenier's theorem, and the c-concave functions are the smooth functions relative to h. This offers a new perspective on lifting Mirror Descent to the space of measures.

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Linear convergence of CMA-ES

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Keywords: CMA-ES, linear convergence, Markov chain

Evolution strategies (ES) are a class of derivative-free optimization algorithms. Among them, the covariance matrix adaptation evolution strategy (CMA-ES) is the state-of-the-art method of this class that demonstrated good empirical performances for nonconvex, multimodal, ill-conditioned or noisy problems. However, establishing a mathematical proof of its convergence is a challenging problem. I will present recent works in which we prove the linear convergence of CMA-ES on ellipsoidal problems (minimizing functions with ellipsoidal level sets). We show also that the inverse Hessian of convex-quadratic functions is approximated by the algorithm. This proof relies on the analysis of an underlying Markov chain, that we prove to be geometrically ergodic.

Comparing Deterministic and Chance Constrained Models for Renewable-Powered Mini-Grids in Unreliable Grid Conditions

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This talk addresses the operation of a renewable-powered mini-grid connected to an unreliable main grid in rural Africa, where energy access rates are low. The system faces four key uncertainties: solar power and load forecasting errors and frequency and outage duration from the main grid. These uncertainties challenge traditional power system operations. We explore three alternatives to the Joint Chance Constrained (JCC) programming approach: Individual Chance Constraint (ICC), Expected-Value Model (EVM), and a deterministic model that disregards both outages and forecasting uncertainties. The JCC model ensures a high probability of meeting local demand during outages by maintaining adequate reserves for Diesel generation and battery discharge, offering the most robust solution. In contrast, the ICC model provides less robust dispatch by ensuring reliability only at individual time steps, and the simpler EVM focuses on average values, further reducing robustness. A case study of a real mini-grid in Lake Victoria, Tanzania, demonstrates that while the JCC model increases costs slightly, it significantly improves dispatch planning for battery and Diesel reserves compared to the other models.

Keywords
Probability functions; SDG-7; Stochastic programming; grid-connected mini-grids.

A Fresh Look at Algorithms for Solving Smooth Multiobjective Optimization Problems

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Keywords: Multiobjective optimization, descent algorithms, (weak) Pareto efficiency

We propose a new approach for constructing practical algorithms for solving smooth multiobjective optimization problems based on determining decreasing directions via suitable linear programming problems, exploiting a characterization for descent directions provided in [2]. The presented iterative method is specialized for unconstrained, and linearly constrained multiobjective optimization problems. In all cases, the objective function values sequence is decreasing with respect to the corresponding nonnegative orthant, and every accumulation point of the sequence generated by the algorithm is a substationary point to the considered multiobjective optimization problem, becoming, under convexity assumptions, a weakly Pareto efficient solution. Different to similar algorithms from the literature (see [1]), the ones proposed in this work involve decreasing directions that are easily computable in polynomial time.

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A Speed Restart Scheme for an Inertial System with Hessian-Driven Damping and Constant Coefficients

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Keywords: Convex optimization, Speed restart scheme, Differential equations, First-order methods

We study a speed restart scheme [1, 2] in order to improve the convergence rate of accelerated gradient schemes. We propose a second order differential equation with constant coefficients

$$\ddot{x}(t) + \alpha\dot{x}(t) + \beta\nabla^2 f(x(t))\dot{x}(t) + \gamma\nabla f(x(t)) = 0,$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is a convex function of class $\mathcal{C}^2$, which attains its minimum value, $\alpha > 0, \beta \geq 0$ and $\gamma > 0$ are three parameters. The gradient of function f is Lipschitz continuous with constant $L > 0$.

It is shown in [3] that suitable choices for α, β and γ provides exponential decay of the value function when f is $\mathcal{C}^2$ strongly-convex with Lipschitz gradient. In our case, we establish a linear convergence rate for the function values along the restarted trajectories without assuming that the strong convexity of the objective function and it holds for any value of $\alpha > 0, \beta \geq 0$ and $\gamma > 0$. We also report numerical experiments which show that dynamical system with speed restarting scheme together improve the performance of the dynamics.

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An enumerative formula for the spherical cap discrepancy

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Keywords: spherical cap discrepancy, uniform spherical point sets, enumerative formula

Among many criteria for uniformity of point sets on the unit sphere, the *spherical cap discrepancy* plays a prominent role due to the possibility of estimating the integration error of functions defined on the sphere. This aspect is of much importance, for instance, when dealing with chance constraints in optimization methods and using the well-established method of *spherical-radial decomposition*. Unfortunately, contrary to other criteria, the spherical cap discrepancy is not defined by a closed formula but rather as a supremum of local discrepancies which are discontinuous functions of their parameters. Therefore, already the shere computation of this discrepancy is a challenge and usually approximations or relaxations are employed. In [1], a formula has been established which reduces the computation of the discrepancy to the enumeration of finitely many expressions. This allows one, for instance, to get precise numerical values as long as the dimension of the sphere is not too large (e.g., 6) and the size of the point set is moderate (100 to 1000 depending on dimension). Not surprisingly, there is an issue of complexity involved. However, we see two major potentials of the mentioned formula: first, it may serve in moderate dimensions for calibration purposes, when approximating algorithms with nicer complexity have to be evaluated on the basis of their deviation from true values. Second, and more importantly, the reduction to finitely many expressions may be exploited in order to derive analytic al properties (continuity, Lipschitz continuity, Clarke subdifferential) of the spherical cap discrepancy as a function of the point set. This may be considered as a first step towards optimal quantization of the uniform distribution on the sphere. The talk illustrates the mentioned enumerative formula, provides some numerical results and addresses some structural results of the discrepancy of the discrepancy as a function of the point set.

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Nesterov momentum for convex functions with interpolation: is it faster than Stochastic gradient descent ?

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Keywords: Stochastic convex optimization, Stochastic gradient descent with momentum, Machine learning, Convergence

Considering convex functions, the deterministic Nesterov accelerated gradient algorithm (NAG) significantly improves over the gradient descent in term of convergence speed. However, the literature has been rather pessimistic on the possibility to keep this improvement when replacing the gradient by stochastic approximations, because of sensitivity of the algorithm to errors on the gradient computation [1, 2]. We study the possibility to achieve accelerated convergence of the Stochastic Nesterov Accelerated Gradient algorithm (SNAG) in the favorable setting of interpolation. We start presenting the non accelerated convergence results we can obtain considering convex and smooth functions. Then, specializing to the case of minimizing a sum of functions, we show that the average correlation between gradients plays a key role in order to achieve acceleration with SNAG. Thereby, we show that the correlation between gradients impacts on the benefit of increasing batch size on the convergence speed.

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Online Inventory Problems: Theory and Practice

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Keywords: Online learning, inventory control, online convex optimization, newsvendor problem, regret analysis.

This talk will present our work [1] which addresses multi-product online inventory control. This problem consist of a manager that makes sequential replenishment decisions based on partial historical information in order to minimize its cumulative losses.

The literature on this problem focused on the design of specialized algorithms that handle specific loss structures and dynamics while providing theoretical guarantees under an unrealistic i.i.d. demand assumption [2, 3, 4]. Our motivation is to go beyond these models. Building on top of Online Convex Optimization (OCO), we start by providing a more general framework for inventory problems. Then, we propose MaxCOSD, an online algorithm that has provable optimal guarantees even for problems with non-i.i.d. demands and stateful dynamics, including for instance perishability. Finally, we argue for the necessity of non-degeneracy demand assumptions to allow learning.

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The Stochastic Central Path : Sampling the Optimal Solution of a Convex Program with Random Parameters

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Keywords: Stochastic Programming, Logarithmic Barrier, Monte Carlo Markov Chains, Langevin Dynamics

Some methodological approaches for solving two-stage stochastic programs depend on efficiently computing (the expectation of) some optimal dual variables, which are typically given as solution of a linear program with random coefficients, i.e.

$$\min_x \quad c(\xi)^\top x \quad \text{s.t.} \quad A(\xi)x \leq b(\xi) \tag{LP-R}$$

where the matrix $A(\xi)$ and vectors $b(\xi)$ and $c(\xi)$ depend on a random parameter ξ.

We present a geometric approach for generating samples of the optimal solution to the above problem. Our work builds upon ideas first presented in [1]. We assume that ξ has a log-concave smooth probability distribution so that it can be simulated by Langevin-type dynamics, as in [2]. We consider the log-barrier regularized problem with parameter $\mu > 0$,

$$\min_x c(\xi)^\top x - \mu \sum_i \log(b_i(\xi) - A_i(\xi)) \ .$$

We call the map sending μ to the unique optimal solution $x^*(\mu)$ the *stochastic central path*. For a fixed parameter μ, we show that $x^*(\mu)$ is the steady-state solution of a stochastic differential equation (SDE). By discretizing this SDE, we can simulate the regularized optimal solution of Problem (LP-R).

The method carries over to more general classes of convex programs with explicit barriers. We present preliminary numerical experiments.

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Sampling and Estimating the Set of Pareto Optimal Solutions in Stochastic Multi-Objective Optimization

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Keywords: Stochastic Optimization, Multi-objective Optimization, Stochastic Approximation, Inference

The framework of stochastic multi-objective programming allows for the inclusion of uncertainties in multi-objective optimization problems at the cost of transforming the set of objectives into a set of expectations of random quantities. Of the family of Robins-Monro algorithms, the stochastic multi-gradient algorithm (SMGDA) gives a solution to these types of problems without ever having to calculate the expected values of the objectives or their gradients [1]. However, a bias in the algorithm causes it to converge to only a subset of the whole Pareto front, limiting its use. To sample along the whole Pareto front, we reduce the bias of the stochastic multi-gradient calculation using an exponential smoothing technique, and promote the exploration of the Pareto front by adding non-vanishing noise tangential to the front.

Then, given a set of points which are potentially located on the Pareto front of a stochastic multi-objective optimization program how can we assess dominance? Working only with previously collected minima, the primary challenges to overcome are to perform probabilistic comparison between points with no knowledge of the true value of the expected quantity of interest and to infer the design points that correspond to likely undominated points in objective space. We solve this problem through a dual modeling approach, building a nearest neighbor model to approximate the objectives and a probabilistic model in design space to assess their likely dominance.

We prove that our algorithm, Stochastic Tangential Pareto Dynamics (STPD), generates samples in a concentrated set containing the whole Pareto front and We analyze the the performance of our postprocessing method as the sample size, noise level, and dimension of the underlying stochastic multi-objective program change in toy problems.

