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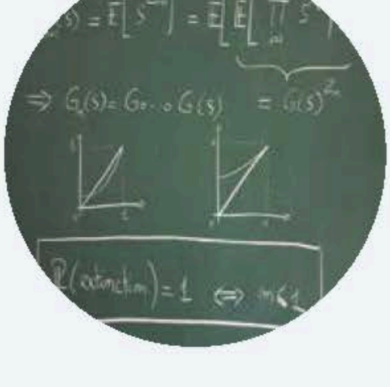

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LM-CMA-ES: an alternative to L-BFGS for large-scale black-box optimization

Ilya Loshchilov

Abstract The limited memory BFGS method (L-BFGS) of Liu and Nocedal (1989) is often considered to be the method of choice for continuous optimization when first- and/or second- order information is available. For instance, it is quite popular in the fast growing field of deep learning algorithms, where it is competitive to stochastic gradient approaches (SGDs). The use of L-BFGS can be complicated when gradient information is not available and therefore should be numerically estimated. The accuracy of this estimation (e.g., by finite differencing methods) is often problem-dependent that may lead to premature convergence of the algorithm when a black-box scenario is considered.

In this paper, we demonstrate an alternative to L-BFGS, the limited memory Covariance Matrix Adaptation Evolution Strategy (LM-CMA-ES) proposed by Loshchilov (2014). The LM-CMA-ES is a stochastic derivative-free algorithm for numerical optimization of non-linear, non-convex optimization problems. The algorithm has an important property of invariance w.r.t. strictly increasing transformations of the objective function, such transformations even performing from one iteration to another do not compromise (in contrast to the L-BFGS) algorithm's ability to approach the optimum. The LM-CMA-ES demonstrates a comparable performance on 100-, 1000- and 100.000- dimensional benchmark problems.

Keywords LM-CMA-ES · L-BFGS · large-scale optimization · black-box optimization

Mathematics Subject Classification (2000) MSC 68T20 · MSC 65Y20

1 Introduction

In a black-box scenario, knowledge about an objective function $f : \boldsymbol{X} \to \mathbb{R}$, to be optimized on some space $\boldsymbol{X}$, is restricted to the handling of a device that delivers the value of $f(\boldsymbol{x})$ for any input $\boldsymbol{x} \in \boldsymbol{X}$ [5]. The goal of black-box optimization if to find solutions with small (in the case of minimization) value $f(\boldsymbol{x})$, using the least number of calls to the function f [5]. In continuous domain, $f : \mathbb{R}^n \to \mathbb{R}$, where n is the number of variables.

The use of gradient-based approaches in the black-box scenario is complicated by the fact that gradient information is not available and therefore should be estimated with costly finite difference methods (e.g., $n+1$ function evaluation per gradient estimation for finite differencing and $2n$ for central differencing).

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The latter procedures are problem-sensitive and require a priori knowledge about the problem at hand, e.g., scaling of f and decision variables.

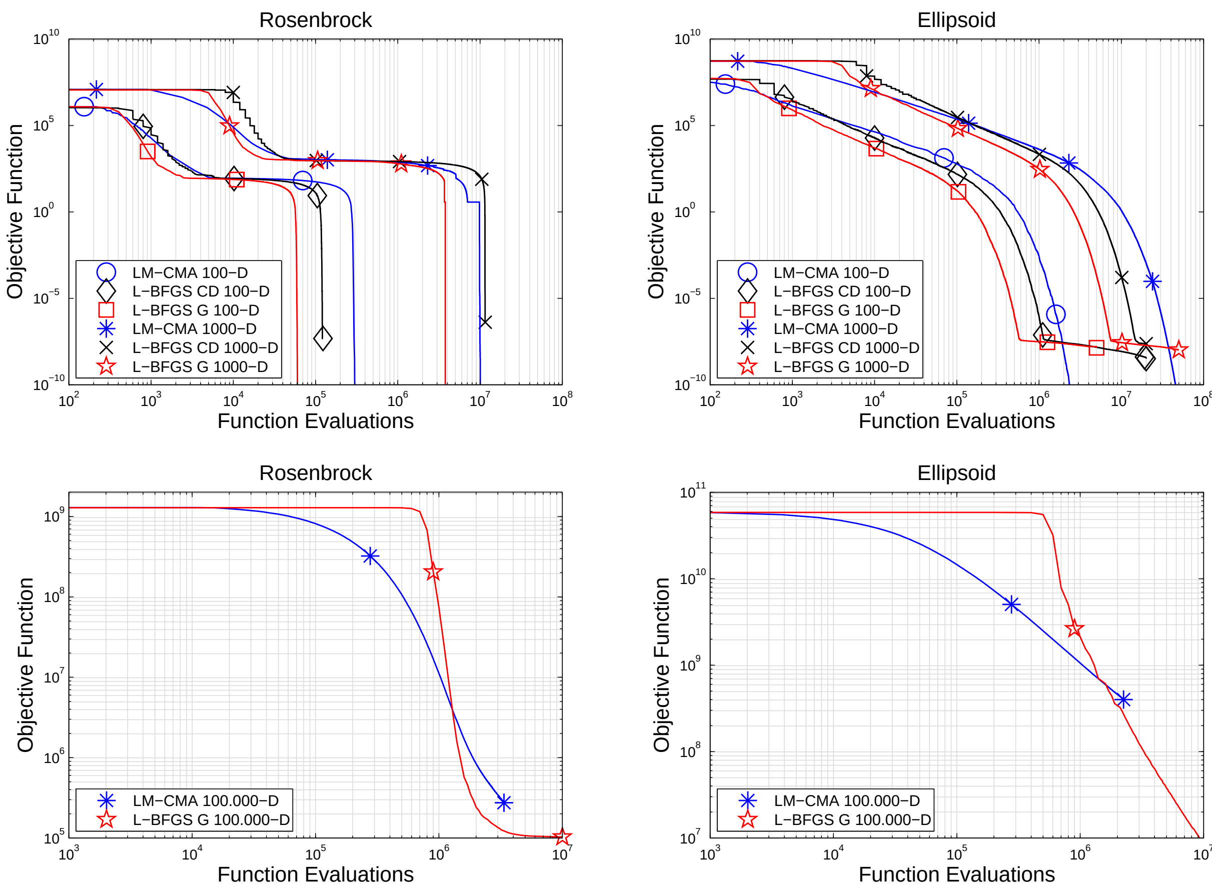

Fig. 1 The trajectories show the median of 15 runs of LM-CMA-ES, L-BFGS G (L-BFGS with exact gradients provided at the cost of $n+1$ evaluations per gradient) and L-BFGS CD (L-BFGS with gradients estimated by central differencing) on 100-, 1000- (**Top**) and 100.000- dimensional (**Bottom**) Rosenbrock (**Left**) and Ellipsoid (**Right**) problems. For all subfigures except for **Top-Left**, the presence of a marker in the end of the trajectory corresponds to the maximum number of function evaluations used, for **Top-Left** it corresponds to an early stopping of the algrotihm.

The limited memory Covariance Matrix Adaptation Evolution Strategy (LM-CMA-ES [3]) is a recent extension of CMA-ES [1] to large-scale optimization. In the original CMA-ES, at each iteration t, $\lambda = 4 + \lfloor 3 \ln n \rfloor$ candidate solutions are sampled as follows

$$\boldsymbol{x}_k^t = \mathcal{N}\left(\boldsymbol{m}^t, {\sigma^t}^2 \boldsymbol{C}^t\right) = \boldsymbol{m}^t + \sigma^t \mathcal{N}\left(\boldsymbol{0}, \boldsymbol{C}^t\right), \quad k = 1, \ldots, \lambda, \tag{1}$$

where the mean $\boldsymbol{m}^t \in \mathbb{R}^n$ of the distribution can be interpreted as the current estimate of the optimum of function f, $\boldsymbol{C}^t \in \mathbb{R}^{n \times n}$ is a (positive definite) covariance matrix and σ^t is a mutation step-size. These λ solutions are evaluated according to f. The new mean $\boldsymbol{m}^{t+1}$ of the distribution is computed as a *weighted sum* of the best $\mu = \lfloor \lambda/2 \rfloor$ individuals out of the λ ones. Weights are used to control the impact of selected individuals, with usually higher weights for top ranked individuals.

The original CMA-ES has $O(n^2)$ time and space complexity per function evaluation which precludes its applications to large-scale problems with $n \gg 1000$. The main novelty of LM-CMA-ES is to sample candidate solutions according to a covariance matrix associated with a Cholesky factor A, where $\boldsymbol{A}^t {\boldsymbol{A}^t}^T = \boldsymbol{C}^t$ [6]. The Cholesky factor A and its inverse are incrementally reconstructed from a limited number m of direction vectors. These vectors represent so-called evolution path vectors of the CMA-ES and are defined as a smoothed change of the mean of the mutation distribution. This simple approach is inspired by L-BFGS [2] with the difference that gradients are estimated in a stochastic way and no line search is used. The LM-CMA-ES has $O(mn)$ time and space complexity per function evaluation.

2 Results and Discussion

The empirical performances of LM-CMA-ES and L-BFGS are investigated on two well-know benchmark problems: Ellipsoid $f_{Elli} = \sum_{i=1}^{n} 10^{6\frac{i-1}{n-1}} \boldsymbol{x}_i^2$ and Rosenbrock $f_{Rosen} = \sum_{i=1}^{n-1} \left(100.(\boldsymbol{x}_i^2 - \boldsymbol{x}_{i+1})^2 + (\boldsymbol{x}_i - 1)^2\right)$ for $n = 100, 1000, 100000$ and $\boldsymbol{x} \in [-5, 5]^n$. While f_{Elli} is separable, both algorithm do not exploit separability. The L-BFGS is investigated in two settings: i) when exact gradients are provided but at the cost of $n+1$ function evaluations per gradient (L-BFGS G) and ii) when gradients are estimated by central differencing at the cost of $2n$ evaluations. The minFunc implementation of L-BFGS by Mark Schmidt available at `http://www.di.ens.fr/mschmidt/Software/minFunc.html` is used in its default settings. For all algorithms, the number of gradient/direction vectors stored $m = 4 + \lfloor 3 \ln n \rfloor$, more specifically, 17 for 100-D, 24 for 1000-D and 38 for 100.000-D.

Figure 1 shows that LM-CMA-ES is outperformed by L-BFGS G by at most a factors of 4-5 (on 1000-D Ellipsoid) but is numerically more stable (can reach $f(\boldsymbol{x}) = 10^{-10}$ on Ellipsoid). It should be noted that L-BFGS G represents the case when exact gradients are provided, however, in practice, estimates obtained by finite differencing lead to an even earlier premature convergence (data not shown) while estimates obtained by central differencing (L-BFGS CD) are twice more expensive.

The results on 100.000-dimensional problems show that LM-CMA-ES outperforms L-BFGS G on the first $10n - 15n$ function evaluations. This observation suggests that LM-CMA-ES can be viewed as an alternative to L-BFGS when n is large and the available number of function evaluations is limited. While it can provide a competitive performance in the beginning, it is also able to learn dependencies between variables to approach the optimum. However, the latter is achieved with a loss factor compared to L-BFGS, which, we believe, can be reduced thanks to an online adaptation of internal hyper-parameters of the algorithm as suggested for CMA-ES in [4]. Most importantly, LM-CMA-ES is invariant to order-preserving transformation of the objective function and thus is potentially more robust than L-BFGS.

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The Balanced Vertex k-Separator Problem

Denis Cornaz · Fabio Furini · Mathieu Lacroix · Enrico Malaguti · A. Ridha Mahjoub · Sébastien Martin

Abstract Given an indirected graph $G = (V, E)$, a *Vertex k-Separator* is a subset of the vertex set V such that, when the separator is removed from the graph, the remaining vertices can be partitioned into k subsets that are pairwise edge-disconnected. In this paper we focus on the Balanced Vertex k-Separator Problem, i.e., the problem of finding a minimum cardinality separator such that the sizes of the resulting disconnected subsets are balanced. We present a compact Integer Linear Programming formulation for the problem, and present a polyhedral study of the associated polytope. We also present an Exponential-Size formulation, for which we derive a column generation and a branching scheme. Preliminary computational results are reported comparing the performance of the two formulations on a set of benchmark instances.

Keywords Mixed Integer Programming · Balanced Vertex k-Separator Problem

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Regularized Best Responses, Reinforcement Learning, and Applications to Routing

Panayotis Mertikopoulos

Motivated by mirror descent methods in optimization, we investigate a class of reinforcement learning dynamics in which each player plays a "regularized best response" to a score vector consisting of his actions' cumulative payoffs. Regularized best responses are single-valued regularizations of ordinary best responses obtained by maximizing the difference between a player's expected cumulative payoff and a (strongly) convex penalty term. In contrast to the class of smooth best response maps used in models of stochastic fictitious play, these penalty functions are not required to be infinitely steep at the boundary of the simplex; in fact, dropping this requirement gives rise to an important dichotomy between steep and nonsteep cases. In this general setting, our main results extend several properties of the replicator dynamics such as the elimination of dominated strategies, the asymptotic stability of strict Nash equilibria and the convergence of time-averaged trajectories to interior Nash equilibria in zero-sum games. Time permitting, we will also discuss applications to traffic routing in packet-switched networks with splittable flows.

Joint work with William H. Sandholm

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Colorful linear programming, Nash equilibrium, and pivots

Frédéric Meunier · Pauline Sarrabezolles

Abstract Let $\mathbf{S}_1, \ldots, \mathbf{S}_k$ be k sets of points in $\mathbb{Q}^d$. The *colorful linear programming problem*, defined by Bárány and Onn (*Mathematics of Operations Research*, **22** (1997), 550–567), aims at deciding whether there exists a $T \subseteq \bigcup_{i=1}^k \mathbf{S}_i$ such that $|T \cap \mathbf{S}_i| \leq 1$ for $i = 1, \ldots, k$ and $\mathbf{0} \in \operatorname{conv}(T)$. They proved in their paper that this problem is NP-complete when $k = d$. They leave as an open question the complexity status of the problem when $k = d+1$. Contrary to the case $k = d$, this latter case still makes sense when the points are in a generic position.

We solve the question by proving that this case is also NP-complete. The proof is inspired by the proof of the NP-completeness of the linear complementarity problem and uses some relationships between colorful linear programming and complementarity problems that we explicit in this paper. We also show that if P=NP, then there is an easy polynomial-time algorithm computing Nash equilibrium in bimatrix games using any polynomial-time algorithm solving the case with $k = d+1$ and $|\mathbf{S}_i| \leq 2$ for $i = 1, \ldots, d+1$ as a subroutine. We also show that we can adapt algorithms proposed by Bárány and Onn for computing a feasible solution T in a special case and get what can be interpreted as a "Phase I" simplex method, without any projection or distance computation.

Keywords Bimatrix games · colorful linear programming · complexity · linear complementarity problem · pivoting algorithm

Mathematics Subject Classification (2000) MSC 49M37 · MSC 52C99 · MSC 65K05 · MSC 68Q15 · MSC 90C05

A set of points is said to be *positively dependent* if it is nonempty and contains $\mathbf{0}$ in its convex hull. Given a configuration of k sets of points $\mathbf{S}_1, \ldots, \mathbf{S}_k$ in $\mathbb{R}^d$, a set $T \subseteq \bigcup_{i=1}^k \mathbf{S}_i$ such that $|T \cap \mathbf{S}_i| \leq 1$ for $i = 1, \ldots, k$ is said to be *colorful*. The *colorful linear programming problem*, called Colorful Linear Programming and defined by Bárány and Onn [4], is the following.

Colorful Linear Programming
Input. A configuration of k sets of points $\mathbf{S}_1, \ldots, \mathbf{S}_k$ in $\mathbb{Q}^d$.
Task. Decide whether there exists a positively dependent colorful set for this configuration of points.

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The study of colorful linear programming was originally motivated on the one hand by its particular status in the landscape of the complexity theory and on the other hand by applications in discrete geometry. One of the purposes of our work is to show its applicability on some classical problems in optimization. It is already known that the usual linear programming is the special case of Colorful Linear Programming when $\mathbf{S}_1 = \cdots = \mathbf{S}_k$. We will outline links with the computation of Nash equilibria in bimatrix games and with the linear complementarity programming.

In their paper, Bárány and Onn proved among several other results that the colorful linear programming problem is NP-complete (Theorem 5.1 of their paper) using a reduction of the partition problem to the case when $k = d$. In a comment of their theorem, they note that it "would be interesting to determine the complexity of deciding the existence of a positively dependent colorful set when $k = d+1$ but the $\mathbf{S}_i$ are not necessarily positively dependent". This question is motivated by the colorful Carathéodory theorem, found by Bárány [2], whose statement is: *When each of the $\mathbf{S}_i$ is positively dependent and $k = d+1$, there exists a positively dependent colorful set.* If $k = d+1$ and each $\mathbf{S}_i$ is positively dependent, the colorful Carathéodory theorem implies that the answer to Colorful Linear Programming is always 'yes'. Not requiring each $\mathbf{S}_i$ to be positively dependent but keeping the condition $k = d+1$ is a way to get a true decision problem while slightly relaxing the condition of the colorful Carathéodory theorem. Another motivation is the fact that, contrary to the case $k = d$, the case $k = d+1$ still makes sense when the points are in a generic position. We answer the question by proving a more general result.