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A mean-field-game approach to overfishing

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Keywords: mean field game, reaction diffusion, fishery management, partial differential equations

In this talk, we investigate an instance of the tragedy of the commons in spatially distributed harvesting games. The model we choose is that of a fishes' population that is governed by a parabolic bistable equation and that fishermen harvest. We assume that, when no fisherman is present, the fishes' population is invading (mathematically, there is an invading travelling front). Is it possible that fishermen, when acting selfishly, each in his or her own best interest, might lead to a reversal of the travelling wave and, consequently, to an extinction of the global fishes' population? To answer this question, we model the behaviour of individual fishermen using a Mean Field Game approach, and we show that the answer is yes. We then show that, at least in some cases, if the fishermen coordinated instead of acting selfishly, each of them could make more benefit, while still guaranteeing the survival of the fishes' population. Our study is illustrated by several numerical simulations

Minimal Euclidean Distance Degree of Segre-Veronese Varieties

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Keywords: ED degree, Segre variety, rank-1 approximation problem, Frobenius product, Eckart-Young theorem

The Euclidean Distance degree of a real algebraic variety X counts the number of (complex) critical points of the distance function from a generic point to X. This notion that was introduced in [1] is a measure of the algebraic complexity of the corresponding distance minimization problem. By its definition, ED degree depends on the inner product on the ambient space. A natural problem is to understand for which inner product the ED degree achieves its minimal value. I will discuss this question for the Segre-Veronese variety, for which the above optimization problem is equivalent to the well-known rank-1 approximation problem to a partially symmetric tensor. I will talk about a conjecture asserting that the Frobenius inner product realizes the minimum of the ED degree and discuss results around it. More details can be found in [2].

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A regret minimization approach to fixed point iterations

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Keywords: Fixed points, Regret minimization, AdaGrad

We present a link between regret bounds and fixed point problems with nonexpansive maps. This allows the definition of many new fixed point iterations based on regret minimizing algorithms with corresponding convergence guarantees derived from the regret bounds. This approach recovers the famous Krasnoselskii-Mann iterations [Kra55, Man53] as corresponding to Online Gradient Descent [Zin03]. In particular, we transpose the celebrated AdaGrad algorithm [MS10, DHS11] to obtain a fixed point iteration with strong adaptive properties.

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Continuous-Time Optimal Control Problem under Signal Temporal Logic Constraints

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Keywords: Optimal control, signal temporal logic, nonlinear system

Signal temporal logic (STL) [1] is a formal language specifying time-dependent properties of continuous signals in dynamic systems, such as sequentially triggered events, safety and stability guarantees. It allows expressing complex mission requirements, such as temporal constraints, in optimal control problems for robotics, self-driving cars, and cyber-physical systems. The current methods discretize and approximate these problems and solve them using mixed integer linear programming [3] or successive convexification algorithm [2]. None of these frameworks, however, are able to provide rigorous justifications on the convergence of the discretized and approximate problem towards the original one, specified by the non-local and non-smooth STL constraints. In this talk, we introduce a novel approach for solving continuous-time optimal control problems under STL constraints that guarantee robustness. Specifically, our approach introduces auxiliary variables according to the STL constraints, which are used to augment the system's state and extend its dynamics. Based on these auxiliary variables, the auxiliary functions are defined to construct state constraints, so that the original system's behavior, as specified by the STL, is accurately represented through an augmented system with state constraints. Therefore, we solve directly the optimal control problem of augmented system with state constraints, instead of the original problem with STL constraints. In addition, we theoretically and numerically demonstrate the feasibility of this strategy, as well as the solutions' convergence to that of the original problem.

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Quantum Optimization with Permutation Group-Structure Ansatze

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Keywords: Quantum Computing, Variational Algorithm, Optimization, Permutations

We devise a novel quantum variational algorithm for the resolution of permutation problems. In particular, our focus is on quadratic binary optimization problems with specific constraints. The set of feasible solutions is constituted exclusively of vectors derived from a given binary vector wherein the coordinates have been permuted. Via a suitable amplitude encoding step, the algorithm uses only a number of qubits that scales logarithmically with the number of variables. By efficiently designing the parametrized permutation ansatz via group theory results inspired from those of Bataille [1], the number of spanned permutations scales exponentially with respect to the number of parameters in the variational ansatz. The latter is a free tuning knob and can be adjusted by the implementer. These features render the proposed algorithm suitable for implementation on a large scale.

This research benefited from the support of the FMJH Program PGMO, under project: P-2023-0009.

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Tikhonov Regularization in Continuous and Discrete Time Optimization

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Keywords: Nonsmooth convex optimization, damped inertial dynamics, Hessian-driven damping, Tikhonov regularization, strong convergence.

In a Hilbert setting we study a second order in time differential equation, combining viscous and Hessian-driven damping and a Tikhonov regularization term. The dynamical system is related to the problem of minimization of a nonsmooth convex function. In the formulation of the problem as well as in our analysis we use the Moreau envelope of the objective function and its gradient and heavily rely on their properties. We show that such systems preserve the well-known fast convergence properties of the function and Moreau envelope values along the trajectories and, moreover, the strong convergence of the trajectories to the element of the minimal norm from the set of all minimizers of the objective could be obtained. In order to address the discrete counterpart, numerical algorithms featuring Tikhonov regularization are also considered and the same set of the results is obtained. The talk will be a brief overview of the well-known results in this area concluding with the presentation of our contribution ([1, 2]) to this topic, including several numerical examples.

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Explicit Convergence Rate of the Proximal Point Algorithm under R-Continuity

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Keywords: R-continuity, metric regularity, metric subregularity, calmness, Proximal Point Algorithm, convergence rate)

The paper provides a thorough comparison between R-continuity and other fundamental tools in optimization such as metric regularity, metric subregularity and calmness. We show that R-continuity has some advantages in the convergence rate analysis of algorithms solving optimization problems. We also present some properties of R-continuity and study the explicit convergence rate of the Proximal Point Algorithm (**PPA**) under the R-continuity.

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A doubly nonlinear evolution system with threshold effects associated with dry friction

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Keywords: fast convex optimization; damped inertial dynamic; dry friction; time scaling; averaging; Hessian-driven damping.

We investigate the asymptotic behavior of inertial dynamics with dry friction in convex optimization. We first focus on a doubly nonlinear first-order evolution inclusion involving two potentials: the differentiable objective function and the nonsmooth dry friction potential. The dry friction term acts on a combination of the velocity vector and the gradient of the objective function, ensuring any stationary point corresponds to a critical point. To facilitate the convergence of the objective's gradient, we explore the dual formulation, which possesses a Riemannian gradient structure. Using the acceleration technique by Attouch, Bot, and Nguyen, we derive from the first-order dynamics second-order systems involving dry friction, vanishing viscous damping, and implicit Hessian-driven damping, which exhibit fast convergence properties. This presentation is based on [1].

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Online Moment Constrained Optimal Transport applied to Electric Vehicle Charging

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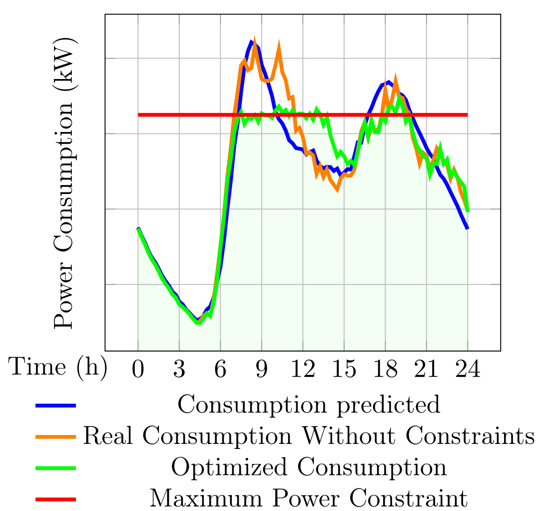
Keywords: Mean Field Control, EV Charging, Optimal Transport

The widespread adoption of electric vehicles (EVs) necessitates their efficient charging management to minimize impact on the power grid and optimize electricity consumption. We propose a mean-field optimization approach to handle a large population of EVs and addresses challenges like stochastic arrival and departure patterns. A Mean Field Control problem is formulated as an optimal transport problem as in [1], including both hard constraints and grid objectives.



We introduce an online version of this moment-constrained optimal transport problem to handle real-time EV charging station management scenarios. An algorithm is proposed to solve this problem sequentially and we discuss how the optimal solution at the previous time step offers a good initialization for the current one. The ElaadNL dataset [2] containing 10,000 EV charging transactions in the Netherlands is used for empirical validation, demonstrating the effectiveness of the proposed model: Our algorithm takes as input EV arrival and departure predictions at the start of the day (corresponding to the consumption denoted in blue in the figure) and at each time step, discovers the actual states of the arriving EVs and chooses a plugging policy. The nominal global consumption is shown in orange, while the green curve corresponds to the result of our online algorithm, which respects the maximum power constraint (in red) while minimizing a given criterion (here, the difference between the arrival time and the connecting time of EVs). This algorithm also shows a certain robustness with regard to prediction quality.

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Capra-Cutting Plane Method Applied to Sparse Optimization

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Keywords: abstract cutting plane method, generalized convexity, sparse optimization

In 1960, Kelley proposed a cutting plane method [2] to minimize a continuous convex objective function over a compact set. This iterative method consists at each step in minimizing a polyhedral approximation of the objective function before improving the approximation by a valid affine cut. In generalized convexity, the affine functions giving the cuts are replaced by some family of base functions. This base functions are chosen so that the objective function is their supremum, making it abstract convex. In this framework, the Kelley's algorithm has been generalized to continuous abstract convex functions [4, 5]. We continue the generalization of the cutting plane method by providing a convergence result that can be applied to lower semicontinuous objective functions.

This convergence result is motivated by the Capra-convexity results [1] on the ℓ_0 pseudonorm, which is lower semicontinuous. As explicit formulas for the Capra-subdifferential of the ℓ_0 pseudonorm have been calculated [3], we can now implement Capra-cutting plane methods for the sparse problem of minimizing ℓ_0 over a compact set.

We present numerical results of this method on small instances of the problem $\min_{Ax=b} \ell_0(x)$.

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Multi-Agent Contextual Combinatorial Multi-Armed Bandits with Linear Structured Super Arm: application to energy management optimization in Smart Grids

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Keywords: Smart grid, Renewable Energy, Multi-Agent, Reinforcement Learning, Bandits

The integration of electric vehicles (EVs) and renewable energy sources (RES) into future electrical grids presents both significant challenges and opportunities for energy management. While RES production is an incentive for increased demand at specific times of the day (e.g., at noon for photovoltaic production), the distribution system operator (DSO) should also prevent grid congestion. The optimal strategy for integrating each EV with regard to the DSO constraints in a manner that maximizes the local use of RES is an NP-hard problem, as it requires the resolution of a mixed-integer linear programming problem (MILP) [1]. Moreover, due to weather and human behaviors, this optimization should be done under uncertainty. In particular, learning algorithms from the family of Bandit algorithms have been studied in [2], [3] to handle uncertainty. In light of the decentralized algorithms developed in [2], we propose a Multi-Agent Contextual Combinatorial Multi-Armed Bandits (MA-CC-MAB) approach with linearly structured super arms. By leveraging contextual information such as time-varying renewable energy generation, grid load conditions and day of the week, each agent will selfishly and dynamically schedule its charging intervals while considering grid constraints. The use of linearly structured super arms enables efficient exploration and exploitation in a high-dimensional combinatorial action space, while decentralization will address scalability issues inherent to large systems. The use of contextual information allows adapting to different conditions of the environment, ensuring efficient decision-making under uncertainty. We show performance of this algorithm on the IEEE LVTF network model (55 agents) with PV production, using pandapower python library for the load flow simulation. This work is conducted as part of the EDEN4SG project funded by the French National Agency for Research (ANR), under grant agreement ANR-22-CE05-0023.

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The dual charge method for the multimarginal optimal transport with Coulomb cost

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Keywords: Multimarginal optimal transport, Kantorovich duality, Quantum chemistry

The multimarginal optimal transport occurs in statistical physics as well as in quantum chemistry, where it is used to describe strongly correlated systems of electrons, see *e.g.* [3, 2]. In the recent years, efforts have been put into fashioning efficient numerical methods to solve this problem, see *e.g.* [4], which is notoriously hard to solve from a computational point of view. In this talk, I will introduce a specific discretization to tackle this problem numerically in the case of Coulomb interaction, and which can be interpreted as the dual version of the so-called *Moment-Constrained Optimal Transport* (MCOT) introduced in [1].

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Distributionally Robust Standard Quadratic Optimization with Wasserstein Ambiguity

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Keywords: Stochastic optimization, quadratic optimization, distributionally robust optimization, portfolio optimization

The standard quadratic optimization problem (StQP) consists of minimizing a quadratic form over the standard simplex. If the quadratic form is neither convex nor concave, the StQP is NP-hard. This problem has many interesting applications ranging from portfolio optimization to machine learning.
Sometimes, the data matrix is uncertain but some information about its distribution can be inferred, e.g. the first two moments or else a reference distribution (typically, the empirical distribution after sampling). In distributionally robust optimization, the goal is to minimize over all possible distributions in an ambiguity set defined based upon above mentioned characteristics. We explore the StQP under distributionally robust chance constraints and an ambiguity set based upon maximal Wasserstein distance to the sampled empirical distribution.

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Universal Approximation of Dynamical Systems by Semi-Autonomous Neural ODEs

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Keywords: neural ODEs, universal approximation, Barron space, transport equations

In this presentation, we introduce semi-autonomous neural ordinary differential equations (SA-NODEs), a variation of the classical NODEs, employing fewer parameters. We investigate the universal approximation properties of SA-NODEs for dynamical systems from both a theoretical and a numerical perspective. Within the assumption of a finite-time horizon, under general hypotheses we establish an asymptotic approximation result, demonstrating that the error vanishes as the number of parameters goes to infinity. Under additional regularity assumptions, we further specify this convergence rate in relation to the number of parameters, utilizing quantitative approximation results in the Barron space. Based on the previous result, we prove an approximation rate for transport equations by their neural counterparts. Our numerical experiments validate the effectiveness of SA-NODEs in capturing the dynamics of various ODE systems and transport equations. Additionally, we compare SA-NODEs with classical NODEs, highlighting the superior performance and reduced complexity of our approach.

Minimal sparsity for scalable moment-SOS relaxations of the AC-OPF problem

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Keywords: Global optimization, Optimal power flow, moment-SOS relaxations, sparsity

The AC-OPF (Alternative Current Optimal Power Flow) [2] problem aims at minimizing the operating costs of a power grid under physical constraints on voltages and power injections. The problem is solved daily by electrical grid stakeholders, usually in very large scale. Its mathematical formulation results in a large nonconvex polynomial optimization problem. It is hard to solve optimally in general, but it can be tackled by a sequence of SDP (Semidefinite Programming) relaxations corresponding to the steps of the moment-SOS (Sums-Of-Squares) hierarchy [3].

Unfortunately, the size of these SDPs grows drastically in the hierarchy, so that even second-order relaxations exploiting the correlative sparsity pattern [4] of AC-OPF are hardly numerically tractable for real-life instances - with thousands of power buses.

Our contribution [1] lies in a new sparsity framework, termed minimal sparsity, inspired by the specific structure of power flow equations. Despite its heuristic nature, numerical examples show that minimal sparsity allows the computation of highly accurate second-order moment-SOS relaxations of AC-OPF, while requiring far less computing time and memory resources than the standard correlative sparsity pattern. Thus, we manage to compute second-order relaxations on test cases with about 6000 power buses, which we believe to be unprecedented.

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Regularized Optimal Transport: disentangling suboptimality and entropy

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Keywords: Optimal transport, Entropy regularization, Schrödinger problem

In this talk, we study the convergence rates of regularized optimal transport problems as the regularization parameter ε tends to zero. The focus is on the entropy-regularized optimal transport problem, where transport plans γ_ε converge towards the unregularized solution γ_0 and the corresponding cost (c, γ_ε) converges to (c, γ_0).

We establish that, under compactly supported marginals and infinitesimally twisted cost functions, the Wasserstein distance $W_2(\gamma_\varepsilon, \gamma_0)$ asymptotically behaves as $C\sqrt{\varepsilon}$, while the suboptimality $(c, \gamma_\varepsilon) - (c, \gamma_0)$ scales linearly with ε. In the quadratic cost case, we relax the compactness assumption to a moment of order $2 + \delta$, which broadens the applicability of the results.

Furthermore, in the presence of a Lipschitz transport map for the unregularized problem, the Wasserstein distance between the plans converges to zero at the rate $\sqrt{\varepsilon}$. For cases where the marginals have finite Fisher information, we prove a refined result: $(c, \gamma_\varepsilon) - (c, \gamma_0) \sim d\varepsilon/2$, and we offer an expansion of the entropy $H(\gamma_\varepsilon)$.

These findings disentangle the distinct roles played by the cost function and the entropy term in the regularized setting, providing a clearer understanding of their contributions to convergence behavior.

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Tikhonov Regularized Exterior Penalty Methods For Hierarchical Variational Inequalities

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Keywords: Hierarchical Optimization, Krasnoselskii-Mann Iteration, Prox-Penalization

In a real Hilbert space setting, this paper addresses the numerical solution of hierarchical problems of the form

$$\mathrm{VI}(G, \mathrm{Zer}(A + F))$$

where $G : \mathcal{H} \to \mathcal{H}$ is a given operator, describing the upper-level variational problem reading as the constrained variational inequality

$$\text{Find } x^* \in \mathrm{Zer}(A + F) \text{ s.t.: } \langle G(x^*), x - x^* \rangle \geq 0 \quad \forall x \in \mathrm{Zer}(A + F).$$

The feasible set of this variational problem is represented in terms of the solution set of another (lower-level) variational inequality involving maximally monotone operators A and F. This family of hierarchical optimization problems is notoriously difficult to solve but at the same time very rich in terms of concrete applications in economics, machine learning, engineering and signal processing. In particular, purely hierarchical convex bilevel optimization problems and certain multi-follower games are particular instances of nested variational inequalities of the above form [1, 2]. Working within a real Hilbert space setting, we develop a prox-penalization algorithm with strong convergence guarantees towards a solution of the nested VI problem. In the special case of a bounded domain in the lower level problem, we also prove rates of convergence towards a solution of the lower level problem. Our proof method builds on general Krasnoselskii-Mann iterations [3] involved in a double-loop architecture, and is thus very flexible. We present various application that fit into our framework and present also some preliminary numerical results.

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Computing Usage Values for Prospective Studies in Energy Systems Using Spatial Decomposition

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Keywords: Interconnected energy system modelling, stochastic multistage optimization, spatial decomposition.

The growing penetration of renewable energy requires additional storage to manage system intermittency. As a result, there is increasing interest in evaluating the opportunity cost of stored energy, or usage values, that can be derived by solving a multistage stochastic optimization problem. We aim to compute these usage values for each country in the interconnected European electrical system, mathematically modeled as an oriented graph, where nodes represent countries and arcs represent interconnection links. In large energy systems, spatial complexity adds to the temporal complexity, making classic stochastic dynamic programming techniques inadequate. To address this, we apply Dual Approximate Dynamic Programming (DADP) [1], which decomposes the global multistage stochastic optimization problem into smaller nodal problems and a transport problem.

The global multistage stochastic optimization problem is stated as the aggregation of nodal problems linked with a transport problem. The nodal problems allocate local production to meet local demand and handle energy imports/exports, while the transport problem manages energy flows across interconnections. The coupling constraint (Kirchhoff's law) links the two. DADP approximates a lower bound of the global problem by dualizing this constraint, allowing the nodal problems to be solved individually (locally) using stochastic dynamic programming techniques for a given dual value of the coupling constraint (the decomposition price). The transport problem, on the other hand, is solved analytically since it lacks temporal coupling.

This approach produces nodal usage values that depend solely on the nodal state, independent of other nodes. To validate this, we present a case study that models three countries - France, Germany, and Switzerland - each with a representative production mix and the interconnection capacities between them. By solving the global problem, we study the dependency of usage values across nodes. Additionally, we demonstrate that the decomposition price corresponds to the nodal marginal price, which is the dual variable of the nodal energy balance. This insight helps develop an initialization of the algorithm specifically suited to the system.

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Representation results for differential games and related neural-network approximations

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Keywords: neural networks, feedback controls, non-anticipative strategies, differential game, two-player game, deterministic optimal control, state constraints, front propagation

Abstract

We give new representations formula for the value of differential games in a finite horizon context [1]. This leads to improved error estimates for some semi-discrete approximations. These representations can then be used to look for non-anticipative strategies in feedback form and to develop schemes on mesh-free approximations such as neural network schemes.

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On the Duality between Frank–Wolfe and Cutting-Plane Algorithms

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Keywords: Duality in Convex Optimization, Frank–Wolfe Algorithm, Conditional Gradient Algorithm, Cutting-Plane Algorithm, Level-Set Method, Nonsmooth Optimization.

The Frank–Wolfe Algorithm [2], or Conditional Gradient Algorithm, is a well-known algorithm from convex optimization. It is used to find an approximate minimizer of a convex and C^1 function f on a closed convex bounded subset K of a Hilbert space $\mathcal{H}$, where we only assume being able to minimize inner products of the form $\langle \mu, x \rangle$ for $x \in K$, and this for any $\mu \in \mathcal{H}$.
the Cutting-Plane Algorithm allows for the minimization of a convex differentiable function g on a closed convex bounded subset D of $\mathcal{H}$, where we assume being able to minimize a convex piecewise-affine lower approximation of g.
I am going to present my results obtained during the first two years of my PhD [1], which show the duality, in the sens of Fenchel-Rockafellar, bewteen the Level-Set Method [3], which is a more robust variant of the Cutting-Plane Method, and an algorithm which can be seen as a extension of the Frank–Wolfe Algorithm for functions of the form $f + h$, where f is a convex and C^1 function and h is a nonsmooth convex function. I also give theoretical guarantees obtained for these methods, as well as a result on their convergence speeds.