Theorem 1 Colorful Linear Programming *with k sets of points $\mathbf{S}_1, \ldots, \mathbf{S}_k$ in $\mathbb{Q}^d$ is* NP-*complete, even if $k - d$ is a fixed integer and if each $\mathbf{S}_i$ has cardinality at most* 2.

Polynomially checkable sufficient conditions ensuring the existence of a positively dependent colorful set exist: the condition of the colorful Carathéodory theorem is one of them. More general polynomially checkable sufficient conditions when $k = d+1$ are given in [1,7,8]. However, Theorem 1 implies that there is no polynomially checkable conditions that are simultaneously sufficient and necessary for a positively dependent colorful set to exist when $k = d+1$, unless P=NP.

A problem similar to Colorful Linear Programming was proposed by Meunier and Deza [8] as a byproduct of an existence theorem, the Octahedron lemma [3,6], which by some features has a common flavor with the colorful Carathéodory theorem. The Octahedron lemma states that if each $\mathbf{S}_i$ of the configuration is of size 2 and if the points are in a generic position, the number of positively dependent colorful sets is even. The problem we call Finding Another Colorful Simplex is the following.

Finding another colorful simplex
Input. A configuration of $d+1$ pairs of points $\mathbf{S}_1, \ldots, \mathbf{S}_{d+1}$ in $\mathbb{Q}^d$ and a positively dependent colorful set in this configuration.
Task. Find another positively dependent colorful set.

Another positively dependent colorful set exists for sure. Indeed, by a slight perturbation, we can assume that all points are in a generic position. If there were only one positively dependent colorful set, there would also be only one positively dependent colorful set in the perturbed configuration, which violates the evenness property stated by the Octahedron lemma. In their paper, Meunier and Deza question the complexity status of this problem. We solve the question by proving that it is actually a generalization of the problem of computing a Nash equilibrium in a bimatrix game. The problem of computing a Nash equilibrium in a bimatrix game – called Bimatrix – is known as a "difficult" problem with the powerful machinery of complexity theory. Many problems for which the solution is known to exist belong to a complexity class called PPAD, defined by Papadimitriou in 1992 [9], which contains complete problems for this class: the PPAD-complete problems. Since Bimatrix is PPAD-complete [5], Finding Another Colorful Simplex is PPAD-complete as well.

Proposition 1 Finding Another Colorful Simplex *is* PPAD-*complete.*

Moreover, we show that any algorithm solving Colorful Linear Programming can be used to solve Finding Another Colorful Simplex, which shows that any NP-complete problem is at least as hard as any PPAD-complete problem. It has already been noted that P=NP would imply P=PPAD, see [9]. We give here a concrete example of this implication. We do not know whether such an example was already known.

Theorem 1 and Proposition 1 are our two main results. In addition to them, other results are also obtained.

For instance, we propose an adaptation of two pivoting algorithms by Bárány and Onn for finding a positively dependent colorful set under the condition of the colorful Carathéodory theorem. The first of their algorithms is directly inspired by the proof of the colorful Carathéodory theorem [2] and requires a distance computation at each iteration. The second one is an adaptation of the first and consists roughly in replacing the distance computation by a projection on a segment. These algorithms are not polynomial and the question whether there is a polynomial algorithm finding a positively dependent colorful set under the condition of the colorful Carathéodory theorem is an important open question [4, 6]. We show that the distance computation or the projection can be replaced in the aforementioned algorithms by a classical reduced cost consideration. We get in this way an algorithm similar to the "Phase I" simplex method. Numerical performances of this approach will be investigated in future work.

We also discuss links between colorful linear programming and linear complementarity problems.

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Simulation–based algorithms for the optimization of sensor deployment

François Le Gland · Christian Musso · Sébastien Paris · Dinh-Tuan Pham

Abstract

Keywords sensor deployment · Monte Carlo · dynamic programming · multilevel splitting · Gibbs sampling

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction and context

The problem considered here can be described as follows: a limited number of sensors should be deployed by a carrier in a given area, and should be activated at a limited number of time instants within a given time period, so as to maximize the probability of detecting a target (present in the given area during the given time period). The criterion to be maximized is a probabilistic criterion, because the target behavior and its initial position are not known exactly, however a probabilistic model is available, as a Markov process. There is also an information dissymmetry in the problem: if the target is sufficiently close to a sensor position when it is activated, then the target can learn about the presence and exact position of the sensor, and can temporarily modify its trajectory so as to escape away before it is detected. This is referred to as the target intelligence. Of course, if the target is too close to the sensor position when it is activated, then it cannot escape away and is detected. Conversely, if the target is sufficiently far from the sensor position when it is activated, then nothing happens (the target is not detected, and it cannot learn anything about the sensor position). Notice that because of this information dissymmetry, activating a

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sensor permanently is a costly and also very poor strategy, since the target would necessarily learn this sensor position before it is detected. Activating a sensor at some unpredictable time instants is a more sensible strategy. This is why the optimization problem is not only about where (in which positions) to deploy the sensors but also when (at which time instants) to activate them: in other words, one should solve a space–time optimization problem, and the deployment plan is a sequence of positions and activation times. Finally, notice that not all deployment plans are admissible: for instance, if one sensor position is significantly remote from another sensor position, then it could not be activated immediately after the other sensor has been activated.

To summarize, a deployment plan is a sequence of positions and activation times, that should respect some physical constraints. The criterion to be maximized over all admissible deployment plans is the probability of a target to be detected. The criterion is evaluated by running a large number of target trajectories under a given probabilistic model. Notice however that the probabilistic model for the target behavior depends on the deployment plan, because of the target intelligence.

Two different simulation–based algorithms have been designed to solve jointly this optimization problem, and are described in the next two sections.

2 Sequential dynamic programming algorithm

The first algorithm is sequential by construction with a discrete time model. It is based on the idea of the dynamic programming principle with a infinity dimensional state (functional state). Hereafter, we call "control" the action of dropping a sensor (and activate it) or to reactivate a existing sensor. Initially, a large population of targets is propagated simultaneously At time t_0, this distribution shows M_0 local prominent maxima. At the same time, a control is done (or not) on each maximum if it is a valid control (consistent with the carrier and sensor constrains). The probability of the target to be detected at this time instant is easily evaluated as the fraction of detected targets in the current population. For each distribution, these targets are terminated and the remaining non-detected targets are considered as still alive. This leads to $M_{1/0}$ updated target distributions which represent $M_{1/0}$ potential initial deployments. Repeating this procedure as time grows up, we obtain an increasingly broadly tree where each leaf is a target distribution produced by an unique deployment strategy. Observing that the (complement) non–detection probability is log–additive, we can develop a dynamic programming like algorithm. The strategies are selected with respect to two criteria: the cost (related to the detection probability) and the entropy. The selected distributions (or strategies) belong to the Pareto front. This selection allows to continue deeply in the tree. At the final time, various strategies are produced.

This algorithm is fast, and it usually provides various efficient deployments plans. However, there is no guarantee that the maximum probability of detection is achieved. The iterative algorithm presented in the next section enhances the detection probability in using these approximate sequential suboptimal strategies as initial solutions.

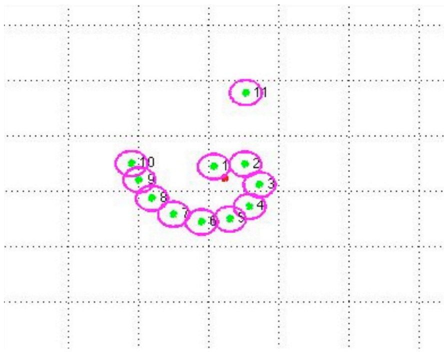
3 Iterative multilevel splitting algorithm

The second algorithm is global (non–sequential) and iterative. A somehow arbitrary probability distribution is introduced initially in the space of solutions (deployment plans). At each iteration of the algorithm, a population of deployment plans is proposed, that all have a probability of detection above some threshold. Recall that for a given deployment plan, the probability of detection is evaluated by running a Monte Carlo simulation with a large number of target trajectories. A new threshold is defined as an empirical quantile of the evaluated probabilities of detection. The deployment plans that have a probability of detection below the new threshold are eliminated, and the deployment plans that have a probability of detection above the new threshold are selected and replicated. This new population is shaked under the action of (a few steps of a) Gibbs sampler that preserves the probability distribution of the population. In particular, all the deployment plans in the shaked population have a probability of detection above the new threshold. The procedure is iterated, until it becomes impossible to increase the threshold. When the algorithm terminates, the current value of the threshold is the proposed approximation of the maximum probability of detection, and the current population of deployment plans provides a set of maximizers.

As with many iterative optimization algorithms with a high dimensional state and with a strongly multimodal criterion, an important issue is the initial condition. This is why the initial population of deployment plans are provided by the output of the sequential dynamic programming algorithm presented in Section 2.

4 Conclusions

Two simulation–based algorithms have been presented, that have been successful applied to an industrial optimization problem (below a deployment illustration).



These two algorithms have different and complementary features. One is fast, and sequential: it proceeds by running a population of targets and by dropping and activating a new sensor (or re–activating a sensor already available) where and when this action seems appropriate. The other is slow, iterative, and non–sequential; it proceeds by updating a population of deployment plans with guaranteed and increasing criterion value at each iteration, and for each given deployment plan, there is a population of targets running to evaluate the criterion. Finally, the two algorithms can cooperate in many different ways, to try and get the best of both approaches. A simple and efficient way is to use the deployment plans provided by the sequential algorithm as the initial population for the iterative algorithm.

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Scheduling under multiple energy resources

Sandra Ulrich Ngueveu · Christian Artigues · Pierre Lopez

Abstract We consider a scheduling problem to which an additional decision level is added regarding the selection of the energy source used to satisfy the total power demand of tasks processed at each instant. Different energy sources are available, with different characteristics in terms of efficiency, power range and dynamics. The objective is to identify the best combination between scheduling and energy resource utilization which minimizes the total duration and energy cost of the project.

Keywords energy-aware scheduling · efficiency functions · column generation · combinatorial optimization · mathematical programming

Mathematics Subject Classification (2000) MSC 49M27 · MSC 65K05 · MSC 90C27

1 Problem statement

Energy considerations are becoming paramount in the resolution of real-world applications. A rising combinatorial optimization challenge is the integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling. The specificity of this work is to consider multiple energy sources with constraints related to their physical, technological and performance characteristics. It is part of the project OREM (Ordonnancement de Ressources Energétiques Multiples) financed by the PGMO. The ambition of the project is to define a new and efficient methodology for the integration of the energy sources characteristics in combinatorial optimization problems.

Previous works, in collaboration with the LAPLACE electrical engineering team and the NEXTER Electronics company, have focussed on the optimization of the allocation of multiple sources of energy to predefined demand curves in hybrid electric vehicles. Non-linearities coming from energy efficiency functions make the allocation problem difficult to solve. Morevoer, flaws in existing modeling hypotheses led to significant gaps between state-of-the-art solutions and optimal ones. Global optimization-based heuristics have been designed to outperform the prior state-of-the-art [6,7]. As an alternative to non-linear modeling yielding suboptimal solutions and important computation time, a new and efficient

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combinatorial modeling was proposed [5]. Although these studies considered the allocation of multiple energy sources and general non-linear efficiency functions, the scheduling of energy consuming activities, that would allow more flexibility for energy management was not considered.

On the other hand, the ROC team at LAAS was historically involved in resource allocation problems and production scheduling applications. Over the years the stakes of such applications have evolved towards a more responsible management of resources. In particular we considered models in production scheduling where the energy can be modulated at every instant of task processing, mainly to avoid peaks of electrical consumption. These models were solved via pure integer linear programming formulations [9] or hybrid constraint and integer programming based methods [1,8]. All our previous works on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

The OREM project aims at explicitely solving in an integrated fashion energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions. Note that in the literature, a few promising mathematical programming-based approaches on similar problems can be found [3,4], either based on MINLP or transformations into approximate MILP. We propose an alternative approach based on piecewise linear lower and upper bounding, rather than approximating, the non linear efficiency functions. Related work in scheduling problems involving non-linear efficiency functions can also be found in the field of scheduling with continuous resources [2], mainly associated with parallel machine scheduling applications and considering theoretical complexity studies of remarkable special cases. In contrast with these studies, we aim at rather proposing mathematical decomposition methods to solve (relatively) general problems.

2 Solution procedure

The solution procedure is based on two main ideas: (i) the bounding of the nonlinear energy efficiency function, then (ii) the reformulation of the problem, which originally is a mixed-integer non-linear problem (MINLP), into two mixed integer linear problems (MILP) using the pair of bounding functions previously defined. The piecewise bounding of a function f of m variables within a tolerance value ϵ consists in identifying two piecewise linear functions denoted $\overline{f}^{\epsilon}$ and $\underline{f}^{\epsilon}$ that verify equations (1) to (3). The two MILP, denoted $\overline{\text{MILP}}$ and $\underline{\text{MILP}}$ respectively, are obtained by substituting f with $\overline{f}^{\epsilon}$ and $\underline{f}^{\epsilon}$, respectively.

$$\underline{f}^{\epsilon}(x) \leq f(x) \leq \overline{f}^{\epsilon}(x), \forall x \in \mathbb{R}^m \quad (1)$$

$$f(x) - \underline{f}^{\epsilon}(x) \leq \epsilon f(x), \forall x \in \mathbb{R}^m \quad (2)$$

$$\overline{f}^{\epsilon}(x) - f(x) \leq \epsilon f(x), \forall x \in \mathbb{R}^m \quad (3)$$

Performing the linearizations before the optimization allows not only the generation of $\overline{f}^{\epsilon}$ and $\underline{f}^{\epsilon}$ with respect to a predefined tolerance value ϵ, but also the minimization of the number of sectors of the resulting piecewise functions and therefore the minimization of the number of additional integer variables in $\overline{\text{MILP}}$ and $\underline{\text{MILP}}$.

Solving a $\overline{\text{MILP}}$ generates solutions that are feasible for the original MINLP, and that have a total cost less than ϵ% higher than the optimal solution cost. Solving a $\underline{\text{MILP}}$ generates solutions that may not be feasible for the original MINLP, but whose total cost is less than ϵ% lower than the optimal solution cost and can help proving the optimality of a solution. Note that both problems share the exact same structure and only differ in terms of the numerical data of their respective piecewise functions. Therefore, a single dedicated resolution method needs to be developped and applied to solve both problems.

The underlying scheduling problem studied considers pre-emptive activities with release dates, due dates, duration and energy requirements. Because of the piecewise linear bounding-based solution method, the efficiency functions can be assumed to be piecewise linear without any loss of generality. Building on that, we proved the NP-hardness of the resulting problem. We aim at defining patterns of resource consumption via the set of activities that can be simultaneously in progress and a procedure for feasible patterns generation with regard to the predefined constraints. A column-generation-based algorithm to exploit the feasible patterns identified is being developed. It is based on a Dantzig-Wolfe decomposition which moves to the subproblem all piecewise linear cost functions. The results obtained will be presented at the conference.

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Stochastic graph partitioning

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Keywords Graph Partitioning · Chance Constrained Programming · Central Limit Theorem · Second-Order Cone Programming · Mixed-Integer Linear Programming · Branch-and-Bound

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

Abstract

We study the stochastic problem of partitioning networks of processes onto a fixed number of nodes. Given a dataflow application, the objective is to assign the tasks to processors in order to minimize the total communications among them without exceeding the limited capacity of each processor in terms of resources (memory footprint, core occupancy, etc.). This problem, as an extension of the Node Capacitated Graph Partitioning problem proposed by C. E. Ferreira and al [1], is obviously NP-hard. We present a novel approach for graph partitioning with capacity constraints with respect to the uncertain resource requirements affecting the weights of the tasks and thus, we have to solve the chance constrained program for which the capacity of each processor is respected for each resource with a minimal probability target.

We focus our studies on the case of individual chance constraints in which different probability levels can be assigned to different constraints. An extension of the central limit theorem was used to approximate our constraints, in which the distribution of uncertain resources is arbitrary, by the new constraints of normal distribution. For probability levels greater than one half, these constraints are convex. We reformulate them in the form of second-order cone constraints.

We then study several mathematical formulations for the problem and various solution techniques. These models were used are the Node-Cluster model and the Node-Node model in which we have succeeded in finding a relaxation for the Node-Node model to make it comparable with the Node-Cluster model in term of problem size. The first technique to solve such problems is the so called Second-Order Cone Programming (SOCP) [3] that amounts to transform each separate chance constraint into a second-order constraint. The two other solution techniques, based on reformulating the problem as a mixed-integer linear program, are the Fortet's classical linearization technique [2] and the linearization technique recently proposed by Hanif D. Sherali and J. Cole Smith [4].

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We have conducted numerical experiments comparing the formulations and various solution techniques in a branch-and-bound approach as well as in the continuous relaxation. The results obtained tend to suggest that, the Node-Cluster model perform better than the Node-Node model in the branch-and-bound algorithm although the later is much better in the quality of the continuous relaxation. These experiments with the solution techniques considered show that in some formulations, the linearization technique of Hanif D. Sherali and J. Cole Smith is more efficient than the SOCP technique, and the Fortet's classical linearization technique is often the most efficient for the sparse graphs. These experiments also show the importance of taking data variations into account as compared to the deterministic version.