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Finite Adaptability in Robust Optimization: Asymptotic Optimality and Tractability

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Keywords: Two-stage robust optimization, finite adaptability, complexity

Of particular importance in operations research are two-stage robust optimization problems

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{y}(\cdot)} \quad & \boldsymbol{c}^\top \boldsymbol{x} + \max_{\omega\in\Omega} \boldsymbol{d}^\top \boldsymbol{y}(\omega) \\ \text{s.t.} \quad & A(\omega)\boldsymbol{x} + B(\omega)\boldsymbol{y}(\omega) \leqslant \boldsymbol{b}(\omega) \quad \forall \omega \in \Omega\,, \end{aligned} \tag{CompAdapt(Ω)}$$

where Ω is the uncertainty set, $A(\cdot)$ and $B(\cdot)$ are input matrices depending on the uncertainty, and $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are deterministic vectors. The value of the variable $\boldsymbol{x}$ has to be determined without knowing the exact $\omega \in \Omega$ that will be selected, contrary to the variable $\boldsymbol{y}(\cdot)$ whose value can arbitrarily depend on ω. These problems, well studied in the literature [3], are relevant when recourse actions are possible after uncertainty realization. A fundamental technique to solve this problem, *finite adaptability*, has been introduced in 2010 by Bertsimas and Caramanis [1]. It consists in restricting the range of $\boldsymbol{y}(\cdot)$ to piecewise constant functions with at most k distinct values:

$$\begin{aligned} \min_{\substack{\Omega_1\cup\cdots\cup\Omega_k \\ \boldsymbol{x},\boldsymbol{y}_1,\ldots,\boldsymbol{y}_k}} \quad & \boldsymbol{c}^\top \boldsymbol{x} + \max_{i\in[k]} \boldsymbol{d}^\top \boldsymbol{y}_i \\ \text{s.t.} \quad & A(\omega)\boldsymbol{x} + B(\omega)\boldsymbol{y}_i \leqslant \boldsymbol{b}(\omega) \quad \forall i \in [k] \;\; \forall \omega \in \Omega_i\,, \end{aligned} \tag{Adapt$_k$(Ω)}$$

where the Ω_i are constrained to form a partition of Ω.

Our contributions are twofold:

- We provide a counter-example to a condition ensuring asymptotical optimality of finite adaptability proposed by Bertsimas and Caramanis, and give a corrected condition [2].
- We show that some special relevant cases of (Adapt$_k(\Omega)$) can be solved in polynomial time. To the authors' knowledge these form the first tractability results in the area of finite adaptability. Our approach relies on new results in discrete geometry dealing with the covering of a polytope with convex sets.

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On Common Noise in Finite State Space Mean Field Games

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Keywords: Mean field games, common noise, master equation, hyperbolic systems

the typical form of a master equation [1] solved by the value function of finite state space mean field game is

$$\begin{cases} \partial_t U + (F(x,U)\cdot\nabla_x)U = G(x,U) \text{ for } (t,x)\in(0,T)\times\mathbb{R}^d, \\ U(0,x) = U_0(x) \text{ for } x\in\mathbb{R}^d. \end{cases} \tag{1}$$

In this talk we are interested in finite state space mean field games affected by a common source of randomness. Namely, we are going to consider that the coefficients of the game depend on the value of a stochastic process $(p_t)_{t\geq 0}$ defined by

$$dp_t = -b(p_t)dt + \sqrt{2\sigma}dW_t,$$

where $(W_t)_{t\geq 0}$ is a m-dimensional Brownian motion. In such a case, we look for a value which also depends on the realization of this stochastic process (p_t). In general we consider instead

$$b \equiv b(x,p,U),$$

as the evolution of this process may also be affected by the decisions of players. Thus, the problem now becomes the one of finding a solution $V:[0,T]\times\mathbb{R}^d\times\mathbb{R}^m\to\mathbb{R}^d$ of

$$\begin{cases} \partial_t U + (F(x,p,U)\cdot\nabla_x)U + b(x,p,U)\cdot\nabla_p U - \sigma\Delta_p U = G(x,p,U) \text{ in } (0,T)\times\mathbb{R}^{d+m}, \\ U(0,x,p) = U_0(x,p) \text{ in } \mathbb{R}^{d+m}. \end{cases} \tag{2}$$

This is a different approach to noise (or randomness) in mean field game from the usual additive common noise acting as a random push. This kind of dependence is extremely natural, consider for instance the case of $(p_t)_{t\geq 0}$ modelling environmental variables, we are then modelling the fact that the decisions of players are impacted by (and can impact) these environmental variables.

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Scenario Tree Reduction via Wasserstein Barycenters

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Abstract

Scenario tree reduction techniques are essential for achieving a balance between an accurate representation of uncertainties and computational complexity for solving multistage stochastic programming problems. In the realm of available techniques, the Kovacevic and Pichler algorithm (Ann. Oper. Res., 2015) stands out for employing the nested distance, a metric suited for comparing multistage scenario trees. However, dealing with large-scale scenario trees can lead to a prohibitive computational burden due to the algorithm's requirement of solving a large-scale linear problem per stage and iteration. This study concentrates on efficient approaches to solving such linear problems, recognizing that their solutions are Wasserstein barycenters of the tree nodes' probabilities on a given stage. We leverage advanced optimal transport techniques to compute Wasserstein barycenters and significantly improve the computational performance of the Kovacevic and Pichler algorithm. Our boosted variants of this algorithm are benchmarked on several multistage scenario trees. Our experiments show that compared to the original scenario tree reduction algorithm, our variants can be eight times faster for reducing scenario trees with 4 stages, 78 125 scenarios, and 97 656 nodes.

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Making Wasserstein gradient flows noisy with Stochastic Moment Dynamics

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Keywords: Wasserstein gradient descent, stochastic optimization, conditioned McKean-Vlasov diffusion, stochastic PDE

In finite dimension, the Langevin diffusion is a noisy version of gradient descent which can be used in order to minimize a non-convex function with many local minima. Following this idea for Wasserstein gradient flows (i.e. gradient descent in the space of probability measures) is not straightforward as there is no clear analogous of the Brownian motion in this setting. We propose a simple process, the Stochastic Moment Dynamics, which is designed so that some desired moments (e.g. mean, variance, etc.) follow some given diffusion (e.g. Brownian motion). This provides controllability properties, enabling to escape from local minima. The resulting process is a random flow over the space of probability measures, which can be described as a conditioned McKean-Vlasov diffusion or a stochastic PDE. In its basic form, it may explode in finite time (for instance, when trying to force the variance, which is non-negative, to be a Brownian motion). In practice it is approximated by interacting particles, influenced by a common noise. We show the convergence of the particle system toward the non-linear process up to explosion time, explain how to remove the well-posedness issue while preserving the controllability properties, and illustrate the process on simple examples.

Slicing Unbalanced Optimal Transport

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Keywords: computational optimal transport, optimization, sliced divergences

Optimal transport (OT) is a powerful framework to compare probability measures, a fundamental task in many statistical and machine learning problems. Substantial advances have been made in designing OT variants which are either computationally and statistically more efficient or robust. Among them, *sliced OT* distances have been extensively used to mitigate optimal transport's cubic algorithmic complexity and curse of dimensionality. In parallel, *unbalanced OT* was designed to allow comparisons of more general positive measures, while being more robust to outliers.

In this talk, we bridge the gap between those two concepts and develop a general framework for efficiently comparing positive measures. We notably formulate two different versions of sliced unbalanced OT, and study the associated topology and statistical properties. We then develop a GPU-friendly Frank-Wolfe like algorithm to compute the corresponding loss functions, and show that the resulting methodology is modular as it encompasses and extends prior related work. We finally present an empirical analysis of our loss functions and methodology on both synthetic and real datasets, to illustrate their computational efficiency, relevance and applicability to real-world scenarios including geophysical data.

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A heuristics for Pickup and Delivery Problem with Cooperative Robots

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Keywords: Warehouse, Traveling Salesman Problem, Pickup and Delivery Problem, Operations Research, Heuristics

We introduce a novel neighborhood structure and an efficient algorithm for constructing initial solutions to the Pickup and Delivery Problem with Cooperative Robots (PDP-CR), a complex optimization problem in automated warehouse. In PDP-CR, we have a fleet M of m identical robots and a set N of n tasks. Every task i is characterized by its pick-up location p_i, destination d_i, process time c_i, and number of robots required r_i. A task i cannot start until r_i robots arrive at its pickup point. This means early-arriving robots must wait for all required robots before starting the task from the pick-up point to the destination. All the robots begin their paths at a starting depot and finish their path at a returning depot. t_{ij} is the time for any robot to move from task i to task j on the graph, corresponding to the time a robot moves from d_i to p_j. The objective of PDP-CR is to minimize the makespan, which is the time the last robot reaches the returning depot. The problem's solutions are represented using a network flow graph, where each unit of flow corresponds to a robot. The proposed neighborhood and heuristics is created by "cut" the graph and redistributing the flows across the cut. With an enhanced version allowing for the insertion of tasks between the cut. This neighborhood is integrated into a tabu search framework and also used for constructing initial solutions to the PDP-CR. We evaluate the performance of the proposed heuristic against an existing Mixed Integer Linear Programming (MILP) model on both small and large datasets. Experimental results show that the heuristic reduces computation time while delivering high-quality solutions, proving its effectiveness and scalability for large-scale applications.

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On the Kalai-Smorodinsky solutions for Bi-objective Spanning Tree Problem

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Keywords: Minimum Spanning Tree Problem, Bi-Objective Combinatorial Optimization, Kalai-Smorodinsky solution

This work explores the Bi-objective Spanning Tree Problem (BSTP) [2], an extension of the classic Minimum Spanning Tree problem with a variety of practical applications. In the BSTP, each edge of a given graph has two distinct weights, and the goal is to find a spanning tree that minimizes both total weights simultaneously. Since BSTP is a specific case of bi-objective optimization, one common interest is enumerating all Pareto-optimal solutions. However, as BSTP is known to be NP-hard [3], no existing methods can enumerate these solutions within polynomial time.

In this study, we introduce a novel approach for identifying preferred Pareto-optimal solutions. Our method seeks to minimize the worst ratio of the two objective values relative to their respective maximum possible values. This approach draws on the well-established *Kalai-Smorodinsky* (KS) solutions [1] concept from cooperative game theory. Specifically, we extend this concept to the discrete case, where solutions can be obtained by optimizing convex combinations of the two objectives.