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Pareto-based definitions of the optimal value and optimal solutions of a fuzzy program

Benoît Pauwels · Serge Gratton · Frédéric Delbos

Abstract Solving an *optimization problem* consists in finding the decisions that best satisfy some predefined preferences – often represented by an *objective function* – on a space of admissible decisions. The optimization problems that are encountered in real world applications are often subject to uncertainties (*e.g.* design in mechanical engineering, molecular modelling, well placement in reservoir engineering), such as imprecision on the values of the objective function or incomplete knowledge of the set of admissible decisions. These uncertainties may be of non-stochastic nature (*e.g.* imprecision in experimental measurements). In that case *fuzzy sets theory* may be more appropriate to model them than probability theory. Furthermore fuzzy arithmetic may be less computationally expensive than stochastic calculation, the latter often involving costly Monte Carlo integrations. This theory has also been reported to be more efficient when little data is available [2].

We consider an optimization program with an objective function defined on a euclidean space $\boldsymbol{X}$ and taking values in the set of fuzzy subsets of $\mathbb{R}$: for all $\mathbf{x}$ in $\boldsymbol{X}$, $\widetilde{f(\mathbf{x})}$ is characterized by a *membership function*:

$$\mu_{\widetilde{f(\mathbf{x})}} : \mathbb{R} \to [0,1] : t \mapsto \mu_{\widetilde{f(\mathbf{x})}}(t).$$

Fuzzy subsets of $\mathbb{R}$ generalize subsets of $\mathbb{R}$ in the classic sense (or *crisp* subsets of $\mathbb{R}$), their membership functions coinciding with their *characteristic functions* – which equal 1 on the subset and 0 elsewhere. Denoting $\boldsymbol{\mathcal{X}}$ the set of admissible points (included in $\boldsymbol{X}$) the program under consideration can be summarized as follows:

$$\begin{aligned} \widetilde{\underset{\mathbf{x}}{\text{minimize}}} \quad & \widetilde{f(\mathbf{x})} \\ \text{subject to} \quad & \mathbf{x} \in \boldsymbol{\mathcal{X}}. \end{aligned} \tag{FP}$$

This kind of program is called a *fuzzy program* – or a *flexible program.* We did not find satisfactory definitions in the literature for both the *optimal value* and the *optimal solutions* of such a program. Papers on this matter usually provide a definition for only one of both. Hence our intent is to fill in this gap by proposing new general definitions for both the optimal value and the optimal solutions –

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consistent with each other – of optimization programs with fuzzy-valued objective functions such as (FP). Following [3,4], the fuzzy program (FP) is reduced to a family of *biobjective programs* (BP_η) indexed by η in $]0,1]$:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \left(\left\lfloor \widetilde{f(\mathbf{x})} \right\rfloor_\eta, \left\lceil \widetilde{f(\mathbf{x})} \right\rceil_\eta \right) \\ \text{subject to} & \mathbf{x} \in \boldsymbol{\mathcal{X}}, \end{array} \tag{BP_η}$$

where, for all $\mathbf{x}$ in $\boldsymbol{\mathcal{X}}$,

$$\left\lfloor \widetilde{f(\mathbf{x})} \right\rfloor_\eta = \inf \left\{ t \in \mathbb{R} : \mu_{\widetilde{f(\mathbf{x})}}(t) \geq \eta \right\} \text{ and } \left\lceil \widetilde{f(\mathbf{x})} \right\rceil_\eta = \sup \left\{ t \in \mathbb{R} : \mu_{\widetilde{f(\mathbf{x})}}(t) \geq \eta \right\}.$$

We use the *Pareto-efficient sets* of the biobjective programs (BP_η), $\eta \in]0,1]$, as a basis to define the fuzzy subset $\widetilde{\text{Argmin}}$ (of $\boldsymbol{X}$) of the optimal solutions of (FP). Then we derive a definition for the (fuzzy) optimal value $\widetilde{val}$ thanks to the *extension principle*, well-known in fuzzy sets theory [1].

Based on these new definitions we investigate further the case when the uncertainty on the objective function values results from the presence of uncertain parameters, here modeled as fuzzy vectors. This study includes fuzzy nonlinear regression, and the particular cases of fuzzy linear and fuzzy quadratic objective functions as well.

Keywords Fuzzy programming · Optimization under uncertainty · Biobjective optimization

Mathematics Subject Classification (2000) 90C70 · 68T37 · 90C29

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Online multi-class classification via Blackwell's approachability

Vianney Perchet

In binary classification, errors of types one and errors of type two might have different and non-comparable importance. Therefore the convex, and linear aggregations of the errors usually made to treat the problem via optimization can be meaningless. We propose to view the binary classification, and even the multi-class classification as a multi-objective optimization problem.

We investigate the online version of this problem, when data to be labelled arrive sequentially and we show how Blackwell's approachability theory can be generalized to fit into this framework and which guarantees are achievable.

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New results in bandit problems with applications

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Bandit problems are sequential and simultaneous optimization/estimation of a noisy function in order to minimize a cumulative loss, therefore at the junction of game theory, optimization, statistics and machine learning. They have a wide variety of potential applications as ad placements, online classification, spam filtering, clinical trials, etc.

New results on the original multi-armed bandit problem, with a finite number of possible actions, are obtained and described. We also investigate new variants of the original framework – that model the aforementioned applications more acurately –, describe the guarantees that are achievable and show how to obtain them.

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Competition in hydro-dominated electricity markets

Andy Philpott · Michael Ferris

Abstract The problem of computing perfectly competitive prices in a hydro-dominated electricity market can be modeled as a multi-stage stochastic optimization problem when inflows are random and markets are complete. We study this problem when markets are incomplete, resulting in a multi-stage stochastic equilibrium problem. When probability distributions are finite, these problems can be attacked using complementarity models. We also compare solutions in open-loop games with those for closed loop games. If agents can strategically arrange hydro storage ahead of time, perfect competition in real-time electricity wholesale spot markets might not be enough to yield an optimal social plan.

Keywords hydro-thermal scheduling · stochastic dual dynamic programming · stochastic equilibrium

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

It is often asserted that partial equilibrium in perfectly competitive electricity spot markets corresponds to the welfare maximizing solution of a social planning problem. This is the political justification for much market reform in the liberalization of electricity markets around the world. In this paper we explore the extent to which this assertion is valid in market settings where hydroelectric generation is dominant, and future inflows to reservoirs are uncertain.

2 The model

Our approach works with a scenario tree $\mathcal{N}$ to represent uncertainty. Each node $n \in \mathcal{N}$ spans a period and corresponds to a realization $\omega(n)$ of reservoir inflows in that period. Thus inflows in each time period

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are assumed to have a finite distribution where node $n \in \mathcal{N}$ has probability $\phi(n)$. The social plan seeks a solution to the problem:

$$\begin{aligned} \text{SSP: } \min \ & \textstyle\sum_{n\in\mathcal{N}} \phi(n) \left(\sum_{j\in\mathcal{T}} f_j(v_j(n)) - \sum_{k\in\mathcal{C}} c_k(d_k(n))\right) \\ & \qquad\qquad + \textstyle\sum_{n\in\mathcal{L}} \phi(n) \sum_{i\in\mathcal{H}} Q_i(x_i(n)) \\ \text{s.t.} \quad & \textstyle\sum_{i\in\mathcal{H}} g_i(u_i(n)) + \sum_{j\in\mathcal{T}} v_j(n) \geq \sum_{k\in\mathcal{C}} d_k(n), \qquad n \in \mathcal{N}, \\ & x_i(n) = x_i(n-) - u_i(n) - s_i(n) + \omega_i(n), \qquad i \in \mathcal{H}, n \in \mathcal{N}, \\ & u(n) \in \mathcal{U}, \quad v(n) \in \mathcal{V}, \quad x(n) \in \mathcal{X}, \quad s(n) \in \mathcal{S}. \end{aligned}$$

Here $v_j(n)$ is the thermal generation of thermal plant $j \in \mathcal{T}$ in node n, $u_i(n)$ is the hydro release of hydro plant $i \in \mathcal{H}$ in node n, and $d_k(n)$ is the consumption of consumer $k \in \mathcal{C}$ in node n. The objective of SSP is to maximize expected social surplus, measured by total consumer welfare $\sum_{k\in\mathcal{C}} c_k(d_k(n)$, net of thermal generation cost $\sum_{j\in\mathcal{T}} f_j(v_j(n))$, and including expected residual water value $\sum_{n\in\mathcal{L}} \phi(n) \sum_{i\in\mathcal{H}} Q_i(x_i(n))$. Released water $u_i(n)$ flowing through a station $i \in \mathcal{H}$ generates electric energy defined by $g_i(u_i(n))$. The flow balance constraints for SSP relate this water flow to the change in reservoir level through the reservoir dynamics, where the reservoir volume $x_i(n)$ is defined in terms of its value $x_i(n-)$ in the parent node $n-$ of node n (i.e. in the previous period).

If markets are complete and perfectly competitive, and all agents share the same underlying probability distribution and are risk-neutral then SSP corresponds to the solution of the complementarity formulation:

$$\begin{aligned} \text{CE: } & u_i, x_i, s_i \in \arg\max \text{HP}(i), \\ & v_j(n) \in \arg\max \text{TP}(j), \\ & d_k(n) \in \arg\max \text{CP}(k), \\ & 0 \leq \textstyle\sum_{i\in\mathcal{H}} g_i(u_i(n)) + \sum_{j\in\mathcal{T}} v_j(n) - \sum_{k\in\mathcal{C}} d_k(n) \perp p(n) \geq 0. \end{aligned}$$

Here $p(n)$ is the market clearing price in node n of the scenario tree. Solutions to CE can be computed in GAMS/EMP as a MOPEC [2].

In practice, markets are not complete. For example, different reservoirs on a river chain might be owned by different agents, and in the absence of contracts to trade water between them, a competitive equilibrium might be inefficient in comparison to a social plan [3]. Uncertainty can also lead to incompleteness when agents wish to trade different risk positions, but the necessary intruments are missing [4].

When markets are incomplete, one must compute equilibria using models like CE, without being able to resort to optimization. This then raises questions of existence and uniqueness of solutions. One approach to demonstrating this is to seek a version of CE that is a non-cooperative game, and then apply classical results (e.g. [5]) that show the existence of a Nash equilibrium. Alternatively one might formulate CE in special cases as a linear complementarity problem (LCP) and prove the existence of a solution by checking certain conditions on the matrices of the LCP (see e.g. [1]).

Our interest in this paper is threefold: first we wish to outline conditions under which competitive equilibria exist for multi-stage competitive equilibrium under inflow uncertainty and market incompleteness.

Second, we wish to quantify the extent to which incompleteness yields inefficiency. We do this using a sequence of experiments with risk-averse agents that involve different choices of risk-hedging instrument.

Our third purpsoe is to study two-stage games using different conjectural variations for the hydro agents in each stage. We show by example that a hydro agent in a perfectly competitive equilibrium has a higher welfare if their initial water level is lower. Thus there is an incentive to spill water in advance of playing competitively. This leads to a form of equilibrium that is competitive in the real-time market, but where agents act strategically in arranging water for later release. Such an equilibrium might be inefficient in comparison to an optimal social plan.

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Capital Investment and Liquidity Management with Collaterized debt

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Abstract We analyse the interaction between dividend policy and investment decision of a cash constrained firm. We depart from the standard literature by allowing the firm to issue collaterized debt. The collaterized debt continuously pays a variable coupon indexed on the firm debt with the covenant that debt value cannot overpass the value of productive assets. Therefore, the liability side of the balance sheet of the firm consist in two different type of owners; shareholders and debtholders. Should the firm be liquidated, the debtholders have seniority over shareholders on the productive assets. The debt is issued in order to finance the working capital requirement or to invest in productive assets with decreasing return to scale. This leads us to study a bi-dimensional singular control problem that we solve quasi-explicitly in some special cases and by means of viscosity solutions in the general case.

Keywords Dividend policy · Investment · Singular control

Mathematics Subject Classification (2000) MSC 49L20 · MSC 49L25 · MSC 49J20 · MSC 35Q91

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Column generation for an electricity production planning problem with stochastic outage durations

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Abstract We propose an approach for solving an electricity production planning with stochastic outage durations, based on a column generation scheme. In particular, we model and solve the subproblem as a constrained shortest path problem.

Keywords Column generation · MDP

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

The problem is to determine the optimal outage dates for refueling the fifty eight nuclear reactors of the French electric company EDF (Electricité de France), over a five to ten years horizon. Besides these dates, the decision variables are the refueling levels for each outage, as well as the production level for each time period for the nuclear and conventional power plants. This problem can be modeled as a large combinatorial problem in mixed integer variables : outage dates take discrete values, while the other decision variables can be continuous. Several additional resource constraints create non-linear phenomenon, the modelisation of which involve binary variables. Various uncertainties affect the parameters of this problem: demand, market prices and volumes, production capacity, outage durations.

The problem has been submitted to the 2010's Euro/Roadef Challenge [2], in a formulation that did not cover stochastic outage durations, although these have a decisive influence on the solution. In this paper we focus on an extension of the formulation and the resolution method proposed in [3], which improves the treatment of uncertainties on the load balance and, notably, covers stochastic outage durations. We represent the duration of the k^{th} outage of power plant i by a random parameter $\theta_{i,k}$. We suppose that

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this parameter is described by a discrete probability distribution $\delta_{i,k}$. One of the specificities of the outage duration uncertainties is that they can be considered as independant from each other. In the following we give an outline of the solution approach (Dantzig-Wolfe decomposition). Section 2 describes the master problem, while Section 3 introduces the proposed method for solving the subproblem.

2 The model of the master problem

For each nuclear power plant $i \in I$, we consider the set of possible production plans R_i. Only a finite subset of R_i is needed in the final solution, leading to a column generation approach. For each plant i, the proposed algorithm dynamically generates production plans $r \in R_i$ by solving a subproblem. The master problem selects one plan for each nuclear power plant by using binary variables λ_r (see constraint (6,7)). The selected plans must respect a set of coupling constraints, which will determine the exact content of a column.

We must first ensure that the balance between demand and production of the nuclear and thermal power plants is satisfied. The number of constraints would be too high if this balance would be satisfied for each possible uncertainties. As a first approximation, we suppose that this balance is only satisfied in expectation. A column associated with a plan r will therefore contain an average production level $\tilde{p}_t^r$ for each time period t. Given additional variables $\tilde{p}(j,t)$ representing the average production of a thermal power plant $j \in J$, we must satisfy constraint (2). To compensate this simplication, and to be sure that the solution will not rely too much on thermal power plants for some scenarios, we add a worst-case balance constraint. For this, we suppose that the subproblem also returns the minimal production $\underline{p}_t^r$ at time t (with respect to all the possible scenarios), and we impose the cumulated minimal production to meet a least a certain amount $DemNMIN_t$ (see constraint (3)).

$$
\begin{aligned}
\min \quad & \sum_{i \in I} \sum_{r \in R_i} \tilde{c}_r \lambda_r + \sum_{j \in J} \sum_{t} c_{j,t} \tilde{p}(j,t) && && (1)\\
\text{s.t.} \quad & \sum_{i \in I} \sum_{r \in R_i} \tilde{p}_r(t) \lambda_r + \sum_{j \in J} \tilde{p}(j,t) = Dem_t && \forall t \in [\![0, T-1]\!] && (2)\\
& \sum_{i \in I} \sum_{r \in R_i} \underline{p}_r(t) \lambda_r \geq DemNMIN_t && \forall t \in [\![0, T-1]\!] && (3)\\
& \sum_{i \in I, r \in R_i} \log(Pr(Success_i^r)) \cdot \lambda_i^r \geq \log(\alpha) && && (4)\\
& \sum_{i \in I, r \in R_i} \eta_{i,r,m} \cdot \lambda_i^r \leq Q_m && \forall m \in M && (5)\\
& \sum_{r \in R_i} \lambda_r = 1 && \forall i \in I && (6)\\
& \lambda_r \in \{0,1\} && \forall r \in \bigcup_{i \in I} R_i && (7)\\
& \tilde{p}(j,t) \in [PMIN_{j,t}, PMAX_{j,t}] && \forall j \in J, \forall t \in [\![0, T-1]\!] && (8)
\end{aligned}
$$

Contrary to the problem considered for the 2010's Euro/Roadef Challenge, we will allow the subproblem to compute a wait-and-see outage date $a(i,k,\theta)$. However, since it now depends on θ, we would like to limit its possible variations in a specific interval $I_{i,k}$ with a given probability. More precisely, for a given plan $r \in R_i$, we define the following event $Success_i^r = \{\theta \mid \forall k,\ a^r(i,k,\theta) \in I_k\}$. We want to garantee that $Pr(Success) \geq \alpha$ where $Success = \bigcap \{Success_i^r \mid \lambda_i^r = 1\}$ is the event representing the success of all the selected plans. If we suppose that the duration probabililities are independant from each other, we can represent the probability of $Success$ as a product of the probabilities of $Success_i^r$. Taking the logarithm (which preserves order), we obtain the linear constraint (4) in the master problem.