We begin by introducing the concept of KS solutions for the BSTP and characterizing its properties. We then propose a weakly polynomial time algorithm to compute these KS solutions for the BSTP. Finally, we present computational results on several instances and discuss the findings. Beyond its application to the BSTP, this work provides an efficient and interpretable method for solving bi-objective combinatorial optimization problems.

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Stochastic Localization via Iterative Posterior Sampling

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Keywords: sampling, diffusion, multi-modal distribution

Building upon score-based learning, new interest in stochastic localization techniques has recently emerged. In these models, one seeks to noise a sample from the data distribution through a stochastic process, called observation process, and progressively learns a denoiser associated to this dynamics. Apart from specific applications, the use of stochastic localization for the problem of sampling from an unnormalized target density has not been explored extensively. This work contributes to fill this gap. We consider a general stochastic localization framework and introduce an explicit class of observation processes, associated with flexible denoising schedules. We provide a complete methodology, Stochastic Localization via Iterative Posterior Sampling (SLIPS), to obtain approximate samples of this dynamics, and as a by-product, samples from the target distribution. Our scheme is based on a Markov chain Monte Carlo estimation of the denoiser and comes with detailed practical guidelines. We illustrate the benefits and applicability of SLIPS on several benchmarks of multi-modal distributions, including Gaussian mixtures in increasing dimensions, Bayesian logistic regression and a high-dimensional field system from statistical-mechanics.

Hybrid Quantum Search on Complex Networks: Noisy Spatial Search with Quantum Stochastic Walks

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Keywords: Quantum Walks, Random Walks, Spatial Search, Optimization

Continuous-time quantum walks are a model of transportation on complex networks modeled as graphs. Like their classical counterpart, they can be used to solve the spatial search problem [1, 2], which corresponds to finding an element in a graph-modeled database. Spatial search on the complete graph is the analog equivalent of Grover's algorithm and its performance depends on the graph's topology and spectral properties [3]. We use the Quantum Stochastic Walk framework [4] to mix both quantum and classical random walks in order to design a hybrid algorithm that can perform quantum search, classical search or a linear combination of the two. Previous work has shown that mixing classical and quantum dynamics can improve transport efficiency for many graphs [5, 6]. In this work we investigate the effects of mixing coherent and incoherent dynamics for spatial search on different types of graphs, and show that under certain conditions the hybrid regime is more efficient than either quantum or classical search.

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Chance-constrained stochastic zero-sum games

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Keywords: Stochastic games, Chance constraints, Random rewards

We consider two-person zero-sum stochastic dynamic games [4] with random rewards. We model such games with a pair of chance-constrained optimization problems, following the method in [2]. The aim of each player is to get the maximum payoff he can guarantee with a given probability $p \in [0,1]$, against the worst possible move from his opponent. We propose equivalent minimax reformulations when the rewards follow a multivariate Gaussian distribution.
A player has risk-seeking attitude if his confidence level p is less than $\frac{1}{2}$, otherwise he is risk-averse. Both cases are discussed separately.
We prove that the risk-seeking problem is equivalent to the constrained optimization of parameterized zero-sum stochastic games, which can be achieved by a fixed point iteration algorithm. We rewrite the risk-aversion problem as a discrete minimax, that can be solved using a linearization method, see [3] and [1]. We prove some convergence results for our algorithms, and illustrate them with numerical experiments.

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A regularized interior-point method for optimization problems with complementarity constraints

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Keywords: Nonlinear programming, interior-point method, complementarity constraints.

We present a regularized primal-dual interior-point method (IPM), based on an Augmented Lagrangian penalty. It is known that the regularization term stabilizes the IPM algorithm, allowing convergence on degenerate optimization problems. In this talk, we aim at solving nonlinear programs with complementarity constraints (MPCC). We revisit recent results that analyze the global convergence of the Augmented Lagrangian method for MPCC problems, and we detail how they apply to our regularized IPM algorithm. Furthermore, the regularized problem exhibits a specific algebraic structure we can leverage in the computation of the Newton step. As a result, we obtain a competitive algorithm that compares with a state-of-the-art solution method. We demonstrate the capability of the algorithm on large-scale security-constraints optimal power flow (SCOPF) problems, which write naturally with complementarity constraints.

Parametric Shape Optimization of Flagellated Micro-Swimmers Using Bayesian Techniques

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Keywords: Shape Optimization, Bayesian Optimization, Flagellated Helical Micro-swimmer, Free-Form-Deformation, Boundary-Element-Method

Micro-swimming is emerging as a significant research area due to its potential applications, particularly in the medical field for tasks like cargo transport and drug delivery [1]. A key factor in the performance of micro-robots is the optimization of their shape, which directly influences their motion.

This talk investigates shape optimization of micro-swimmers with one or two helical flagella. The Boundary Element Method (BEM) is used to simulate the dynamic [4]. Due to the complexity of our model, we focus on parametric shape optimization by using Bayesian Optimization (BO), which overcomes the challenges of computing gradient, constraints treatment and reduce the number of costly function evaluations inherent in swimmer dynamics. Additionally, we employ the Free-Form Deformation (FFD) technique [3], which provides a sufficiently complex admissible shape space. This is integrated with the Scalable Constrained Bayesian Optimization (SCBO) method [2], ideal for high-dimensional constrained optimization problems.

The optimized designs are compared to biological swimmers, revealing a wide variety of efficient swimming strategies, including both *pushers* and *pullers*.

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Quantum Computing for Integer Quadratic Programming

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Abstract : A principled way to solve NP-Hard combinatorial optimization problems by means of a quantum computing algorithm is to reformulate them as a quadratic unconstrained binary optimization (QUBO) problems. Formally, a QUBO is defined by

$$\min_{x\in\{0,1\}^n} x^T Qx + c^T x = x^T (Q + diag(c))\, x$$

where Q is a symmetric matrix $\in \mathbb{R}^{n\times n}$, and $c \in \mathbb{R}^n$. Since $x_i^2 = x_i$ for $i \in \{1, \ldots, n\}$ the equivalence holds where $diag(c)$ denotes the diagonal matrix whose diagonal elements are the entries of the vector c. Once the constrained problem has been formulated as a QUBO ($x \in \{0,1\}^n$), it can be converted to an Ising formulation ($s \in \{-1,1\}^n$ which represents whether the spin $i \in \{1,\ldots,n\}$ is pointing up or down and Q the coupling between pairs of spins) by means of the affine transformation $s \to 2x - 1$. This allows QUBOs to be solved on quantum computers through quantum annealing or variational algorithms like the quantum approximate optimization algorithm (QAOA). While the two formulations are isomorphic, the choice of spin or binary variables affects the way the problem can be expressed ; namely, QUBOs can always be expressed in both expanded and matrix forms (since $x_i^2 = x_i$), while Ising can be expressed in the expanded form, but not in the matrix form since (since $s_i^2 = |s_i|$).

One can distinguish three classes of combinatorial optimization problems i) those that contain no constraints other than requiring the variables to be binary and thus admit a natural QUBO formulation (e.g., max-cut), ii) those that contain constraints that admit an equivalent penalty function and thus can be reformulated as a QUBO model by multiplying them by positive, scalar penalty value chosen sufficiently large (to ensure the penalty term is equivalent to the classical constraint), and iii) linear (in)equality constrained problems such as the quadratic knapsack problem (QKP), assignment problem (QAP), etc. for which the main technique considered since so far is to penalize the quadratic violation of the linear constraints (after adding adequate slack variables for inequality constraints). For this class, the constraints of the original model are systematically moved to the objective function so that any violation of the linear constraints is penalized quadratically. Thus, the QUBO formulation is obtained by penalizing the quadratic violation of the constraints of the original model.

In this paper, we argue that the method for handling iii) should not be limited to penalize the quadratic violation of the constraints of the original model but also to penalize the linear violation of these constraints. In mixed-integer and nonlinear programming, this combined method is well-known augmented Lagrangian method. The augmented Lagrangian variant of the QUBO formulation has been recently considered in [2] to solve the KP using a quantum annealer. In this paper, we characterize penalty terms of the augmented Lagrangian function of general classes of linearly constrained problems that meets the properties of a QUBO. Then, we illustrate its application to the max k-cut problem and other QKP variants that remain challenging with quantum algorithms. Since the handling of inequality constraints by means of slack variables (being represented as a sum of binary variables) increases in turn the number of binary variables to the QUBO reformulation [1] ; we also provide alternatives to formulate the penalisation of inequality violation.

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Optimal Operation and Valuation of Electricity Storages

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Keywords: electricity markets, storage management, indifference pricing, multistage stochastic optimization, SDDP

The increasing proportion of renewable energy increases the uncertainties and seasonalities in the supply and thus, the price of electricity. This creates strong incentives to store energy. This paper applies computational techniques of convex stochastic optimization to optimal operation and valuation of electricity storages. Our valuations are based on indifference pricing which finds optimal hedging strategies and calibrates to the user's initial position, market views and risk preferences.

Indifference pricing has been extensively studied in pricing financial products in incomplete markets (see e.g. the collection [Car09] and the references therein). Applications of indifference pricing to energy have been studied, e.g., in [PTW09] and [CCGV17]. We solve the optimal storage management problem numerically using SDDP which allows for state constraints which are inherent in physical storages. Our approach is applicable to various specifications of storages. We illustrate the approach numerically by studying the sensitivities of indifference prices with respect to storage capacity, charging speed, risk aversion and volatility of spot prices.

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Duality in Convex Stochastic Optimization

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Keywords: multistage stochastic optimization, conjugate duality

We present a general formulation of convex stochastic optimization problems in finite discrete time that covers and extends classical problem formats from mathematical programming, optimal stopping, optimal control and financial mathematics. We study these problems within the general functional analytic duality theory of convex optimization. This will yield a dual problem whose optimum value coincides with that of the primal problem and whose optimal solutions can be used to characterize those of the primal. We demonstrate the general results with applications to stochastic control and financial mathematics. The talk is based on the forthcoming book [1].

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Electricity Dispatch and Pricing using Agent Decision Rules

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Keywords: electricity markets, storage, batteries, energy prices

Models for computing economic dispatch and prices in wholesale electricity market pools are typically deterministic multiperiod mathematical programs that are solved in a rolling horizon fashion. In convex settings with perfect foresight these optimization problems yield dispatch outcomes and locational marginal prices that solve a competitive equilibrium problem. Growing investment in renewable energy has increased the uncertainty in net demand to be met by dispatchable generation. To accommodate this, stochastic programming models formulated using scenario trees have been proposed for economic dispatch. The use of these models in practice is challenging for several reasons. Market participants need to agree on the scenarios used for uncertain parameters in the model, and realizations of these parameters will be different from those in any modelled scenario. When updated in a rolling horizon fashion, stochastic models can misprice the option value of energy storage and the value of changing current dispatch to meet future ramping constraints. This leads to uplift payments that compensate participants for the fact that the system operator forecasts the future incorrectly. We present a class of new economic dispatch models that attempt to overcome these drawbacks, based on agent decision rules (ADRs). Forecasting future outcomes or scenarios passes from the system operator to market participants who implicitly make state-dependent offers of energy through these decision rules. We show how storage and ramping can be priced correctly in convex markets and illustrate the advantages of the new model through simple examples.