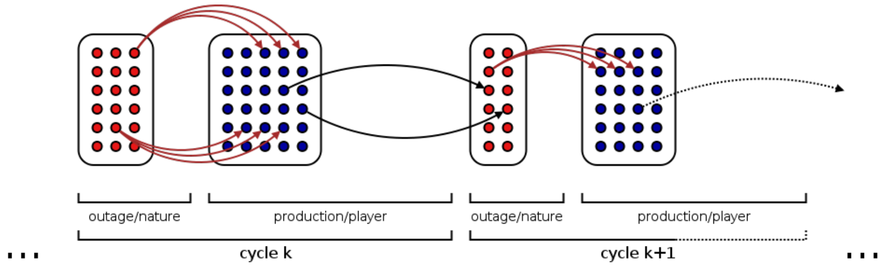
Additional resource constraints limit the possible combinations of plans and are modeled as packing constraints (5). Finally, the objective value (1) consists in the sum of the costs of the selected plans, plus the additional expected production cost of the thermal plants. Given a set of computed columns, the linear relaxation of the master problem gives the value of the dual variables used by the pricing subproblems : $\mu_t \in \mathbb{R}$ associated with the expected balance constraint (2), $\nu_t \geq 0$ with the worst case balance constraints (3), γ with the chance constraint (4), and $\kappa_i \in \mathbb{R}$ with (6). Once the whole set of interesting columns have been generated, we solve the ILP of the master to the optimum.

3 Solving the subproblem

Each plant can be modeled as a Markov Decision Process, whose states represent the time and the corresponding stock level (through some discretization of this stock level). The process alternates between two types of states : those which represent a possible start of an outage, and those which represent the restart of the production. In the former case, the nature decides randomly the outage duration. This leads to a state where the player decides the state in which the next outage will take place. Between these two states, the evolution of the stock is governed by a classical set of equations (see [3] for more details), which allows to compute the optimal refueling and production policy. The original objective is composed of the expected costs of refueling $C_k\tilde{r}_k$ for each cycle k, minus the expected cost $C_T\tilde{x}(T)$ of reselling the remaining fuel at the end of the horizon. Moreover, the dualized constraints of the master induce additional members of the cost function. The pricing subproblem consists then in finding, for each plant i, a feasible production planning r with a strictly negative cost :

$$\min \quad \sum_k C_k\tilde{r}_k - C_T\tilde{x}(T) - \sum_t (\mu_t \tilde{p}_r(t) - \nu_t \underline{p}_r(t)) - \gamma \log(Pr(Success_i^r)) - \kappa_i \tag{9}$$

s.t.



(10)

Without the worst-case balance constraints (3) and the chance constraint (4), the problem can be solved by a classic backward induction. However, these contraints create additional non-linear terms in the objective. We propose an alternative approach, inspired by Errico *et al* [1], which uses a new graph built with two types of nodes. The refueling level is decided when the node represents an outage. The stock level at the next outage is decided when the node represents a production restart. The problem is solved using a constrained shortest path algorithm whose labels consist of a probability distribution on the transit time at this node, and an expected cost from the origin.

4 Conclusions and perspectives

We propose a new method for solving an electricity production planning problem with stochastic outage durations. The preliminary computational results are promising. Nevertheless, the way of dealing with expected balance constraints is not completely satisfactory. In order to garantee a robust solution, we are exploring an approach using both column and row generation (L-shape approach).

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Electricity adjustment market: the best response of a producer

D. Aussel · P. Bendotti · M. Pištěk

Abstract We model the competition of producers in deregulated electricity adjustment market as equilibrium problem with equilibrium constraints (EPEC) with an Independent System Operator (ISO) playing a central role. The main novelty is the proposed analytic formula for the solution to the ISO problem assuming quadratic bid functions of producers. Then, owing to explicit formulae for market marginal price and the price elasticity, a problem of each producer is solved in terms of its best response to the bid functions of other producers and also conditions for existence of such best response are discussed in detail. We use an illustrative example to show all properties of this solution, mainly with regards to their economic interpretation.

Keywords Electricity adjustment market · multi-leader-follower game · analytic solution of ISO

Mathematics Subject Classification (2000) MSC 90C30 · MSC 90C99 · MSC 91C99

1 Introduction

Electricity markets were deregulated and privatized since 1980's in many countries. Soon, many new models appeared in the literature as this topic abounds with open questions from broad spectra of disciplines. The immanent complexity of these markets is due to an Independent System Operator (ISO), a central authority operating in the market determining the electricity prices and handling the electricity dispatch. To this end, the ISO solves an equilibrium problem considering bids of producers and consumers, the properties of the transmission grid, and more generally any other knowledge important to protect the public welfare. Such interaction of market participants is naturally modelled as a bi-level game with the ISO problem in the lower level in the role of a follower, and all producers and consumers on the upper level being leaders. Then, since the problem of each producer is constrained by a solution of the ISO, i.e. another optimization problem, it is a mathematical program with equilibrium constraint (MPEC), see e.g. [12,13]. Next, such collection of mutually coupled MPECs of producers and consumers

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is an equilibrium problem with equilibrium constraints (EPEC), for an analysis of stationary points of EPEC model consult [7].

Basically, there are two variants of electricity market. In the case of *aggregated electricity market* the problems of market clearing and network dispatch are solved at once. On the other hand, *non-aggregated electricity market* consists of two linked markets: the day-ahead and the adjustment market. The first one is based on the determination of a clearing price one day in advance, while the adjustment market may adjusts the price of electricity during the present day. The latter deals with the electricity needs to "complete" the day-ahead market and reacts to demand changes, either due to weather change, or dispatch or network problem. The adjustment market is typically organised as a pay-as-bid market, and therefore is the subject of our present study.

We used the following notation: $D > 0$ is the overall energy demand, $\mathcal{N}$ is finite set of producers (N being its cardinal, $N > 1$), $q_i \geq 0$ represents the non-negative production of the i-th producer. Considering $q \in \mathbb{R}_+^N$ we denote by $q_{-i} \in \mathbb{R}_+^{N-1}$ the vector $q_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_N)$, and the same convention is used also for other vector quantities hereinafter. For $i \in \mathcal{N}$ we denote $a_i, b_i \geq 0$ the coefficients of the i-th producer's bid function $a_i q_i + b_i q_i^2$ and $A_i, B_i \geq 0$ the coefficients of the real production cost function $A_i q_i + B_i q_i^2$.

Each producer provides to the ISO a quadratic bid function $a_i q_i + b_i q_i^2$ given by non-negative parameters $a_i, b_i \geq 0$. Actually in most of electricity markets, only piece-wise linear bid functions or block orders, which are either fully executed or fully rejected, may be allowed. However, quadratic function with non-negative coefficients well captures the typical increasing marginal price of electricity, yet it is amenable to further analysis.

Then, the ISO, knowing the bid vectors $a = (a_1, \cdots, a_N) \in \mathbb{R}_+^N$ and $b = (b_1, \cdots, b_N) \in \mathbb{R}_+^N$ provided by producers, computes the electricity dispatch $q = (q_1, \ldots, q_N) \in \mathbb{R}_+^N$ in order to minimize the total generation cost, that is to solve the following optimization problem

$$\text{ISO(a,b,D)} \qquad \min_q \sum_{i \in \mathcal{N}} (a_i q_i + b_i q_i^2) \quad \text{s.t.} \begin{cases} q_i \geq 0, \ \forall i \in \mathcal{N} \\ \sum_{i \in \mathcal{N}} q_i = D \end{cases}$$

for positive overall demand $D > 0$. It is a well-known fact that this problem admits at least one solution due to the continuity of the criterion and compactness of the optimization domain.

Nevertheless, the electricity market problem in scope in this paper can be ill-posed if the solution set of ISO(a,b,D) contains more than one point. In [8,2] the uniqueness of the response of the ISO(a,b,D) comes from the hypothesis that producers are bidding true quadratic function, that is with $b_i > 0$, for any $i \in \mathcal{N}$, thus implying the strict convexity of the objective function of ISO(a,b,D) problem. Since in our work we need to allow linear bid of producers, an additional assumption is needed to guarantee uniqueness of solution of ISO(a,b,D) problem. On that account, we define the *equity property* assumption

$$\text{(E)} \qquad (a_i, b_i) = (a_j, b_j) \Longrightarrow q_i = q_j,$$

which is supposed to hold for all $i, j \in \mathcal{N}$. This assumption actually formalize that ISO makes no difference among producers. Thus the optimization problem ISO(a,b,D) assuming (E) is as follows

$$\text{E-ISO(a,b,D)} \qquad \min_q \sum_{i \in \mathcal{N}} (a_i q_i + b_i q_i^2) \quad \text{s.t.} \begin{cases} q_i \geq 0, \ \forall i \in \mathcal{N} \\ \left((a_i, b_i) = (a_j, b_j) \Rightarrow q_i = q_j\right), \forall i, j \in \mathcal{N} \\ \sum_{i \in \mathcal{N}} q_i = D. \end{cases}$$

We found solution of this problem, yielding $q_i(a,b,D)$ for all $i \in \mathcal{N}$ together with an explicit formulae for market marginal price and elasticity of the price.

Next, we use these results to solve problem of a producer. We assume that the overall demand D and the set of all producers $\mathcal{N}$ are fixed. Moreover, we suppose that the true production cost function of producer $i \in \mathcal{N}$ is given by $A_i q_i + B_i q_i^2$, with coefficients $A_i \geq 0$ and $B_i > 0$ being known only to producer i. We note that $B_i = 0$ is not realistic since the real marginal cost of electricity production is increasing in q_i. Now, each producer $i \in \mathcal{N}$ aims to maximize his profit $\pi_i(a,b,D)$ given by

$$\pi_i(a,b,D) = (a_i - A_i)\, q_i(a,b,D) + (b_i - B_i)\, q_i(a,b,D)^2 \tag{1}$$

manipulating his own strategic variables $a_i, b_i \geq 0$ with the rest of variables $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$ kept fixed. In other words, the i-th producer's problem $P_i(a_{-i}, b_{-i}, D)$ reads

$$P_i(a_{-i}, b_{-i}) \qquad\qquad \max_{a_i, b_i \geq 0} \pi_i(a,b,D). \tag{2}$$

We search only for such solution to $P_i(a_{-i}, b_{-i}, D)$ that $\pi_i(a,b,D) > 0$, that is we assume that producer will not participate in the market otherwise. Indeed, since we model only one time period here, it makes no sense to participate in the market having non-positive profit. In the real world, producers do sometimes sell electricity below their production cost, but it is only in such situation where the contract spreads over several hours and the overall profit is still positive. To cope with such setting it would be necessary to aggregate the profit $\pi_i(a,b,D)$ over several time instants. However, this is beyond the scope of this paper.

2 Conclusions

Using several simplifications, we obtained a model of electricity markets which is analytically solvable, see [1]. Then, the optimal bid function of a producer corresponds to an idealized setting when the producer has a full information about bid functions of other producers, and he also knows the total electricity demand. Despite these simplifications, the obtained solution provides us with detailed insight into the electricity price formation and thus leads to clear economic interpretation.

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MILP model for a real-world short-term hydro-power unit-commitment problem

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Abstract:
The hydro unit-commitment problem considered is to find a cost-optimal generation schedule for hydroelectric plants connected via a set of reservoirs. On top of often requiring long resolution times, the large-scale MILP intakes real-world data and raises several practical challenges, such as infeasibilities due to data inconsistencies and strict modeling of soft constraints. We illustrate those challenges through numerical experiments and draw some lines of research to grapple with them.

Keywords: hydro unit-commitment, mixed-integer linear programming
Mathematics Subject Classification (2000): MSC 90C11

1) Introduction

We study the daily hydro-power unit-commitment problem tackled by utility company EDF. This problem involves a set of several multi-unit pump-storage plants connected via a network of reservoirs and channels – commonly referred to as a valley. Within a valley, the aim of the problem is to plan the discrete power generation level of its units, while satisfying a set of operational constraints. This discrete modeling implies huge combinatorics that hampers optimization and sometimes even questions the feasibility of our problem.

Several objectives can be considered.
This problem is actually a sub-problem of the overall EDF daily unit-commitment problem which deals with scheduling all the managed plants in order to meet the electricity of its customers. This overall problem is currently solved with the implementation of a decomposition-coordination scheme (described in [1]) which allows breaking the problem down into a set of optimization sub-problems; each sub-problem is limited to the scheduling of a single means of production – a valley in our case – and aims at minimizing operational costs – water usage value in our case. The solution is then obtained iteratively, each iteration requiring the resolution of all sub-problems.
For the hydro sub-problem, one could also consider an operational planner needing to meet a local demand, and who wants to maximize his revenues, and/or who wants to minimize the use of water in the reservoirs. Combining these goals has been studied in the bi-objective approach proposed in [2].

As the short-term horizon is set to a few days, we can consider a deterministic problem where given parameters such as electric demand forecasts (hence dual signals), water inflows and availabilities of plants are non-random. Binary variables are used to model startup and shutdown decisions of units and to express corresponding logical constraints; power-flow functions are either piece-wise linearized or discretized.
These choices lead to a deterministic Mixed-Integer Linear Program (MILP).

2) Our challenges

Solving the current industrial elaborate model with real datasets gives rise to numerical issues, computational difficulties and infeasible instances.
Beyond handling these practical necessary matters, we want to improve the resolution performance (both time and optimality) because the decomposition scheme of the overall problem and the scheduling process mentioned above require the convergence to optimality of all sub-problems in a very limited time-frame.
In addition, we would like to have an even more comprehensive model to be as close as possible to the real description of units' operation.
We would also like to include a better representation of the intrinsic bi-objective nature of our problem.

3) Model description

We consider a valley with several reservoirs and hydro-plants connected along several water streams.
The volume of a reservoir is:
- expressed as the conservation of water of previous time step, given flows of upstream and downstream plants, and external inflows (such as precipitations);
- constrained by the capacity of the reservoir;
- constrained by target values imposed by water management policies and/or valued in the objective with a usage cost to reflect the lost opportunity of discharging water later to generate electricity in the future; we propose to relax and penalize deviations to the target value so as to model the management policy and keep a single-objective MILP.

Within a hydro-plant:
- power-flow curves are partly discretized and piece-wise linearized to model effective discharge, spillage or pumping;
- flow is subject to ramp-up and ramp-down constraints;
- flow is subject to minimum-up and minimum-down constraints;
- pumping and generation cannot be simultaneous.

4) Our approach

The complexity of the model we deal with, the variability of the real datasets we handle and the relative opacity of the solver complicate the understanding of the resolution difficulties. Therefore, we designed a tractable simplified model whose performance could be easily analyzed.
To limit the number of binary variables for the simplified model - we assumed the difficulties were mainly embedded in the huge combinatorial structure – power-curves were approximated with fewer segments and discontinuities. Moreover, two different ways to model the second objective as constraints plus penalty terms added to the first objective function are proposed and compared.

Difficulties remaining for the simplified model can be more easily identified and overcome while difficulties only encountered for the complete model can be tackled with more adapted MILP resolution methods (many being described in [3]).
The characterization of these difficulties will relevantly lead us to simultaneously pursue two very definitely connected but different lines of research:
- modeling and reformulations: formulation strengthening, cuts, decomposition methods, approximations to efficiently provide effective lower bounds on the optimal value, multi-objective formulation;
- heuristics: matheuristics, possibly exploiting the previous formulations/ decompositions/ approximations, to efficiently provide good quality feasible solutions.

5) Computational results for the characterization

Tests were run on 40 instances relative to several valleys at different dates.
We look at three difficulties: feasibility status, numerical stability and resolution time and only present results for the former two.
Regarding resolution time, for reference, respective feasible instances were solved easily under the simplified model (less than 2 minutes approximately), while taking hours or even remaining unsolved after a day under the complete model.

5.1) Infeasibilities

We first compare the number of feasible instances according to the model to better detect and analyze infeasibilities, as shown in Table 1.

	Status			
Model	feasible	intractable	infeasible	Total
simplified	29		11	40
complete	9	11	20	40

Table 1: No. of feasible/infeasible/intractable instances on simplified/complete models

For the simplified model, 10 infeasibilities were analyzed with Cplex 12.4 conflict refiner [4] and are actually due plain to inconsistencies with the real data and need be discarded at our scope.

Additional infeasibilities for the complete model are due to incompatibilities between operational constraints described with integer restrictions and other model elements.
The 11th infeasible instance of the simplified model presented such an incompatibility which was lifted by relaxing the target volume constraint.
We subsequently relaxed the target volume constraint on the complete and compared the number of feasible instances (out of the 30 that have no data inconsistency) with and without relaxation, as shown in Table 2.

	Status			
Target volume	feasible	infeasible	intractable	Total
strict	9	10	11	30
relaxed	26	1	3	30

Table 2 : No. of feasible/infeasible/intractable instances with and without target volume on complete model

For 17 instances, infeasibility or intractability was lifted by relaxing target volumes and confirmed the potential incompatibilities between discrete operations and unattainable mid-run water management policies.
These cases led us to update the modeling by introducing penalization of target volume violations. It would be interesting to model the target volume as a second objective, and to measure the consequences of relaxing the target volumes on the decomposition scheme of the overall problem.

5.2) Numerical issues

To validate the results of the finite-precision solver we use, numerical stability was assessed by looking at the "attention level" metric. A high attention level (>10e6) means that several LP bases of the branch-and-bound nodes were ill-conditioned [3, 4]. As numerical issues arise from data misrepresentations and miscomputations, we tried to rescale or recompute model elements by independently varying volume units, tightening volume bounds, centering volumes, reformulating power-flow curves with efficiency coefficients, and getting rid of dependent variables (power, flow, volume).
Experiments were done on the 30 feasible instances on the simplified model with penalized relaxations of target volumes.

Volume reformulation	(default)				(default)	tightened	centered	no dep. var.
Power/flow reformulation	(default)				yes	(default)		
Change of volume unit	m3	dm3	dam3	hm3	m3	m3		
No. unstable instances	14	11	2	1	15	14	14	1
Total no. of instances	30	30	30	30	30	30	30	30

Table 3: No. of unstable instances with high attention level

Instabilities from the default case are alleviated by specific unit choice and by formulating our model without dependent variables, while other reformulations seem ineffective.
Choosing the combination of reformulations that is most robust to ill-conditioning depends also on other numerical metrics such as the occurrence of rounding errors and resolution time, and on how the results apply with the complete model.

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Integrated Model for Optimal Electric Vehicle Scheduling and Charging in the Business Context.

Ons Sassi · Ammar Oulamara

Abstract Our study deals with the Electric Vehicle Scheduling and Optimal Charging Problem (EVSCP). More precisely, given a fleet of Electric Vehicles EVs and Combustion Engine Vehicles - CVs, a set of tours to be processed by vehicles and a charging infrastructure, the problem aims to optimize the assignment of vehicles to tours and, simultaneously, to minimize the charging cost of EVs in order to avoid costly and carbon-hungry peak demand periods while considering several operational constraints mainly related to chargers, electricity grid and EVs driving range. To solve this problem, we provide a mixed-integer linear programming formulation to model the EVSCP problem and we use CPLEX to solve real test instances. Several business scenarios and extensions to the baseline problem with different new constrains and objectives are proposed, tested and compared to a reference scenario on the basis of costs and CO2 emissions.

Keywords Electric vehicle · Assignment problem · Charging problem · Mixed Integer Programming

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction and Problem Description

The substantial growth of the transport sector in recent years has made it the prime player in energy consumption and greenhouse gas emissions. Nowadays, the governments are more and more conscious of the urgency to conserve the environment by investing in more environmentally friendly and safe modes of transportation, for instance the electric vehicles. However, Electric Vehicles (EVs) are currently facing several weaknesses related to the battery management, the availability of a charging infrastructure and the high costs.

In this paper, we address the Electric Vehicle Scheduling and Optimal Charging Problem (EVSCP) defined as follows: Given a set of tours, a fleet of EVs and Combustion Engine Vehicles (CVs) and a charging infrastructure, the objective is to find the optimal way to allocate tours to vehicles and to minimize the EVs charging cost while satisfying several constraints mainly related to the grid, chargers and EVs batteries capacities. The overall objective is to provide an efficient, scalable and generic decision

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support tool for all vehicle fleet managers.
This study is promoted in the scope of the French National Project Infini Drive, led by La Poste Group, ERDF and seven other companies and research laboratories such as EDF R&D. This R&D project aims at designing, with a progressive approach, an intelligent system that manages the charging infrastructures and allows an economical and ecological sustainable deployment of EVs fleet in the business context.
Optimization problems related to EVs can be classified into four classes: (i) EV use in the context of Smart Grid or Vehicle to Grid (V2G) problems, (ii) charging infrastructure design problems, (iii) controlled EV charging problems, (iv) EV routing problems. On the interaction between the EV and the electricity grid, the reader is referred to [5] and [3]. The problem of the network design of the charging infrastructure is considered in a number of studies including the works [7], [8] and [2]. The controlled EV charging problems consist of a better management of the charging load. The objective of those problems is to integrate the charging load with the traditional load in the electricity grid and minimize the charging cost (see [6] and [4]). The EV routing problem is less considered in the literature. The energy-optimal routing problem is addressed in [1].
To the best of our knowledge, no previous study was devoted to jointly optimize the scheduling and the charging of a fleet of EVs and CVs. This, in part, represents the novelty of our study. Moreover, this paper presents a detailed analysis of EV use and charging under a variety of scenarios, objectives and assumptions. For instance, we consider the impact of EV use on the battery health and we are interested in different charging schemes, charging infrastructure, energy mix, CO2 emissions and total EVs costs of ownership.
To solve the EVSCP, we propose a Mixed-Integer Linear Programming Model (MIP). Since we consider two lexicographical objective function for the baseline problem, we use CPLEX to solve the MIP in two steps. In the first step, the objective is to maximize the EVs kilometers travelled. In the second step, a new constraint which ensures that the number of EVs kilometers travelled is greater than the objective function found in the first step, is added to the MIP. Then, the new MIP is solved with the second objective function. Moreover, we varied the constraints and the objectives to develop a scalable and generic decision support tool for captive fleets management. The new extensions to the baseline problem concern, in particular, (i) the battery cell health; i.e., how to avoid the battery cell degradation after repeated use, (ii) different EV models and different charging levels (level 1 and level 2 chargers), (iii) the charging infrastructure design, (iv) the CO2 emissions minimization using the well-to-wheel ecological analysis and (v) the EV fleet optimization. CPLEX was used to solve the different extensions of our model and we conducted several experiments on real instances of the La Poste Group. Our results show that maximizing the EVs use and controlling EVs charging are significantly important to reduce CO2 emissions and EVs Total Cost of Ownership. Moreover, the results prove that there exists a strong correlation between maximizing EVs use and minimizing CO2 emissions of a fleet of EVs and CVs.

2 Conclusions

The topic of EV routing and optimal charging is becoming increasingly important. In this paper, we considered the problem of optimizing the allocation of tours to EVs and minimizing the charging costs simultaneously. To solve this problem, we proposed a mixed integer linear programming formulation to maximize the EVs use and to minimize the charging costs. An exact method based on two phases was developed to solve the MIP with CPLEX. We also proposed many extensions to the baseline problem formulation to optimize the operational and ownership costs of an EV fleet as well as the CO2 emissions. Different scenarios were proposed, tested on real data instances and compared against a baseline case.

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Improved and Generalized Upper Bounds on the Complexity of Policy Iteration

Bruno Scherrer

Abstract This note provides some upper bounds on the complexity of Policy Iteration algorithms for computing the optimal policy of a Markov Decision process.

Given a Markov Decision Process (MDP) with n states and m actions per state, we study the number of iterations needed by Policy Iteration (PI) algorithms to converge to the optimal γ-discounted optimal policy. We consider two variations of PI: Howard's PI that changes the actions in all states with a positive advantage, and Simplex-PI that only changes the action in the state with maximal advantage.

Like [1], we show that Howard's PI terminates after at most

$$n(m-1)\left\lceil \frac{1}{1-\gamma}\log\left(\frac{1}{1-\gamma}\right)\right\rceil = O\left(\frac{nm}{1-\gamma}\log\left(\frac{1}{1-\gamma}\right)\right)$$

iterations, improving by a factor $O(\log n)$ a result by [2]. We also show that Simplex-PI terminates after at most

$$n^2(m-1)\left(1+\frac{2}{1-\gamma}\log\left(\frac{1}{1-\gamma}\right)\right) = O\left(\frac{n^2 m}{1-\gamma}\log\left(\frac{1}{1-\gamma}\right)\right)$$

iterations, improving by a factor $O(\log n)$ a result by [4].

Under some structural assumptions of the MDP, we then consider bounds that are independent of the discount factor γ: given a measure of the maximal transient time τ_t and the maximal time τ_r to revisit states in recurrent classes under all policies, we show that Simplex-PI terminates after at most

$$\begin{aligned} & n^2(m-1)\left(\lceil \tau_r \log(n\tau_r)\rceil + \lceil \tau_r \log(n\tau_t)\rceil\right) \\ & \qquad \times \left[(m-1)\lceil n\tau_t \log(n\tau_t)\rceil + \lceil n\tau_t \log(n^2\tau_t)\rceil\right] \\ & = \tilde{O}\left(n^3 m^2 \tau_t \tau_r\right) \end{aligned}$$

iterations. This generalizes a recent result for deterministic MDPs by [3], in which $\tau_t \leq n$ and $\tau_r \leq n$. We explain why similar results seem hard to derive for Howard's PI.

Finally, under the additional (restrictive) assumption that the state space is partitioned in two sets, respectively states that are transient and recurrent for all policies, we show that Howard's PI terminates after at most

$$n(m-1)\left(\lceil \tau_t \log n\tau_t\rceil + \lceil \tau_r \log n\tau_r\rceil\right) = \tilde{O}(nm(\tau_t+\tau_r))$$

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iterations while Simplex-PI terminates after

$$n(m-1)\left(\lceil n\tau_t \log n\tau_t\rceil + \lceil \tau_r \log n\tau_r\rceil\right) = \tilde{O}(n^2 m(\tau_t + \tau_r))$$

iterations.

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A Long Term Rolling Horizon Evaluation of Stochastic Unit Commitment in The British Power System

Tim Schulze · Ken I.M. McKinnon

Abstract In recent years the expansion of energy supplies from volatile renewable sources has triggered an increased interest in stochastic optimization models for generation unit commitment. As the UK and central European countries approach their 2020 targets for renewable penetration, the intermittency of renewable energy sources – particularly wind – plays an increasingly important role in day-to-day operational planning. As wind power generation is harder to predict and more volatile than demand, balancing the grid is becoming more and more challenging for system operators. Traditional ways to cope with this problem, i.e. maintaining a constant reserve margin throughout the day, may not be an appropriate tool to deal with this new uncertainty. Consequently, several studies have modelled the unit commitment step of power systems planning as a stochastic mixed-integer optimization problem. The downside of stochastic models, however, is the increased effort in solving the problem. In theory industry standard MIP solvers can cope with this type of model if the objective function is piecewise linear or convex quadratic, and the constraints are linear. But in practice the inclusion of many scenarios slows down the solution process significantly and may even make the problem intractable.

Whether it is worth making the additional effort depends on the performance of both, the traditional approach of maintaining set reserve margins, and the stochastic approach. To address this question, we perform a long term rolling horizon evaluation of deterministic and stochastic unit commitment methods. Two different types of stochastic models have been proposed in this context. Two stage models represent the decision problem a system operator is faced with when large power plants have to be scheduled day-ahead, and their commitment is fixed for the next 24 hours once it has been decided. On the other hand multi stage models represent the more flexible situation of intra-day commitment decisions which can be updated in a rolling horizon fashion, subject to an appropriate notification period. We show how both approaches can be integrated in a rolling planning scheme, and measure their performance on a one year planning horizon. We compare the results obtained from these two models to the deterministic alternative with constant reserve margin, and to a hypothetical plan made under perfect information. This allows us to quantify the value of perfect information (VPI), and the added value of a stochastic solution (VSS).

For the purpose of this evaluation, we use a central planning model of the British National Grid with expected conventional and renewable generation capacities for the year 2020, and corresponding expected

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demand. To simulate wind power scenarios for the stochastic models, we fit correlated ARMA(1,1) time series to historic regional wind forecast errors. In this context, and for evaluation purposes, point forecasts for wind power were synthesized from historical time series, to match the root mean square error (RMSE) of state-of-the-art wind forecasting techniques. For the final evaluation of our models, we use historic wind data from 2010. All stochastic MIP models in this study are solved by a Branch & Price based scenario decomposition method that allows us to reduce the computational effort to a manageable level.

Keywords stochastic unit commitment · rolling horizon planning · wind uncertainty

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

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Estimating Stock Covariance Using Factorial Hidden Markov Models

João Sedoc · Jordan Rodu · Lyle Ungar · Dean Foster · Jean Gallier

Abstract The estimation of stock covariance using multiple time frames is a well studied and difficult problem. We present a novel approach using factorial hidden Markov models (HMMs), to model the multi-horizon covariance. We use fast spectral methods on the factorial HMM to empirically estimate out-of-sample Markowitz portfolio risk. Comparisons on NYSE, AMEX, and Nasdaq stock high frequency data of our new model to other widely used techniques show that our factorial HMM yields superior estimation speed and accuracy.

Keywords covariance matrix estimation · factorial hidden Markov models · random matrices

Mathematics Subject Classification (2000) MSC 62J10 · MSC 15B52 · MSC 15B34

1 Introduction

Stock covariance estimation and factor analysis have a long history in finance. Starting with Markowitz [9, 10], investors recognized asset covariance as an important problem for portfolio management. The variance of expected returns of the assets held in the portfolio depend not only on the expected investment time horizon, but also on longer and shorter term time periods [8]. Thus, the choice of investment time horizon is a critical factor in estimating covariance. Ideally, the model should incorporate both long and short term information. While conceptually intuitive, the rigorous incorporation of long and short horizon data into models has proven to be surprisingly challenging. Furthermore, most of the literature is focused on using daily data, whereas there is now an abundance of detailed records of quotes and transaction prices with nanosecond time resolution, so called high frequency data. Our model targets multi-time horizon aspects of high frequency data.

Our factorial HMM assumes that the data generating mechanism is created by three to five time frames. Each of these time frames are represented as different HMMs which combine to create an estimation of the predicted covariance matrix. The financial interpretation is that we attempt to capture long

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term market dynamics and momentum, medium term mean reversion to factors, and short term market (micro)structure effects. **Our main contribution is the application of factorial HMMs to model multi-scale stock covariance, and the use of spectral methods to estimate these models.**

Before presenting our model specification, we reintroduce the reader to basic HMMs. HMMs consist of a chain of latent states, which at each time point emit an observation. The two primary assumptions are that the underlying state process is Markovian, and that given the hidden states, the observations are independent. A Markovian hidden state sequence is one where, at time t, the probability distribution over the next hidden state at time $t+1$ depends only on the current hidden state at time t, so

$$\Pr(h_{t+1} \mid h_t, \ldots, h_1) = \Pr(h_{t+1} \mid h_t).$$

We can fully specify the HMM by the observation vector $x_t \in \mathbb{R}^n$, the sequence of hidden states $h_t \in \mathbb{R}^m$ where, in our case, $m < n$. The probability of transitioning from one hidden state to the next is encapsulated in a matrix $T \in \mathbb{R}^{m \times m}$, $O \in \mathbb{R}^{n \times m}$ is the emission probability matrix, and $\pi \in \mathbb{R}^m$ is the the initial state distribution.

$$\begin{aligned} \pi_i &= \Pr[h_1 = i], \\ T_{i,j} &= \Pr[h_{t+1} = i \mid h_t = j], \\ O_{i,j} &= \Pr[x_t = i | h_t = j]. \end{aligned}$$

In a factorial HMM the observation is governed by M independent hidden state sequences, such that

$$\Pr[h_{t+1} \mid h_t] = \prod_{i=1}^{M} \Pr[h_{t+1}^{(i)} \mid h_t^{(i)}].$$

Our model further constrains the factorial HMM in order to capture the multi-resolution structure of the financial market and also allow for spectral estimation.

The capital asset pricing model (CAPM) was the first significant breakthrough for the estimation of stock covariance. CAPM can be seen as a single factor model where the factor is the market. Recent factor models have extended CAPM and improved estimation accuracy. With this foundation, Engle [1] analysed multiple time horizons for individual stock return variance using the autoregressive conditional heteroskedasticity (ARCH) model. ARCH was then further extended to multivariate cases [2], applied to high frequency data, and stochastic properties were incorporated. However, CAPM and ARCH lack solutions to intertemporal and non-stationarity of covariance behavior. Several modern techniques attempt to resolve these issues [13]; we present a simpler, extensible, tractable method.

HMMs have been applied with success in many fields, including gene recognition [7], robotics [11], natural language processing (NLP) tasks[12], and more; however, their use in stock forecasting is relatively new [2005] [5] and limited. New techniques for spectral estimation of HMMs, in place of the expectation maximization (EM) algorithm or Gibbs sampling, now allow for fast application of HMMs to large datasets [6, 3]. While currently not used in industrial applications, factorial HMMs have been introduced for learning both short term as well as long term structure [4]. We show that factorial HMMs can be successfully applied to financial time series data.

We use one year of high frequency stock market data for several hundred US equities to estimate our model. Multiple different hidden layers and states are used to accurately estimate the data. To test our model we construct optimal mean-variance portfolios with liquidity constraints. In contrast to factor models, principle components analysis, or macroecomonic models, our model provides multiple time frames and non-stationary estimation of covariance behavior.

2 Conclusions

Our factorial HMM provides a simple and tractable framework for the estimation of stock covariance using multiple time frames. Our proposed method shows improved accuracy and speed, consistent estimation than competing methods. In addition, our factorial HMM model out-performs competing models in out-of-sample portfolio construction with liquidity constraints using high frequency data.

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On the asymptotic behaviour of stochastic optimal control problems governed by stochastic differential equations with small noise intensities

Francisco J. Silva

Abstract In this work we review some classical and new results concerning the asymptotic limit of the value function and optimal solutions for stochastic optimal control problems where the diffusion term is *small*. Special attention will be paid to *large deviation principles* for the optimal dynamics in some special cases.