A Newton-type Method for Constrained Optimization

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Keywords: Constrained Optimization, Newton, Higher Order, Operator Splitting

This work provides a Newton-type method for non-smooth equality and inequality constrained optimization. Feasibility problems can be seen as underdetermined equations, and thus Newton's method can be easily adapted, with the caveat that the output of each iteration is an affine subspace of $\mathbb{R}^n$. Optimization with affine equality constraints simplifies to unconstrained optimization in a lower dimensional subspace. The key insight of our algorithm consists in employing an operator splitting approach in order to combine the two methods, yielding a superlinearly convergent method for general equality constrained optimization. In order to analyze this algorithm, the correct notion of smoothness is that of Newton Differentiability. This notion is based on the work of Qi [1], and is a strictly weaker notion than the standard Fréchet differentiability. The benefit of our algorithm, as opposed to Riemannian Newton or projected Newton type methods is the fact that we do not require any hard to compute geometric information, such as arbitrary projectors onto a set or Riemannian exponential and logarithms. When comparing with SQP, our method can similarly be formulated as a quadratic program, with the advantage stemming from the compatibility of the solution to this program, by solving a linear systems, circumventing the expensive quadratic solver. We provide numerical results showcasing our algorithm compared to a suit of classic solvers implemented in the Julia programming language. Finally, for an inequality constrained optimization problem, we adapt our algorithm by replacing equalities with inequalities, but this raises difficulties by converting linear systems into non-linear systems. In order to solve this problems, we use ideas similar to Riemannian geometry to define a notion of pseudo-invertability, sufficient for showing superlinear convergence of our algorithm.

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Restoring Definiteness by Advancing Towards an Inner SDP Solution (at the Edge of the SDP Cone)

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Keywords: positive semidefinite (SDP) matrix, SDP error, minimum eigenvalue function

What means restoring definiteness? Given a non-SDP matrix S, we seek an SDP matrix that is (very) close or related to S; if S is almost SDP, we simply remove some error from a matrix S that should have been SDP in theory. I needed this problem inside an optimization algorithm called Projective Cutting Planes presented at PGMO-DAYS 2023 (or preprint CoRR abs/2311.09365). The more general context was provided by [1], where we find that "in practice the matrices obtained may lack definiteness due to missing or asynchronous observations or by the nature of their construction, such as in stress testing in finance or when correlations are determined within groups and then the groups are joined together. To ensure the validity of the subsequent analysis the indefinite approximation needs to be replaced by a valid covariance or correlation (SDP) matrix, which we call a replacement matrix. This is needed in a very wide variety of applications including modeling public health [8] and dietary intakes [36], determination of insurance premiums for crops [12], simulation of wireless links in vehicular networks [37], reservoir modeling [26], oceanography [32], and horse breeding [35]."

In both references above, we are provided a target SDP matrix $X \succeq \mathbf{0}$ and the problem reduces to optimizing $t^* = \max\{t : X + tD \succeq \mathbf{0}\}$, where where $D = S - X$. If X is deeply SDP—*i.e.*, if $\lambda_{\min}(X)$ is (far) above 0—we can apply the Cholesky decomposition $X = KK^\top$. We then determine $D' = K^{-1}DK^{-1^\top}$ and the problem is rewritten as $\max\{t : KI_nK^\top + tKD'K^\top \succeq \mathbf{0}\}$. By congruence, this reduces to $\max\{t : I_n + tD' \succeq \mathbf{0}\}$, which gives the solution $t^* = -\frac{1}{\lambda_{\min}(D')}$.

The problem becomes hard at the edge of the SDP cone, when both $v_0 = \lambda_{\min}(X)$ and $v_s = \lambda_{\min}(S)$ are very close to zero. If $v_0 \ll 10^{-5}$, then matrix K^{-1} can contain some huge values and the above calculation is too error-prone. Moreover, most solvers and most people consider X is SDP if $v_0 \geq -10^{-6}$, because of numerical imprecisions are always present when computing $\lambda_{\min}$. The Cholesky decomposition (or related ones) simply fails if $v_0 < 0$. I propose circumventing this as follows. Let $\alpha = 10^{-5} - v_0$ and raise X to $X_\alpha = X + \alpha I$, so that $\lambda_{\min}(X_\alpha) = 10^{-5}$. We can now use the Cholesky approach described above to find $t^*_\alpha = \max\{t : X_\alpha + tD \succeq \mathbf{0}\}$, so that $\lambda_{\min}(X_\alpha + t^*_\alpha \cdot D) = 0$. We develop $\lambda_{\min}(X + t^*_\alpha \cdot D) = \lambda_{\min}(X_\alpha - \alpha I_n + t^*_\alpha \cdot D) = -\alpha < 0$. This means that the sought t^* is between 0 and t^*_α. Instead of using the bisection method (or a dichotomy) as in [1], we exploit the fact that the function $\lambda_{\min}$ is concave. The linear function f such that $f(0) = v_0$ and $f(t^*_\alpha) = -\alpha$ underestimates $\hat{\lambda}_{\min}(t) := \lambda_{\min}(X + t \cdot D)$. Let t^1_α the unique real such that $f(t^1_\alpha) = -10^{-6}$ (the SDP tolerance). We have $\hat{\lambda}_{\min}(t^1_\alpha) \geq f(t^1_\alpha)$. In case of equality, the problem is solved. Otherwise, we replace $(0, v_0)$ with $(t^1_\alpha, \hat{\lambda}_{\min}(t^1_\alpha))$ and repeat.

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Improved learning rates in multi-unit uniform price auctions

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Keywords: Auctions, Online Learning, Bandits Algorithms

Motivated by the strategic participation of electricity producers in electricity day-ahead market, we study the problem of online learning in repeated multi-unit uniform price auctions, with adversarial opposing bids. The main contribution of this paper is the introduction of a new modelling of the action space. Indeed, we prove that a learning algorithm leveraging the structure of this problem achieves a regret of $\tilde{O}(K^{4/3}T^{2/3})$ under bandit feedback, improving over the bound of $\tilde{O}(K^{7/4}T^{3/4})$ previously obtained in the literature. This improved regret rate is tight up to logarithmic terms. Inspired by electricity reserve markets, we further introduce a different feedback model under which all winning bids are revealed. This feedback interpolate between the full-information and bandit scenario depending on the auctions' result. We prove that, under this feedback, the algorithm that we propose achieves regret $\tilde{O}(K^{5/2}\sqrt{T})$.

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Strategic geometric graphs through mean field games

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Keywords: Mean Field Games, Geometric Graphs, Riemannian manifolds

We exploit the structure of geometric graphs on Riemannian manifolds to analyze strategic dynamic graphs at the limit, when the number of nodes tends to infinity. This framework allows to preserve intrinsic geometrical information about the limiting graph structure, such as the Ollivier curvature. After introducing the setting, we derive a mean field game system, which models a strategic equilibrium between the nodes. It has the usual structure with the distinction of being set on a manifold. Finally, we establish existence and uniqueness of solutions to the system when the Hamiltonian is quadratic for a class of non-necessarily compact Riemannian manifolds, referred to as manifolds of bounded geometry.

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How Difficult is it to Check if a Multilevel Optimization Problem is Unbounded?

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Keywords: Computational Complexity, Bilevel Optimization, Multilevel Optimization

Despite their widespread applications, bilevel problems are known to be challenging to solve in practice, as even in the linear case bilevel problems are strongly NP-hard. However, unboundedness in these problems is often overlooked, with research typically assuming boundedness of their feasible sets. This presentation addresses this gap by exploring unboundedness in linear bilevel optimization and its computational complexity. We introduce the notion of direction of unboundedness to linear bilevel programs, and show that the decision problem of knowing whether a linear bilevel program with linking upper-level constraints is unbounded is NP-complete. Additionally, we prove that determining unboundedness in a k-level problem is Σ^p_{k-1}-hard. Finally, we detail an algorithmic approach to detecting unboundedness in a bilevel setting.

Management of a Battery on Short-Term Electricity Markets

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Keywords: Stochastic Dynamic Programming, Energy Storage Systems, Intraday Market, aFRR Market

Our work aims to quantify the benefit of storage flexibilities such as a battery on several short-term electricity markets. We especially focus on two different markets, the intraday market (ID) and the activation market of the automatic Frequency Restoration Reserve (aFRR), also known as the secondary reserve. We propose algorithms to optimize the management of a small battery ($\leq 5\ MWh$) on these markets. In this talk, we first present the modeling of the problem, then we show some theoretical results and numerical simulations.

Smart Charging and Optimization of Personalized Flexibility Services for Electric Vehicles's Users

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Keywords: Electric vehicles, charging station, price menu design, linear program, bi-level optimization

To address climate change, energy providers are accelerating the electrification of various sectors, particularly the transportation sector. Electric vehicles (EVs) play a key role in combating climate change, they help to reduce the greenhouse effect. As a result, EVs have been extensively researched. The increasing number of EVs may cause an overload on the electrical infrastructure. To prevent this, we encourage EVs to be flexible. An EV is considered flexible when it accepts a charging time longer than the minimum time required to complete its charging. To encourage EVs to be flexible, we offer them a price menu design that includes multiple options with different durations required to meet the energy demands of the EVs and their corresponding charging prices. This allows EVs to choose the option that best suits their needs. Therefore, we assume that the charging fee decreases when an EV chooses to remain longer to encourage them to do so. Note that every EV will have a customized menu from the charging station. The price menu design was inspired by article [1]. Also, our goal is to maximize the profit of the station by increasing its revenue from charging the vehicles and minimizing the cost associated with this charging. In our study, this cost is divided into two parts: the purchase cost of electricity and a penalty the station must pay for exceeding the subscribed power. To achieve this, we study the optimal way to share power between the EVs over our time horizon. On the other hand, EV will choose an option from the price menu such that it maximizes its utility. Our problem is a bi-level optimization problem that we can transform into a single-level optimization problem. We study and implement three methods that allow us to make this transformation: the classical KKT transformation, the optimal value transformation, and a reformulation from article [1]. We then compare the results of these methods.

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Illuminating the Diversity-Fitness Trade-Off in Black-Box Optimization

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Keywords: Benchmarking, Diversity, Multi-modal Optimization, Niching, BBOB

In real-world applications, users often favor structurally diverse design choices over one high-quality solution. It is hence important to consider more solutions that decision-makers can compare and further explore based on additional criteria. Alongside the existing approaches of evolutionary diversity optimization, quality diversity, and multimodal optimization, this paper presents a fresh perspective on this challenge by considering the problem of identifying a fixed number of solutions with a pairwise distance above a specified threshold while maximizing their average quality.