Keywords Stochastic control · Vanishing viscosity · Asymptotic results · Large deviation principle

Mathematics Subject Classification (2000) MSC 93E20 · MSC 34K25 · MSC 60F10

1 Introduction

In this work we consider the following unconstrained stochastic optimal control problem

$$\begin{array}{l} \inf_{(x,u)} \mathbb{E}\left(\int_0^T \ell(x(t),u(t))\mathrm{d}t + g(x(T))\right) \\ \text{s. t.} \quad \mathrm{d}x(t) = f(x(t),u(t))\mathrm{d}t + \varepsilon \mathrm{d}W(t), \quad x(0) = y. \end{array} \tag{P_ε}$$

In the above notation $\varepsilon \in [0,\infty[$, $y \in \mathbb{R}^n$, $\ell : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}$, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $W(\cdot)$ is a standard n-dimensional Brownian motion defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let $\mathbb{F}$ be the filtration generated by W, satisfying the *usual conditions* (see e.g. [13]), and take as admissible controls u any process adapted to $\mathbb{F}$. If f satisfies a global Lipschitz property, then the Stochastic Differential Equation (SDE) in (P) is well defined in the *strong sense* (see e.g. [13]). Assumptions for functions ℓ and g, in order to give a meaning to the cost, can be found in the classical monographs [8,15]. Note that when $\varepsilon = 0$ the problem becomes deterministic, since we are assuming that ℓ, g, f do not depend directly on the randomness $\omega \in \Omega$.

Our aim in this work is to establish some simple relations between (P_ε) and (P_0). Regarding the asymptotic behaviour for the optimal cost v_ε of problem (P_ε) around the optimal cost v_0 of problem (P_0), important achievements can be found in e.g. [7,3,11,10]. In [7,3] precise power expansions of v_ε around v_ε are provided under some structural conditions. For more general problems, error estimates for $|v_\varepsilon - v_0|$ are obtained in [3,11,10]. We review some of these estimates in Section 2.

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In this note we focus our attention mainly on the limit behaviour, as $\varepsilon \downarrow 0$, of solutions $(x_\varepsilon, u_\varepsilon)$ of problem (P_ε), if they exist. As a model example, we consider strongly convex Linear Quadratic (LQ) problems (see [4]), and we provide error estimates for $\|(x_\varepsilon, u_\varepsilon) - (x_0, u_0)\|$ in some adequate norm (see [14]). The novelty is that we also prove a Large Deviation (LD) principle for the solutions based on the classical result by Freidlin-Wentzell (see [9]). For this particular unconstrained LQ case, the result is a simple consequence of classical Riccati theory (see [15, Chapter 6]). In Section 3 we discuss some current research which are beyond the result mentioned above, including control constrained problems and some nonlinear problems.

2 Some asymptotic results

We begin by reviewing some standard results about the estimate $|v_\varepsilon - v_0|$ in the general case. More precise results, in the form of a power expansion of v_ε around v_0 (but under stronger conditions) can be found in [7,3]. It is well known (see e.g. [8]) that for all $\varepsilon \in [0, \infty[$, the value function $v_\varepsilon(t, y)$, defined as the optimal cost of (P_ε) when the initial time 0 is changed to $t \in [0, T]$, satisfies a Dynamic Programming Principle (DPP). From this DPP, a Hamilton-Jacobi-Bellman (HJB) equation can be derived and it is proved, under rather general conditions, that v_ε is the unique viscosity solution of the equation. Moreover, for $\varepsilon > 0$ and in our simple framework with the diffusion part of the dynamic being $\varepsilon \mathrm{d}W(t)$, it can be proved that v_ε is a classical solution of the associated HJB equation. Then, using classical arguments based on relaxed limits (see [2, Chapter 6, Section 3]), it is proved that v_ε converges to v_0 uniformly over any compact set $K \subseteq \mathbb{R}^n$. Moreover, using the technique of doubling of variables (which is the fundamental tool to prove the uniqueness of the solution of the HJB equation) or using probabilistic methods (see [15, Chapter 4, Corollary 4.2]), the following estimate can be derived

$$\sup_{(t,y)\in[0,T]\times\mathbb{R}^n} |v_\varepsilon(t, y) - v_0(t, y)| = O(\varepsilon). \tag{1}$$

Under the additional assumption of uniform semiconcavity of v_ε (see [5]) the above estimate can be importantly improved to

$$\sup_{(t,y)\in[0,T]\times\mathbb{R}^n} |v_\varepsilon(t, y) - v_0(t, y)| = O(\varepsilon^2). \tag{2}$$

Formally, estimate (2) suggest that the value function v_0 of the deterministic problem is not affected up to the first order by the addition of a viscosity term. This can be confirmed, as it is proved in [12] (see also [1] for a simple proof in the convex framework).

Now, we consider the limit behaviour for the solutions of strongly convex LQ problems. We assume that

$$\ell(x, u) := \tfrac{1}{2}u^\top Ru + \tfrac{1}{2}x^\top Cx, \quad g(x) := \tfrac{1}{2}x^\top Mx, \quad f(x, u) = Ax + Bu + b,$$

where R, C, M, A, B and b are *deterministic* matrices of suitable dimension. The strong convexity character of problems (P_ε) ($\varepsilon \in [0, \infty[$) is imposed through the conditions: R is positive definite and C, M are positive semidefinite. These conditions imply that for every $\varepsilon \in [0, \infty[$, problem (P_ε) admits a unique solution $(x_\varepsilon, u_\varepsilon)$. Using the stochastic Pontryagin principle, the following estimates (and in particular a convergence result) can be proved.

Proposition 1 *For all $\varepsilon \in [0, \infty[$ the following estimates hold true:*

$$\mathbb{E}\left(\sup_{t\in[0,T]} |x_\varepsilon(t) - x_0(t)|^2\right) + \mathbb{E}\left(\int_0^T |u_\varepsilon(t) - u_0(t)|^2 \mathrm{d}t\right) = O(\varepsilon^2).$$

Now, consider the following Riccati equation

$$\begin{aligned} -\dot{P} + PA + A^\top P + C - PBR^{-1}B^\top P = 0, \quad P(T) = M \\ \dot{\phi} + A^\top \phi - PBR^{-1}B^\top \phi + Pb = 0, \quad \phi(T) = 0. \end{aligned} \tag{3}$$

By [15, Chapter 6, Theorem 7.2] we have that (3) admits a unique solution (P_0, ϕ_0), which is the same for all $\varepsilon \in [0, \infty)$. In addition, Theorem 6.1 in [15, Chapter 6] implies that

$$u_\varepsilon(t) = -\Upsilon(t)x_\varepsilon - \gamma(t), \quad \text{with} \quad \Upsilon(t) := R^{-1}(t)B^\top(t)P_0(t) \quad \text{and} \quad \gamma(t) := R^{-1}(t)B^\top(t)\phi_0(t). \tag{4}$$

Therefore, setting $\hat{A}(t) = A(t) - \Upsilon(t)$ and $\hat{b}(t) := b(t) - B(t)\gamma(t)$ for all $\varepsilon \in [0, \infty)$ we obtain that

$$\mathrm{d}x_\varepsilon(t) = \left[\hat{A}(t)x_\varepsilon(t) + \hat{b}\right] + \varepsilon \mathrm{d}W(t), \quad x_\varepsilon(0) = x. \tag{5}$$

Thus, by the results in [9], x_ε satisfies a large deviation principle (see [6]) with *rate function*

$$I : C^1([0,T], \mathbb{R}^n) \to \mathbb{R}_+ \quad \text{with} \quad I(f) = \int_0^T \left| \dot{f}(t) - \left(\hat{A}(t)f(t) + \hat{b}(t)\right)\right|^2 \mathrm{d}t. \tag{6}$$

3 Conclusions

The very simple LD result presented above has the major drawback that it is based on Riccati theory for LQ problems. As a matter of fact, since this theory cannot be extended to more general settings (even by just adding a simple control constraints $u \geq 0$ in (P_ε) and (P_0)), a priori it is not clear how to obtain LD principles for more general classes of problems. The current research, which aims to extend the above results to constrained problems and non-convex cases, is based on the fact that for $\varepsilon > 0$ the problem the function v_ε is regular enough, and so a feedback law can be constructed directly in terms of Dv_ε. However, the main issue here is that for the unperturbed problem in general Dv_0 is not well defined. On the other hand, if the optimal trajectory x_0 for the deterministic problem lives on a region of strong regularity (see [7]) then a LDP holds again by the Freidlin-Wentzell's result. We expect that a more general study can be done by means of the stochastic Pontryagin principle (see e.g. [15, Chapter 3]) and some recent results for LDP of coupled Forward-Backward Differential equations.

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Applications of Derivative Free Optimization methods

Delphine Sinoquet and Frédéric Delbos

Abstract
Optimization takes place in many IFPEN applications: inferring the parameters of numerical models from experimental data (earth sciences, combustion in engines, chemical process), design optimization (wind turbine, risers, networks of oil pipelines), optimizing the settings of experimental devices (calibration of engines, catalysis). These optimizations consist in minimizing a functional that is complex (nonlinearities, noise, depending on mixed continuous and discrete variables) and expensive to estimate (solution of a numerical model based on differential systems, experimental measurements), and for which derivatives are often not available, with nonlinear constraints, and sometimes with several objectives among which it is necessary to find the best compromise.
We develop our own optimization tools or adapt existing ones in order to meet the needs of our applications as well as possible. In this talk, we illustrate the main encountered difficulties of several applications and we describe the dedicated derivative free optimization methods.

Keywords Derivative free optimization Smooth nonlinear optimization, Mixed-integer nonlinear optimization. Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

Optimal Short-term Unit Commitment Problem in Hydro Valleys

Wim van Ackooij · Claudia D'Ambrosio · Antonio Frangioni · Claudio Gentile · Frédéric Roupin · Raouia Taktak · Kostas Tavlaridis-Gyparakis

Abstract In this paper, we study a crucial problem in energy management, the Unit Commitment subproblem dedicated to hydro valley management. The problem consists in finding an optimal short-term hydro scheduling for a hydro valley composed of head-dependent reservoirs. The strong non-linearity of the problem as well as the combinatorial aspect of some constraints make it of high degree of complexity even in the case of a single reservoir. We describe a MILP formulation for the problem. We also discuss efficient methods of resolution of large-scale real-world instances of the problem.

Keywords hydro valley · short-term hydro scheduling · head-dependent reservoirs · mixed integer programming · decomposition techniques · Lagrangian relaxation

1 Introduction

In energy management, the Unit Commitment (UC) problem aims at computing the optimal production schedule for a hydro-thermal energy mix. This schedule is then executed to meet customer demand in real time the next day. The general hydro-thermal unit commitment is a crucial problem that has been widely studied in the literature. Different versions of the problem and many methodological approaches

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to tackle it are detailed in the surveys [10, 7, 9]. A recent survey dealing with the uncertain version of the problem is proposed in [6]. In order to solve the UC problem efficiently in finite time, decomposition methods based on Lagrangian relaxation are employed. These methods then require that various smaller sub-problems are resolved quickly. One such a sub-problem is the optimization of a hydro valley.

In this work we are only interested to the hydro scheduling sub-problem in its deterministic version. When continuous, such a problem is easily solved to optimality by any current LP solver. However, the introduction of combinatorial elements as a result of the quest for feasible schedules, leads to far tougher hydro valley problems. This is especially true for some of the larger French hydro valleys [4]. In the literature, many variants of the hydro scheduling problem in hydro valleys have been studied using various approaches of resolution. Mixed Integer Linear and Non-linear programming formulations have been proposed to model and solve the hydro short-term scheduling problem. In [1], the authors give an enhanced linearization technique to model and solve the problem in case of one reservoir. The case of several reservoirs has been further studied (see for instance [5]). Non-linear approaches of resolution are also proposed to solve the problem [2]. However, this kind of problem is very difficult to solve using the usual mathematical programming techniques. Moreover, solving large-scale real-world problems with the required efficiency using the standard MI(N)LP tools is almost impossible. Therefore, different methods of decomposition and relaxations (Lagrangian relaxation [3], Benders decomposition [11])as well as dynamic programming [8] have been used to provide solutions with provable high accuracy in a limited amount of time.

2 Problem formulation

We consider a given hydro valley. Each reservoir of the valley is associated to one or more turbines, which can produce power by using the water's potential energy, and one or more pump, that can transfer water back to the reservoir by using some power. In this context, complicated constraints, derived from physical requirements and market demands, have to be satisfied. The objective is minimizing the operational costs.

From a modeling viewpoint, the most important decision variables represent :

- Water flow passing through each turbine at each time period, i.e., variables that control each turbine.
- Water flow passing through each pump at each time period, i.e., variables that control each pump.
- Other important variables are the binary variables that model the on/off status of each turbine and pump.

The decision variables influence the following dependent variables :

- Water volume in each reservoir at each time period.
- Power output from each turbine at each time period.

In general the relationship between the independent and dependent variables is represented by nonlinear functions, see, for example, the power generated by a turbine as function of the water flow passing through the turbine and of the water level of the reservoir. These are part of the constraints that have to be satisfied. Other important constraints are of a technical sort. For example, turbines and pumps cannot be active in the same time period, flow is conserved at each reservoir at each time period, simple bounds on the variables are given, etc. In a real-world situation, additional and/or different variables and constraints could be taken into account. The cost depends on the water volume too.

3 Solution approaches

This problem is practically difficult to solve because of the complicated constraints involved, and the large size of real instances. Moreover, because of the presence of continuous and binary variables and general nonlinear constraints, the mathematical programming formulation describes a Mixed Integer (Non) Linear Programming (MI(N)LP) problem. This kind of problems are among the most difficult to solve in mathematical programming and can be attacked in different ways. Solving these large-scale, real-world problems with the required efficiency by straightforward application of standard MI(N)LP optimization tools is impossible. Therefore, specialized methods are required to provide solutions with provable high accuracy in a limited amount of time.

We first propose to use an efficient linearization technique in order to linearize the objective function and the constraints. The problem is hence modeled as a MILP. Then, we explore decomposition techniques and Lagrangian relaxation to efficiently tackle the problem. More precisely, we relax the linking constraints of the model. Further improvement techniques may also be interesting. One can, for instance, think about strengthening the formulation by cutting planes, improving the decomposition and using enhanced approximations, in order to provide efficient lower bounds of the optimal value.

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Langrangian Heuristics based on Disaggregated Bundle Methods for the EdF Unit Commitment Problem

Wim van Ackooij · Antonio Frangioni · Kostas Tavlaridis-Gyparakis

Abstract Our work focuses on the implementation of Lagrangian Heuristics for solving large scale Unit Commitment Problems, and more specifically for the case of EdF, one of the main electrical operators in France. In our approach we take into account the highly complicated Unit Commitment model of EdF and we exploit the already available sophisticated solution algorithms for the solution of the Lagrangian dual problem (inexact disaggregated Bundle methods) in order to construct Lagrangian heuristics capable of directly obtaining good primal feasible solution during the "dual phase", rather than employing an entirely separate "primal phase" using Augmented Lagrangian techniques.

Keywords Unit Commitment · Disaggregated Bundle Methods · Langrangian Heuristics

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

The Unit Commitment Problem (UC) in electrical power production is in general a very difficult problem to solve, as it requires to co-ordinate the operations of a large set of rather different generators over a given time horizon to satisfy a prescribed energy demand. The case of EdF, the main electrical operator in France and one of the major operators worldwide, is particularly challenging, as it refers to the very large scale of problem, with a very large number of generating units with very different operational characteristics. Above and beyond these difficulties, the EdF UC has specific load and safety requirements (the latter mainly represented as reserve constraints) that couples the generating units together; in addition, the operational constraints of some specific types of units (e.g. nuclear ones) are highly non-convex and/or combinatorial. All of the above already result to a highly challenging problem; furthermore, the operating constraint demands a highly efficient solution approach that can provide good feasible solutions in "unreasonably" small time with respect with the problem's size and complexity.

The current solution strategy in EdF uses Lagrangian relaxation, which has a long successful story in UC (e.g. [2,14][13, §3.3]) due to the fact that relaxing the demand constraints results to a separable problem that can be decomposed into a set of independent sub-problems one for each production unit,

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that can then be more easily solved e.g. using highly specialized algorithms [8]. The current solution process, that has started being developed on '90s under the guidance of researchers such as A. Renaud, C. Lemaréchal and C. Sagastizábal (e.g. [1, 4, 5]), is actually subdivided in two phases. In the first (dual) phase Lagrangian relaxation is used to decompose the system into individual thermal units and individual hydro valleys (multiple plants, turbines, reservoirs); the corresponding Langrangian dual is solved by a highly refined bundle method. This phase basically serves to compute optimal Lagrangian multipliers and a "convexified" (continuous) solution, which act as a starting point for the second (primal) phase where the original problem is reformulated to an Augmented Langrangian, which is solved following the auxiliary problem principle and the Uzawa method. While used with success in operations, this method is not without issues; for once the primal phase exhibits rather slow converge to a feasible solution, even after that a "fake" unit is added that ensures primal feasibility by producing any required amount of power at a hight cost. Moreover, the complex nature of the subproblems requires them to be solved inexactly; while this is quite well understood for the dual phase (e.g. [12]), the same cannot be said for the primal phase, that actually would require both exact solution methods *and* convexity in order to have guarantees. Finally, there is an unwelcome "instability" of the dual signal, in the sense that it is often found by practitioners not to "agree" with the schedule produced by the procedure.

2 The Proposed Approach

In order to improve on the current state-of-the-art, we intend to develop Lagrangian heuristics that, during the dual phase, use the available information to construct good feasible solutions, hopefully allowing us to entirely dispense with the primal phase. For this to work it is crucial to be able to exploit all the information that is generated during the solution process of the Lagrangian dual. Fortunately, this is abundant, and in fact even more so than the information that is generated while, say, solving a continuous relaxation, upon which the heuristics embedded in standard MILP solvers rely. Since the latter have recently shown to be very good at providing accurate solutions for "simplified" versions of UC (e.g. [9, 10]), one can be confident that the extra information provided by the Lagrangian dual can be put to good use. Indeed, it is well-known [6, 7] that most approaches for solving the Lagrangian dual, and in particular Bundle methods, are capable to provide both a set of integer solutions that are feasible to the non-relaxed constraints, and a "convexified" solution that is continuous but satisfies *all* constraints of the problem, comprised those that are relaxed in a Lagrangian way. In particular, the latter is obtained as a convex combination of the former, with multipliers that are available with no extra computational effort at each iteration of the Bundle algorithm.

This information has already been used in the past to drive Lagrangian heuristics for UC. In particular, in [6] the fractional "convexified" commitment solution was proposed to be interpreted as the "probability" that an instance should be on and used to construct feasible solutions with a greedy approach. In [3] this was generalized to a process that rather tried to modify the current integer infeasible commitment schedule by combining the Lagrangian costs of the thermal units (from the convexified solution) to form a priority list of the thermal units, that is used inside the heuristic to find a feasible thermal schedule. This has been shown to work well even in presence of ramping constraints [9], provided that ramp-constrained subproblems are solved [8], although specialized heuristics taking ramps into account can further somewhat improve the results [11].

However, all the above heuristics were tested on "simplified" academic versions of the UC problem, where the thermal units have relatively simple operational constraints (if compared with those of some of the most complex units in EdF UC), and the hydro units are completely continuous. The latter in particular is crucial for the heuristic, because one can then directly use the "convexified" solution of hydro units to define a residual energy demand to be satisfied by the thermal units; these have much less inter-temporal binding constraints (as opposed to those of hydro units related to the total amount of water in the reservoir), and this greatly simplifies the task of developing heuristics, as these only have to take into account "local" information.

Unfortunately, extending these ideas from the "classical" UC formulations (with no nuclear units and continuous hydro units) to the much more complicated structure of the EdF case is quite challenging. In particular, the following issues clearly arise:

– hydro units being no longer continuous, one cannot directly use the convexified hydro solution and simply remove them from the picture (least re-solving an Economic Dispatch once the commitment is found to perform that little last bit of optimization);
– thermal units being much more complicated and varied than in the "simplified" academic version, the greedy heuristic should be adapted to each of the different unit types, which would likely make it rather complex in certain cases;
– even if one would be willing to do the latter, each minor change in the model of the units might in principle require complex changes in the logic of the combinatorial heuristic.

To address all these issues, we plan to exploit the already-mentioned fact that MILP-based heuristics have recently become particularly successful in quickly finding very good-quality solutions to UC formulations (e.g. [10]). In particular, the idea is then to generalize the Lagrangian heuristic [3, 11] by finding ways to exploit the availability of both the convexified and the integer solutions that do not require unit-specific approach, but rather hinge solely on the availability of a MILP model for each unit and on some characteristic of the problem that is common to all units, e.g., the fact that decisions are spread along the same temporal horizon. We believe that the past success of Lagrangian heuristics, where the information provided by the bundle has been proved a very efficient guidance for obtaining good primal solutions, combined with the recent significant increase of effectiveness of general-purpose MILP-based heuristic, will put us in a position of being able to construct heuristics that are at the same time efficient, effective, and resistant to changes to the modeling of units, or even to introduction of entirely new types of units (storage, renewables, . . .). Finally, by naturally having the Lagrangian price signal as an input to the heuristic it should be possible to devise the heuristic in such a way that it produces solutions that "better agree" with the dual signal; this should partly be "free", since the heuristic will take as starting point the integer solutions, which necessarily "fully agree" with the dual one at the point where they are computed.

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Scalable Robust Pricing for margin-constrained revenue management

Arnau Tibau-Puig · Laurent El Ghaoui

Abstract In this paper we consider the problem of pricing a large set of products so as to maximize the revenue while maintaining the aggregate gross margin above a pre-specified level. To account for the variability of real demand, we assume that the sales of each item vary linearly with its price and both the baseline demand as well as the price elasticity suffer from interval uncertainty. Under this model, we show that the robust counterpart of the nominal pricing problem is a tractable convex problem. We introduce a scalable nested-bisection algorithm that is amenable to parallellization and enables the pricing of a large number of items, a desired feature for on-line retailers with potentially large catalogs.

Keywords Robust Optimization · Revenue Management · Dynamic Pricing

1 Introduction

Traditional dynamic pricing addresses the problem of finding a pricing policy that maximizes the revenue obtained from selling a fixed amount of products under a fixed amount of time. This understanding of dynamic pricing, also known as "revenue management", is justified by the problems in the airline and hotel industry that motivated this field of research and is typically concerned with the following setting: a seller wants to price a fixed total capacity of a perishable resource, such as seats in an airplane, to a price-sensisitive demand pool while allowing the prices to vary until the end of the time horizon (e.g. the moment then the corresponding airport check-in counter closes). The problem of revenue management is then stated as that of selecting an offering policy, defined by a collection of tuples of units sold and price per unit at each time point so that the cumulative revenue is maximized while making sure that the available capacity is exhausted at the end of the time horizon [4, 6].

In this paper, we consider a different formulation of the revenue management problem that is better aligned with "steady-state" online retail operations. First, we assume that the resources we want to price are non-perishable, in the sense that, while the capacity at any given time is limited, it is also non-depleted over time thanks to an effective replenishment policy. Second, we suppose that, within the time horizon considered for our pricing decision, demand is time invariant. Third and last, we are

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interested in pricing strategies that not only maximize the revenue, but also maintain two other quantities within pre-specified levels: (i) the gross margin, defined as the ratio between profit and revenue, and (ii) the number of price increases at any given point. The first requirement responds to the retail industry's need to control the global operating margin above satisfactory levels, and is fundamentally different from traditional revenue management formulations where the current inventory is considered to be a sunk cost and hence revenue and margin grow together, rendering the need to control for margin unnecessary. On the other hand, the limitation on price increases reflects the will to mitigate the negative impact of frequent price increases on the customers' trust.

Within the above framework, we formulate the dynamic pricing problem as a fixed-price optimization problem, and show that under a separable linear demand model, this problem can be cast as a simple convex Quadratic Constrained Quadratic Program (QCQP).

Like any other model-based pricing algorithm, our approach is sensitive to reality being different from what our demand model predicts. These deviations can have a severe impact on margin, especially in situations were the algorithm decides to hedge losses from products sold at a low margin with profit from products operating at a higher margin level. If the realized sales are different from the predicted ones (e.g. the items sold at low margin sell more than expected) the margin can fall well below the nominal expected level. To mitigate this problem, we augment the separable linear demand model to incorporate interval uncertainty on both the baseline sales as well as the price elasticity. We show that the robust counterpart (in the sense of [1]) of the nominal pricing problem can be cast as a convex problem with separable objective and two separable constraints. We then exploit the structure of this problem to derive a nested bisection algorithm on the dual problem that is scalable and easy to parallellize using an iterative map-reduce framework such as Spark [11].

Since our main motivation is scalability, there are a number of important issues that are left out of the scope of this paper. For instance, we do not consider stochastic and/or time-variant demand models that typically result into complex Dynamic Programming formulations even for the single product case [8,5, 7,10]. We also don't consider the much more challenging problem of jointly pricing and estimating the demand model such as in [9,2,3].

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Optimization of heating and condensation system of a water-condensed type washer dryer regarding energy consumption

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Abstract As the technology is developed more in time, home appliances which are manufactured to make our lives easier become more efficient, cheaper and useable. Washing machines and dryers which are two important products among white appliances have been conventionally used by consumers for a long time. Recently, these products are functionally combined in one machine and called as a washer dryer. The key component of washer dryer is a condenser which provides condensation of saturated vapor inside air during the process of removing humidity from clothes. The purpose of this study is optimization of the heating and condensation system in which water is sprayed on drying air directly of water – condensed type washer dryer regarding energy consumption. In order to achieve that purpose, a prototype was set up and related equipment was integrated into the prototype so as to store measurement data. The power of heater device, flow rate of fan and the design of nozzle orifice have been studied and the energy consumption of the washer dryer was minimized while keeping the water consumption of the condensation system and heating durations of the dryer steady. Then, tests results were analysed and discussed.

Keywords Washer dryer · Water-condensed · Condensation · Drying · Heater · Fan · Nozzle · Optimization of drying process · Optimization of condensation process

Mathematics Subject Classification (2000) MSC 80A05 · 80A20 · MSC 76T99

1 Introduction

At the present time, the systems, which condense steam inside air to water, are used in drying and cooling units. The washer dryer is a drying unit comprising of a motor rotating drum, a fan providing air circulation, a heater heating air, a nozzle spraying water and a valve directing water to the nozzle (Fig. 1). There is a strong link between energy consumption and drying time. Energy consumption increases due to work of components in the drying unit as mentioned above when drying time is longer. Therefore, drying time directly affects energy consumption, and drying time could be reduced if the efficiency of condenser is improved.

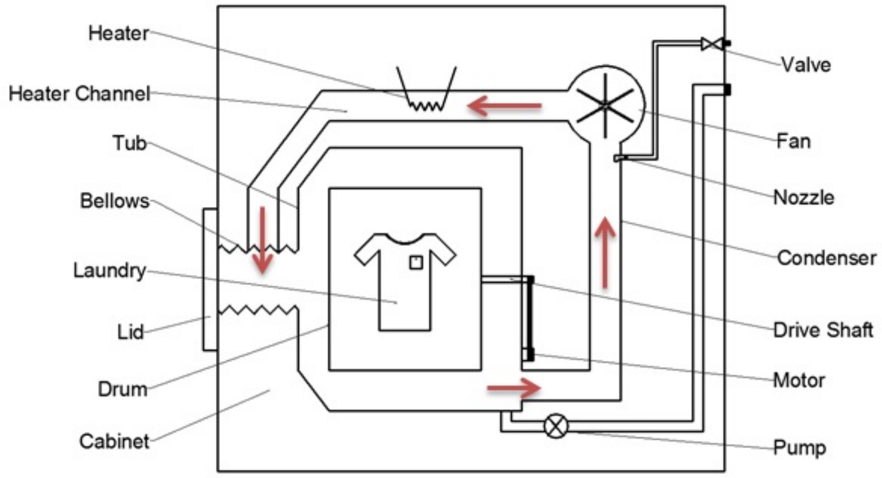

Fig. 1. Schematic diagram of the test apparatus.

The efficiency of the condenser unit depends on many parameters. Structural design of nozzle has so significant influence on heat transfer that a small nozzle diameter gives rise to turbulent flow for same mass flow rate due to higher mean velocity, therefore; better heat transfer occurs during condensation process [1,2]. However, reducing nozzle diameter brings about lime scale and a blockage of the nozzle resulting from tap water. For this reason, the nozzle diameter is restricted in the washer dryer. The other important parameter affecting condensation is a liquid to humid air mass flow rate ratio. Condensation level increases with increasing liquid to humid air mass flow rate ratios. Hence, the highest efficiency has been obtained while using lowest air flow rate and highest water flow rate [3,4]. On the other hand, decrease in air flow rate can result in longer drying time due to temperature and velocity drop. It can be also said that condensation is a cooling process. Effects of spray inclination and gravity in spray cooling systems have been investigated in previous studies [5,6]. Inclination angle of the nozzle orifice relative to the cooled surface would affect the heat removal negatively after a certain degree. It has been also shown that the gravity vector has no obvious effect on cooling because the sprayed water has much higher momentum. Moreover, the fact that temperature affects heat transfer is known [7]. Increasing heater power causes temperature rise which positively influences heat transfer taking place between drying air and a laundry via convection.

In this work, effects of heater power, fan flow rate and nozzle design on energy consumption and drying performance are analyzed (Fig. 2). Then, optimal energy and water consumption is investigated by considering these effects. A reference of heater power is set as 1200 watt, and test parameters are created by increasing the heater power %1,083 and %1,167 times. Similarly, a reference of fan flow rate is set as 18 liter/second, and test parameters are created by increasing the fan flow rate %1,11 and %1,39 times. Moreover, reference inclination angle of nozzle orifice is set as 15°, and test parameters are created by increasing the inclination angle 3 and 6 times. Generally, heater power, fan flow rate and inclination angle of nozzle orifice change within the ranges mentioned above in washer-dryers.

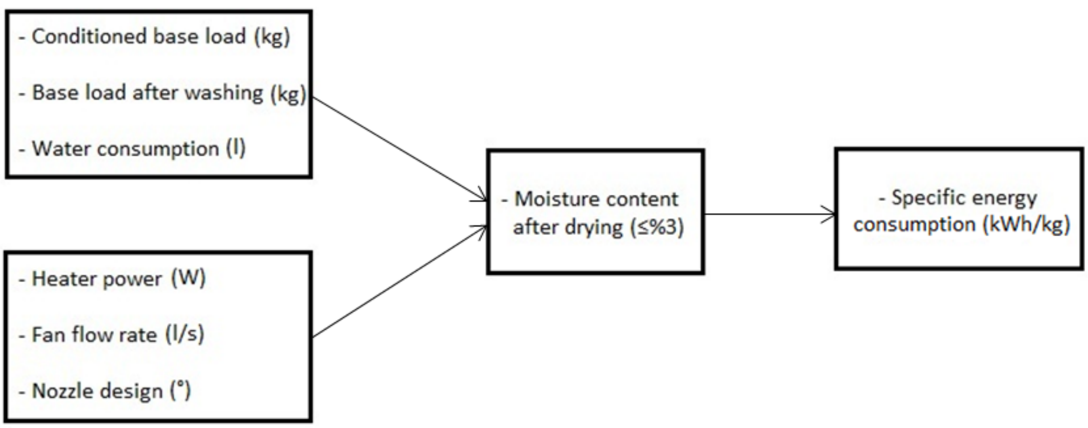

Fig. 2. Solution algorithm.

2 Conclusions

Tests have been conducted according to washer – dryer standards EN50229:2007. The effects of power of heater, fan flow rate and nozzle design, which are thought to affect drying performance, have been separately investigated. Optimization of power of heater, fan flow rate and nozzle design has been made by considering drying performance and specific drying energy consumption. Energy consumption increases whereas drying performance gets better if heater power is increased. Also, it was observed that fan flow rate affects drying temperature. As a result of the study, optimal energy consumption was obtained with the heater power increased %1,083 times of the reference heater power and fan flow rate increased %1,11 times of the reference fan flow rate when considering energy consumption and drying performance. Since effects of nozzle design on energy consumption and drying performance couldn't be clearly seen, its effects on temperature have been observed. Thus, a nozzle which has the reference inclination angle was determined. By using the current results, a model which consumes less energy and water in a predetermined period and protects the nature has been developed.

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Exact Algorithms for Solving Stochastic Games

Kristoffer Arnsfelt Hansen · Michal Koucký · Niels Lauritzen · Peter Bro Miltersen · Elias P. Tsigaridas

Abstract Shapley's *discounted stochastic games*, Everett's *recursive games* and Gillette's *undiscounted stochastic games* are classical models of game theory describing two-player zero-sum games of potentially infinite duration. We describe algorithms for exactly solving these games. When the number of positions of the game is constant, our algorithms run in polynomial time.

Keywords Stochastic Games · Shapley Games · Gillette Games · Everett Games · Polynomial Systems

1 Introduction

Shapley's model of finite *stochastic games* [14] is a classical model of game theory describing two-player zero-sum games of (potentially) infinite duration. Such a game is given by a finite set of positions $1, \ldots, N$, with a $m_k \times n_k$ *reward* matrix (a^k_{ij}) associated to each position k, and an $m_k \times n_k$ *transition* matrix (p^{kl}_{ij}) associated to each pair of positions k and l. The game is played in rounds, with some position k being the *current* position in each round. At each such round, Player I chooses an action $i \in \{1, 2, \ldots, m_k\}$ while simultaneously, Player II chooses an action $j \in \{1, 2, \ldots, n_k\}$, after which the (possibly negative) *reward* a^k_{ij} is paid by Player II to Player I, and with probability p^{kl}_{ij} the current position becomes l for the next round.