We obtain first insight into these objectives by performing a subset selection on the search trajectories of different well-established search heuristics, whether specifically designed with diversity in mind or not. We emphasize that the main goal of our work is not to present a new algorithm but to look at the problem in a more fundamental and theoretically tractable way by asking the question: *What trade-off exists between the minimum distance within batches of solutions and the average quality of their fitness?* These insights also provide us with a way of making general claims concerning the properties of optimization problems that shall be useful in turn for benchmarking algorithms of the approaches enumerated above.

A possibly surprising outcome of our empirical study is the observation that naive uniform random sampling establishes a very strong baseline for our problem, hardly ever outperformed by the search trajectories of the considered heuristics. We interpret these results as a motivation to develop algorithms tailored to produce diverse solutions of high average quality.

Optimization-based efficient algorithms for robot dynamics simulation and control

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Keywords: Rigid-body dynamics, Optimal control, Contact dynamics

We present our recent progress on efficient algorithms for simulating rigid-body dynamics. With simulation being the primary computational bottleneck while solving robot optimal control problems and training reinforcement learning-based controllers, speeding it up is a key problem.

Equality-constrained dynamics problems are equivalently formulated as an equivalent quadratic programming (QP) [1] problems using Gauss' principle of least constraint. We solve these QP problems for kinematic trees using the augmented Lagrangian method and by exploiting the tree structure to derive the constrained articulated body algorithm (constrainedABA) [2]. ConstrainedABA makes no constraint independence assumptions, has linear computational complexity in the robot's degrees-of-freedom and the constraint dimension, and leads to a speed-up of over two times for high dimensional robots like humanoids compared to the cubic-complexity algorithms prevalent in existing simulators.

The talk will also present latest research on extending constrainedABA to kinematic graphs with kinematic closed loops, and on solving the nonlinear complementarity problems associated with frictional contact simulation efficiently using constrainedABA as the innner solver for both an ADMM-based [3] and an interior-point based approach.

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Operating a battery at minimum cost under reserve commitment constraints

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Keywords: Electricity storage, intraday market, reserve commitment, stochastic control

Consider an electric battery whose owner has committed with a transmission system operator (e.g., RTE in France) to making at its disposal a "reserve" of electricity along a given day: every hour, a random quantity will be discharged from or charged to the battery; the commitment is on the range of this random variable. To respond at best to this commitment, the owner of the battery can buy or sell electricity every hour on the "intraday" market, at prices following a random process. Moreover, there is a maximal energy level S, a bound $C^{\max}$ on the maximal variation of this level, and an extra final cost ψ anticipating the future; the exact value of both the reserve activation R_t and the price P_t is only revealed at time t. The problem consists in finding a policy minimizing the expected cost of ensuring satisfaction of the commitment. To the authors' knowledge, this problem has not been addressed yet (see [1, 2] for related questions). It can be written as

$$\begin{array}{lll} \text{minimize} & \mathbb{E}\left[\sum_{t=1}^{T} P_t x_t + \psi(s_T)\right] & \\ \text{s.t.} & s_t = s_{t-1} + x_t - R_t & \forall t \\ & s_t \overset{\text{a.s}}{\in} [0, S] & \forall t \\ & |x_t - R_t| \overset{\text{a.s}}{\in} [-C^{\max}, C^{\max}] & \forall t \\ & \text{non-anticipative } s_t, x_t & \forall t\,. \end{array}$$

We propose an algorithm that computes good solutions when the problem is feasible. This algorithm, based on dynamic programming and discretization of the state space, relies on a preliminary lemma characterizing beforehand the feasible values of the energy level for every hour. The quality of the solutions is evaluated thanks to another theoretical contribution that provides closed-form expressions for the optimal policies when $C^{\max} = +\infty$. This result is also interesting for its own sake since there are real-world situations where this condition on $C^{\max}$ is met. Our work also shows, via toy examples and simulations, that a good knowledge of the distribution of R_t allows to decrease dramatically the cost of operating the battery.

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Dynamic two-stage programming for the stochastic lot-sizing problem with inventory bounds and lost sales

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Keywords: Multistage stochastic lot-sizing, Dynamic programming, Benders decomposition

The present work introduces a novel approach designed to solve the multi-stage stochastic lot-sizing problem with inventory bounds and lost sales. This optimization problem aims at minimizing the production costs of an item supplied to a customer according to an uncertain demand, while managing a storage of limited capacity and penalties in case of stock-out.

It is motivated by use-cases in the green hydrogen production industry, where electricity from both uncertain renewable assets and a costly connection to the distribution grid is used to produce a low-carbon hydrogen. Such a configuration coupled with on-site storage enables power plants to adjust to variations in renewable power sources. A multi-stage stochastic problem has to be studied in order to correctly represent the successive production decisions and uncertainty realizations throughout the planning horizon [1]. The missed opportunity to produce from the uncertain renewable sources can be alternatively represented as a loss induced by a default in demand supply. Hence, it is similar to the stochastic lot-sizing problem with inventory bounds and lost sales.

To solve this problem, we propose an approach based on dynamic programming and the exact resolution of several two-stage stochastic programs. At each stage, the optimal production costs of all future stages depending on the final inventory value of the current stage are represented by a cost-to-go function. From the last stage to the first one, we dynamically estimate such cost-to-go functions and efficiently solve the stochastic lot-sizing problem in the process.

At each step of that dynamical process, a two-stage problem is solved to minimize the cost of production at current and future stages according to the cost-to-go function. To further improve its computational efficiency when accounting for a large number of scenarios, an approximate model is introduced and a Benders decomposition is implemented. In particular, an efficient polynomial algorithm for the resulting dual sub-problem is proposed to fasten its solving time.

Numerical results demonstrate the benefits of this approach, both for theoretical models and for use-cases in the green hydrogen production industry.

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Scenario-Based Decomposition for Optimal Power Curtailment with Priority-Level Producers

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Keywords: stochastic optimal power flow, scenario/dual decomposition, mixed-integer programming, augmented lagrangian, ADMM

The use of renewable energy sources (RES) is becoming increasingly important as the world transitions to a more sustainable and environmentally friendly energy system. However, the variability and intermittency of RES present significant challenges to the efficient and reliable operation of power systems. One mechanism for addressing these challenges is power curtailment, which is initiated by Distribution System Operators (DSOs) based on the priority levels assigned to producers through their contract type. In this work, we propose a novel formulation of the power curtailment problem incorporating producers' curtailment priority levels and associated rules, modeled with the use of binary variables. Uncertainties related to RES production and consumption are addressed through a probability maximization problem, leading to stochastic alternating-current power flow equations. The resulting optimization problem features two types of nonconvexity: one related to binary variables and the other arising from alternating-current power flow equations. By employing a scenario decomposition method we can separate these issues and break down the stochastic component to a deterministic alternating-current optimal power flow (AC-OPF) problems per scenario. This approach results in a mixed-integer linear programming (MILP) problem that captures prioritization rules and parallelizable AC-OPFs that ensure technical feasibility. The proposed methodology is tested in a 33-bus distribution network for several dual update rules (ADMM, proximal Lagrangian) aimed at ensuring consensus for different scenarios.

BOBILib: Bilevel Optimization (Benchmark) Instance Library

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Keywords: Mixed integer linear bilevel optimization, Benchmarking, Instance library, Problem instances, Computational optimization

In this talk, we present the BOBILib, a collection of more than 2500 instances of mixed-integer linear bilevel optimization problems. The goal of this library is to make a large and well-curated set of test instances freely available for the research community so that new and existing algorithms in bilevel optimization can be tested and compared in a standardized way. The library is sub-divided into instances of different types and also contains a benchmark instance set. Moreover, we present a new data format for mixed-integer linear bilevel problems that is less error-prone compared to an older format that will now be deprecated. We provide numerical results for all instances of the library using available bilevel solvers. Based on these numerical results, we select the benchmark instance set, which provides a meaningful basis for experimental comparisons of solution methods in a moderate time. Each instance, together with a solution file if a feasible point or an optimal solution is known, can be downloaded at `https://bobilib.org`. This is joint work with Johannes Thürauf, Thomas Kleinert, Ivana Ljubic, and Ted Ralphs.

On Problem-based Scenario Reductions

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Keywords: Scenario Reduction, Stochastic Optimization, Two-stage Stochastic Optimization

In stochastic optimization, a random variable is often modeled as a discrete probability distribution with n atoms. The number of atoms is a common bottleneck for solving an associated stochastic optimization problem.

Scenario reduction aims to construct an approximate discrete probability distribution with $m \ll n$ atoms. With the downstream optimization problem in mind, scenario reduction differs from usual clustering methods, like k-means, as the value of the stochastic problem with the reduced distribution remains close to the original optimal value.

Recent works [1, 2] build problem-based metrics to evaluate distances between scenarios. In these works, costly problem-based metrics are built by solving every deterministic problem when one fixes the realization of the random variable. Then scenario reduction is done using these metrics. They have shown that problem-based metrics allow scenario reduction methods to provide better reduced discrete distribution.

We propose to iteratively compute and refine problem-based metrics while solving more and more relevant stochastic problems with reduced distributions. Thus, we alleviate the cost of computing such metrics while retaining the quality of the reduced distributions after scenario reduction.

Presentation of these old and new methods, their stability properties along with extensive numerical comparisons will be presented.

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Rational SOS certificates of any polynomial over its zero-dimensional gradient ideal

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Keywords: Polynomial non-negativity, sum of squares, zero-dimensional ideal, rational univariate representation

Given a multivariate polynomial with rational coefficients, we present rational certificates of non-negativity based on sum of squares decomposition, under the assumption that its gradient ideal is zero dimensional. We improve the prior work of Victor Magron, Mohab Safey El Din, Trung-Hieu Vu [1] by considering a bigger ring of decomposition which allows us to drop the radical condition on the gradient ideal. This is a joint work with Matías Bender and Elias Tsigaridas.

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Distributed Economic Dispatch in Power Networks Harnessing Data Center Flexibility

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Keywords: Data centers, flexibility, economic dispatch, Lagrangian decomposition.

In this work, we focus on the interaction of Data Centers (DCs) with electric power networks in an economic dispatch problem, harnessing the ability of DCs to serve as flexible loads that can alter their power consumption to alleviate network congestion. Since DCs can transfer workload between each other, controlling their output via some demand response mechanism that respects power generation and network constraints, while also accounting for DC Quality of Service (QoS) can achieve system-wide benefits. From the DC perspective, we aim to explore the benefits of their incorporation in an economic dispatch problem, and demonstrate that they can achieve significant cost savings for the entire system (data and power networks), enabling the IT sector to contribute to societal sustainability efforts.

Our main contributions are as follows. First, we leverage results from queuing theory to model DCs and form QoS-based cost functions — signifying how well a DC can carry out its workload given an amount of active servers, for which we provide convexity guarantees under certain conditions. Second, we integrate DCs in a centralized economic dispatch problem that determines, apart from power generation, DC workload shifting and server utilization, while respecting power network constraints. Third, we provide a tractable decentralized formulation of the economic dispatch problem employing Lagrangian decomposition and a primal-dual algorithm, which can cater for both the power network constraints, and, most importantly, the DC workload shifting, in a distributed manner that scales for the "coupled" data and power networks. Fourth, we present experimental results on a standard power network that provide useful insights on the system-wide benefits from harnessing DC flexibility in the economic dispatch problem, emphasizing on the trade-offs between the DC locations, their efficiencies, and QoS costs.

Continuous-time Optimal Control for Trajectory Planning of Autonomous Vehicles under Uncertainty

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Keywords: vehicle autonomous systems, trajectory planning, chance-constraint optimisation, continuous-time optimal control, stochastic modelling.

In this paper, we present a continuous-time optimal control framework for generating reference trajectories for autonomous vehicles. The method developed involves chance constraints for modelling uncertainty. A previous work ([1]) presented such a model in discrete time and designed for urban driving scenarios only. We extend those results in continuous time. It generates reference trajectories on urban driving scenarios with faster computation and better capacity to capture uncertainty. Our model is less likely to violate the problem's constraints in risky scenarios, and is also robust for optimal control on long term horizons in national roads and highways.

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Stabilization and Optimal Control of an Interconnected SDE – Scalar PDE System

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Keywords: Stochastic systems, Partial differential equations, Interconnection, Backstepping, Optimal control

Interconnected systems are frequently advocated to model complex systems such as energy transportation networks [1]. In this context, Stochastic Differential Equations (SDE) are key for capturing hazardous uncertain fluctuations that impact nominal dynamics. At the same time, Partial Differential Equations (PDE) play a crucial role in modeling potentially degrading second-order physical effects. Recently, there has been a notable surge of interest in integrating these two approaches to derive very accurate dynamics models, yielding sophisticated interconnected SDE–PDE systems. In this talk, we will introduce novel, so far unexplored methodologies we have recently developed to control such complex systems [2, 3]. Specifically, we will focus on solving optimal control problems subject to interconnected SDE–PDE dynamics. The objective consists of steering the SDE to a desired target area in expectation, while minimizing the variance of the SDE state by controlling the PDE at the boundary. For this, we will start by introducing the problem of boundary control of PDE via backstepping-type methods. These methods are particularly effective for designing stabilizing boundary controllers. Then, we will show that, for a certain, yet fairly large class of interconnected SDE–PDE systems, backstepping enables transforming the problem into a more manageable input-delayed SDE. Finally, we will present strategies to control input-delayed SDE with additive noise, detailing methods that guarantee variance reduction.

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Compact Knapsack: a Semidefinite Approach

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Keywords: Combinatorial optimization, Knapsack, Cuts, Semidefinite programming

Context The min-knapsack with compactness constraint problem (min-KCP) is a variant of the classical knapsack problem, introduced in [1], to capture applications in statistics. Given items $i \in [\![n]\!]$ with cost c_i and weight w_i, the goal is to select objects $S \subseteq [\![n]\!]$ minimizing the cost of the selection, while ensuring that the total weight exceeds a given constant q, in such way that the selection is **compact**: for a given $\Delta \in \mathbb{N}$, the distance separating two consecutive selected objects $i, j \in S$ must not exceed Δ. For example, this constraint is not verified by items 1 and 4 on Figure 1; adding item 2 yields the compact selection proposed on Figure 2.

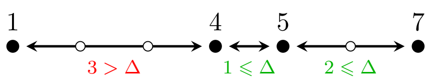

Figure 1: a **non**-compact selection with $\Delta = 2$ and $n = 7$

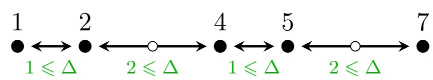

Figure 2: a compact selection with $\Delta = 2$ and $n = 7$

Approach This problem can easily be modeled as a 0, 1-integer linear program, as proposed by [1]. In this work, we propose a semidefinite formulation of this problem, and explain how to strengthen the basic semidefinite relaxation in order to get better bounds, using useful techniques in [2, 4, 5]. Finally, we will show how the semidefinite model compares with the classical known cuts for the knapsack problem, such as cover inequalities from [3].

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Analog Quantum Computing for Combinatorial Optimization Problems

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Keywords: analog quantum computing, optimization problems

Coherent and programmable arrays of neutral atoms provide a natural platform for solving combinatorial optimization problems, as they can be precisely manipulated using lasers and external fields, allowing for fine control over their quantum states. In the context of graph-structured problems, the atoms can represent the nodes, interactions between atoms encode the cost functions, and the system natural evolution can drive it toward a quantum state that represents the (near-)optimal solution.

Different pairs of electronic levels can effectively encode a two-level system (or qubit). Here, we focus on the case where these two states are the electronic ground state $|g\rangle \equiv |0\rangle$ and a s-Rydberg level $|r\rangle \equiv |1\rangle$. In this setup, atoms can be arbitrarily positioned in space, such that the effective Hamiltonian of the system at time t is expressed as:

$$H(t) = \Omega(t) \sum_{u=1}^{|\mathcal{V}|} \hat{\sigma}_u^x - \Delta(t) \sum_{u=1}^{|\mathcal{V}|} \hat{n}_u + \sum_{u<v=1}^{|\mathcal{V}|} U_{uv} \hat{n}_u \hat{n}_v, \tag{1}$$

where the amplitude (giving the Rabi frequency) $\Omega(t)$ and detuning $\Delta(t)$ of the laser can be controlled over time, and the interaction strength $U_{uv} \propto |\mathbf{r}_u - \mathbf{r}_v|^{-6}$ is a function of the distance between atom u and atom v. This choice of energy levels has already been employed in encoding Quadratic Unconstrained Binary Optimization (QUBO) problems, proving successful in several cases.

In recent years, the neutral atom technology has demonstrated significant theoretical and experimental advancements, positioning itself as a front-runner platform for running quantum algorithms. One unique advantage of this technology is the ability to arbitrarily reconfigure the geometry of the qubit register from shot to shot, enabling native embedding of graph-structured problems at the hardware level. This capability has profound implications for solving complex optimization and machine learning tasks. By driving qubits, quantum states are generated that retain the complex properties of graphs, providing either direct solutions or serving as resources in hybrid quantum-classical schemes.

Here, we review recent advancements in quantum algorithms for graph problems running on neutral atom Quantum Processing Units (QPUs). We also discuss newly introduced embedding and problem-solving techniques and outline ongoing hardware improvements, with a focus on increasing the scalability, controllability, and computation repetition rate of neutral atom QPUs.

The Share of Decarbonization Efforts between French Regions : A Bilevel Optimization Model

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Keywords: decarbonization, game theory, bilevel optimization

In 2019, the French Conseil Economique et Social (Economic and Social Council) raised the issue of the lack of coordination between regions, which risks (i) missing national emission targets and (ii) preventing the specific features of each region from being exploited [env]. Most of the works on the share of decarbonization efforts operates at the global scale [BH08], or European scale [PJ22]. To tackle smaller geographical areas like French regions, under the authorities of a national level, we introduce a multi-followers Stackelberg game between a national decarbonization planner and regional planners. The equilibrium is reached by solving a bilevel optimization problem. The national planner wants to reduce the cost of decarbonization as much as possible, while meeting the European targets of a 50% reduction of greenhouse gas (GHG) emissions by 2030 and carbon neutrality by 2050. Firstly, she imposes emission reductions on regional planners and invests in subsidies or infrastructure to facilitate their decarbonization. Secondly, regional planners wish to maximize the utility of their region's inhabitants, while respecting the emissions limits set by the national planner. Our focus is on reducing energy-related GHG emissions in passenger transport (TP) and residential buildings (BR). Solving this problem requires a reformulation of the followers' (regional planners') problems using KKT conditions, and a linearization of the bilinear objective terms.

In this presentation, we will describe this Stackelberg game, and analyze its results on our case study: the sharing of BR and TP decarbonization efforts between French regions.

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Inertial Dynamics and Accelerated Gradient Methods for Strongly Convex Functions

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Keywords: Accelerated gradient descent, inertial system, strongly convex functions

We introduce an inertial system with Hessian driven damping, which plays an important role in fast convex optimization. Two energy functions are provided for the analysis of accelerated gradient methods for strongly convex functions. By doing so, we recover convergence rate results ranging from $\mathcal{O}\left(\exp\left(-k\sqrt{\frac{\mu}{L}}\right)\right)$ to $\mathcal{O}\left(\exp\left(-2k\sqrt{\frac{\mu}{L}}\right)\right)$, which is the fastest known so far. Through the analyses, one can gain an intuitive understanding of the acceleration mechanism behind the accelerated gradient methods. These analyses can also be extended to convex functions satisfying quadratic functional growth condition, where new convergence rate results are derived.

Optimal Strategy against Straightforward Bidding in Clock Auctions

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Keywords: Auction · Bidding Strategy · POMDP · Optimal Control

We study a model of auction representative of the 5G auction in France. We determine the optimal strategy of a bidder, assuming that the valuations of competitors are unknown to this bidder and that competitors adopt the straightforward bidding strategy. Our model is based on a Partially Observable Markov Decision Process (POMDP). We show in particular that this special POMDP admits a concise statistics, avoiding the solution of a dynamic programming equation in the space of beliefs. We illustrate our results by numerical experiments, comparing the value of the bidder with the value of a perfectly informed one.

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Runtime Analysis of the SMS-EMOA for Many-Objective Optimization

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The widely used multiobjective optimizer NSGA-II was recently proven to have considerable difficulties in many-objective optimization. In contrast, experimental results in the literature show a good performance of the SMS-EMOA, which can be seen as a steady-state version of the NSGA-II that uses the hypervolume contribution instead of the crowding distance as the second selection criterion.

This paper [1] conducts the first rigorous runtime analysis of the SMS-EMOA for many-objective optimization. To this aim, we first propose a many-objective counterpart, the m-objective mOJZJ problem, of the popular bi-objective OneJumpZeroJump benchmark. We prove that SMS-EMOA computes the full Pareto front of this benchmark in an expected number of $O(\mu M n^k)$ iterations, where n denotes the problem size (length of the bit-string representation), k the gap size (a difficulty parameter of the problem), $M = (2n/m - 2k + 3)^{m/2}$ the size of the Pareto front, and μ the population size (at least the same size as the largest incomparable set). This result together with the existing negative result for the original NSGA-II shows that, in principle, the general approach of the NSGA-II is suitable for many-objective optimization, but the crowding distance as tie-breaker has deficiencies.

Our main technical insight, a general condition ensuring that the SMS-EMOA does not lose Pareto-optimal objective values, promises to be useful also in other runtime analyses of this algorithm.

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