During play of a stochastic game, a sequence of rewards is paid by Player II to Player I. There are three standard ways of associating a *payoff* to Player I from such a sequence, leading to three different variants of the stochastic game model:

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Shapley games. In Shapley's original paper, the payoff is simply the sum of rewards. While this is not well-defined in general, in Shapley's setting it is required that for all positions k, $\sum_l p_{ij}^{kl} < 1$, with the remaining probability mass resulting in termination of play. Thus, no matter which actions are chosen by the players, play eventually ends with probability 1, making the payoff well-defined except with probability 0. We shall refer to this original variant of the stochastic games model as *Shapley games*. Shapley observed that an alternative formulation of this payoff criterion is to require $\sum_l p_{ij}^{kl} = 1$, but *discounting* rewards, i.e., penalizing a reward accumulated at time t by a factor of γ^t where γ is a *discount factor* strictly between 0 and 1. Therefore, Shapley games are also often referred to as *discounted* stochastic games. Using the Banach fixed point theorem in combination with the von Neumann minimax theorem for matrix games, Shapley showed that all Shapley games have a value, or, more precisely, a *value vector*, one value for each position. Also, the values can be guaranteed by both players by a *stationary strategy*, i.e., a strategy that associates a fixed probability distribution on actions to each position and therefore does not take history of play into account.

Gillette games. Gillette [10] requires that for all k, i, j, $\sum_l p_{ij}^{kl} = 1$, i.e., all plays are infinite. The total payoff to Player I is $\liminf_{T\to\infty}(\sum_{t=1}^{T} r_i)/T$ where r_t is the reward collected at round t. Such games are called *undiscounted* or *limiting average* stochastic games. In this paper, for coherence of terminology, we shall refer to them as *Gillette games*. It is much harder to see that Gillette games have values than that Shapley games do. In fact, it was open for many years if the concrete game *The Big Match* with only three positions that was suggested by Gillette has a value. This problem was resolved by Blackwell and Ferguson [3], and later, Mertens and Neyman [13] proved in an ingenious way that all Gillette games have value vectors, using the result of Bewley and Kohlberg [2]. However, the values can in general only be approximated arbitrarily well by strategies of the players, not guaranteed exactly, and non-stationary strategies (taking history of play into account) are needed even to achieve such approximations. In fact, *The Big Match* proves both of these points.

Everett games. Of generality between Shapley games and Gillette games is the model of *recursive games* of Everett [9]. We shall refer to these games as *Everett games*, also to avoid confusion with the largely unrelated notion of recursive games of Etessami and Yannakakis [7]. In Everett's model, we have $a_{ij}^k = 0$ for all i, j, k, i..e, rewards are *not* accumulated during play. For each particular k, we can have either $\sum_l p_{ij}^{kl} < 1$ or $\sum_l p_{ij}^{kl} = 1$. In the former case, a prespecified payoff b_{ij}^k is associated to the termination outcome. Payoff 0 is associated with infinite play. The special case of Everett games where $b_{ij}^k = 1$ for all k, i, j has been studied under the name of *concurrent reachability games* in the computer science literature [6,4,12,11]. Everett showed that Shapley games can be seen as a special case of Everett games. Also, it is easy to see Everett games as a special case of Gillette games. It was shown in Everett's original paper that all Everett games have value vectors. Like Gillette games, the values can in general only be approximated arbitrarily well, but unlike Gillette games, stationary strategies are sufficient for guaranteeing such approximations.

2 Our Results

We consider the problem of exactly solving Shapley, Everett and Gillette games, i.e., computing the value of a given game. The variants of these two problems for the case of *perfect information* (a.k.a. *turn-based*) games are well-studied by the computer science community, but not known to be polynomial time solvable: The tasks of solving perfect information Shapley, Everett and Gillette games and the task of solving Condon's *simple stochastic games* [5] are polynomial time equivalent [1]. Solving simple stochastic games in polynomial time is by now a famous open problem. As we consider algorithms for the more general case of imperfect information games, we, unsurprisingly, do not come up with polynomial time algorithms. However, we describe algorithms for all three classes of games that run in polynomial time when the number of positions is constant and our algorithms are the first algorithms with this property. As the values of all three kinds of games may be irrational but algebraic numbers, our algorithms output real algebraic numbers in *isolating interval representation,* i.e., as a square-free polynomial with rational coefficients for which the value is a root, together with an (isolating) interval with rational endpoints in which this root is the only root of the polynomial. To be precise, our main theorem is:

Theorem. *For any constant N, there is a polynomial time algorithm that takes as input a Shapley, Everett or Gillette game with N positions and outputs its value vector using isolating interval encoding. Also, for the case of a Shapley games, an optimal stationary strategy for the game in isolating interval encoding can be computed in polynomial time. Finally, for Shapley as well as Everett games, given an additional input parameter $\epsilon > 0$, an ϵ-optimal stationary strategy using only (dyadic) rational valued probabilities can be computed in time polynomial in the representation of the game and* $\log(1/\epsilon)$.

We remark that when the number of positions N is constant, what remains to vary is (most importantly) the number of actions m for each player in each position and (less importantly) the bitsize τ of transition probabilities and payoffs. We also remark that Etessami and Yannakakis [8] showed that the bitsize of the isolating interval encoding of the value of a discounted stochastic game as well as the value of a recursive game may be exponential in the number of positions of the game and that Hansen, Koucký and Miltersen [12] showed that the bitsize of an ϵ-optimal strategy for a recursive game using binary representation of probabilities may be exponential in the number of positions of the game. Thus, merely from the size of the output to be produced, there can be no polynomial time algorithm for the tasks considered in the theorem without some restriction on N. Nevertheless, the time complexity of our algorithm has a dependence on N which is very bad and not matching the size of the output. For the case of Shapley games, the exponent in the polynomial time bound is $O(N)^{N^2}$ while for the case of Everett games and Gillette games, the exponent is $N^{O(N^2)}$. Thus, getting a better dependence on N is a very interesting open problem.

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Approximation of backward stochastic differential equations using Malliavin weights and least-squares regression

Gobet, E. · Turkedjiev, P.

Abstract We design a numerical scheme for solving a Dynamic Programming equation with Malliavin weights [1] arising from the time-discretization of backward stochastic differential equations with the integration by parts-representation of the Z-component by [2]. When the sequence of conditional expectations is computed using empirical least-squares regressions, we establish, under general conditions, tight error bounds as the time-average of local regression errors only (up to logarithmic factors). We compute the algorithm complexity by a suitable optimization of the parameters, depending on the dimension and the smoothness of value functions, in the limit as the number of grid times goes to infinity. The estimates take into account the regularity of the terminal function. As an example, we apply the algorithm to price a spread option with multiple underlying assets under trading constraints.

Keywords Backward stochastic differential equations · Malliavin calculus · dynamic programming equation · empirical regressions · non-asymptotic error estimates

Mathematics Subject Classification (2000) 49L20 · 60H07 · 62Jxx · 65C30 · 93E24

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Stochastic games with partial observation and Borel evaluation

Xavier Venel

The aim of this presentation is to study two-player zero-sum stochastic games with partial observation. At each stage, both players choose some actions. This generates a stage payoff then a new state and new signals are randomly chosen according to a transition function.

There are several ways to study the long term behavior of these games. A lot of attention has been given to two of these approaches: the asymptotic behavior of the n-stage game and the uniform value which focuses on what payoff a player can guarantee independently of the length of the game. A recent counter-example of Ziliotto (2013) with symmetric information showed that when the players are not informed of the state, the values of the n-stage games may not converge.

In this presentation, we come back to a point of view coming from the literature of game determinacy (Gale and Stewart 1953) and adopted by Maitra and Sudderth (1992): from the sequence of stage payoff, we can define an evaluation on the set of infinite histories and study the existence of the value in the induced normal form game.

We provide several counterexamples to the existence of the value and several positive results. In particular, there exists a value for any Borelian evaluation in stochastic games with symmetric information.

Joint work with Hugo Gimbert, Jérôme Renault, Sylvain Sorin and Wieslaw Zielonka,

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Transmission lines switching in electric power networks by means of nonlinear stochastic programming

Francesco Piu · Alois Pichler · Asgeir Tomasgard · Maria Teresa Vespucci

Abstract

Switching off selected transmission lines of an electricity network can lead to savings in the total production costs. This fact is gaining increasing interest since new transmission lines are required to access power production places not exploited in the past (e.g., off-shore wind parks). This offers the opportunity to redesign the power network and to incorporate new switching possibilities. The problem is to identify the transmission lines with the highest savings potential. We employ stochastic programming to face the problem and we study how to achieve a tractable problem size.

Keywords: line switching, stochastic programming

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

Switching off selected transmission lines of an electricity network can lead to savings in the total production costs. This, perhaps surprising, fact has gained increasing interest in the recent past, as the overall profitability of a given network can be increased. Energy is often produced in different places than in the past, for example in off-shore wind parks as in Germany, Denmark and many other countries. This situation offers the opportunity of re-designing the existing power flow network and to incorporate switching possibilities in the network. The central problem consists in finding and identifying those transmission lines, which provide the highest savings potential, while the power supply has to be secure at the same time in the whole area. This paper employs stochastic programming to elaborate the difficulties of the whole problem. We combine the analysis of operational and investment decisions. We employ stochastic programming to study the switching investments under uncertainty.

Investment decisions, required for creating remote and automatic switches, consider several possible scenarios that are modeled as random variables (power demand and generation) on a probability space. The investment decision problem is formulated as a two stage, nonlinear stochastic optimization problem characterized by an inner and an outer minimization. Given a power demand and generation scenario, in the inner minimization (second stage) we identify the set of switching actions that minimizes the active power production cost. In the outer minimization (first stage) we identify the set of switching investment opportunities that minimize the expected active power production cost over a (large) set of power demand and generation scenarios. The outer (combinatorial) minimization identifies and locates the lines which are promising for future switching possibilities. The inner minimization represents the Optimal Transmission Switching (OTS) problem. The OTS models the response of the system operator to manage the power flow network. Given some demand in the area of power supply and a supply from the power plants, the system operator adjusts the network by switching transmission lines on and off with the purpose to save total production costs. The OTS problem formulation is

strongly related with the well known Optimal Power Flow (OPF) problem. The OTS problem introduces a set of binary variables (switching possibilities) in one of the two different OPF formulations: direct current (DC) OPF or alternating current (AC) OPF (computationally expensive).
It was realized recently, that the problem of transmission switching is a genuinely nonlinear problem that should be based on the OPF in its nonlinear form (AC OPF) rather than its much simpler linear approximation (DC OPF). The reason is that the error accepted when passing from the nonlinear to the linear problem often exceeds the amount of savings.
Several authors report that the classical Mixed Integer Non Linear Programming formulation of the OTS (based on the AC OPF) is computationally intractable for real-life instances.
Possibilities of how to reduce computational intractability are elaborated.
Following recent studies, we propose a heuristics that estimate the economic impact of line switching by looking at values of primal and dual variables (shadow prices).

2 Conclusions

The proposed heuristic greatly improve computational times by ranking switching possibilities. The ranking of switching actions require the solution of a corresponding set of AC OPF instances. In order to locate promising switching possibilities and identify the corresponding investment decisions, we need to solve a large set of AC OPF instances. The efficient solution of these instances is a crucial task solved by exploiting a dedicated fortran code (developed jointly with RSE). Once we the inner nonlinear minimization problem is solved for a large set of scenarios, we are able to solve the outer combinatorial minimization.

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Limit game: asymptotic analyis of two person zero-sum dynamic games

Guillaume Vigeral

We define and study the "limit game", that is a game played on the time interval $[0, 1]$ seen as the limit of games played at the discrete time $\frac{k}{n}$ as n goes to infinity, for some classes of zero-sum stochastic games. An interesting feature of the limit game, when it exists, is that it permits to construct simple almost optimal strategies for games with long duration. An example is the case of the Big Match in the dark : in the limit game the optimal play for Player 1 is to reach absorption with probability t at time t ; mimicking this behavior in discrete time games induces optimal strategies. Hence, even if there is no uniform value (the optimal strategies has to depend on the precise duration of the game), the optimal play at some stage depend on the horizon only through the relative length of the past with respect to the future. It is thus intringing to determine wether such invariants exist for more general games : does the "expected accumulated payoff at half horizon under optimal play" converges when the horizon goes to infinity ? If yes, is this quantity half the payoff at the end of the game ? Is there some invariance for stage occupation as well ? We recall how we answered these questions (with Xavier Venel) in the dynamic programming case, and we settle them also for finite absorbing games (using an explicit formula due to Laraki) while showing an example in the compact framework where no limit game exists. . The general case is still open : two of the most interesting frameworks being finite stochastic games, as well as games with imperfect information on both sides (what can we say of the quantity of information revealed at half the horizon under optimal play ?).

Joint work with Sylvain Sorin

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Links between no-regret dynamics and fictitious play

Yannick Viossat

No-regret dynamics and fictitious play are two of the most studied models of heuristic learning in games. We will show that they are related: the solutions of a wide class of no-regret dynamics are perturbed solutions of a continuous version of fictitious play, in the sense of the theory of perturbed differential inclusions. This allows to relate the asymptotic behavior of these processes in some classes of games.

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HERBS: Hydro-Electric Reservoir Bidding System

Faisal Wahid

Abstract In a market environment, electricity producing firms compete to supply electricity to the national grid. A major challenge for hydro-electric producers, participating in an electricity market, is supplying offers which maximise the monetary value of the water. Electricité de France (EDF) faces a similar problem to produce optimal offer curves to an intra-day adjustment market among their hydro assets. The adjustment markets are used to balance the realtime volatility of demand and supply economically. It involves participants to submit offers to indicate to the TSO (Transmission System Operator) how sensitive they are to increasing or reducing their generation from their original schedule. We have developed a suite of stochastic optimization models, called HERBS (Hydro-Electric Reservoir Bidding System), that produce offer curves for each of EDF's hydro valleys in the adjustment market. HERBS suite of models range in a rolling horizon stochastic Mixed Integer Program (SMIP) model and Stochastic Dynamic Program model with a time inhomogeneous Markov chain price process. Furthermore we apply several novel a fusion of Dantzig-Wolfe and Stochastic Dual Dynamic Programming approach to reduce computational difficulties faced by the presence of discrete variables in these models. We evaluate the models and the approach with respect to their compuational efficiency; quality of the solution and present our findings.

Keywords Stochastic optimisation · hydro bidding

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

In a market environment electricity producing firms compete to supply electricity to the national grid. A major challenge for hydro-electric producers, participating in an electricity market, is supplying offers which maximise the monetary value of the water. Hydro stations face the complexity in estimating their marginal cost, with respect to the future opportunity cost of generation because of their ability to store the water for later use. Additionally they face operational challenges to efficiently manage the hydro valley (see figure 1).

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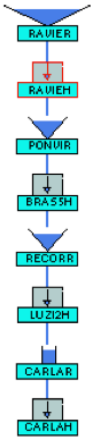

Fig. 1 Example of a hydro valley

Electricité de France (EDF) faces a similar problem to produce optimal offer curves to an intra-day adjustment market among their hydro assets. The adjustment markets are used to balance the real-time volatility of demand and supply economically. It involves participants to submit offers to indicate to the Transmission System Operator (TSO) how sensitive they are to increasing or decreasing their generation from their original schedule (see figure 2).

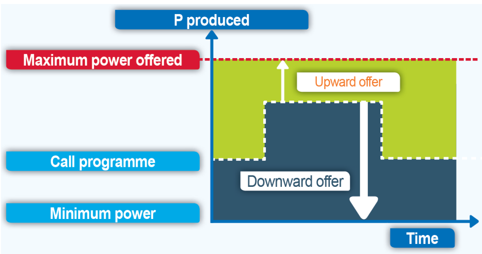

Fig. 2 Example of adjustment offers

Everyday EDF produces a production schedule for their river chains using a program called APOPHIS. In APOPHIS the time steps are of 30 minutes duration. APOPHIS has two phases. In the first phase, APOPHIS is solved by decomposition to give a system clearing price over a time horizon of 96 half hour periods. The second phase of APOPHIS constructs a feasible dispatch for each river chain that is obtained by solving a mixed integer program (MIP). This chooses generation and reserve quantities at each station on the river to maximize the daily revenue from the river chain earned at the system clearing prices. In between solves of APOPHIS the dispatch schedule for the current day is implemented. This might involve some adjustment of a small number of units in real time as demand changes over the day.

We have developed a suite of stochastic optimization models, called HERBS (Hydro-Electric Reservoir Bidding System), that produce offer curves for each of EDF's hydro valleys. These range in a rolling horizon stochastic Mixed Integer Program (SMIP) model (see model 1) and Stochastic Dynamic Program model with a time inhomogeneous Markov chain price process.

The mixed integer program formulation of HERBS (model 1) is over *half-hour* periods $t = 1, 2, \ldots, T$, where we are computing a balancing offer for a particular period. Let $x(t)$ denote a vector of reservoir storages in each node at the beginning of period t, and $w(t)$ a vector of uncontrolled reservoir inflows (in cubic metres) that have occurred in period t. We let $u(t)$ be a vector of flow rates (cubic metres per hour) in the arcs in the network, and $\pi(t)$ a vector of electricity prices at time t. We require $u \in \mathcal{U}$, and $x \in \mathcal{X}$, that might involve binary choices. The optimal generation plan with perfect information on prices is obtained by solving model 1.

$$\begin{array}{rl} \text{MIP:} \ \max & \sum_{t=1}^{T} \sum_{r=1}^{R} \pi^r(t) q^r(t) \\ \text{s.t.} & x(t+1) = x(t) - Au(t) + w(t), \\ & q^r(t) = \sum_{k \in r} u_k(t), \\ & u \in \mathcal{U}, x \in \mathcal{X}. \end{array} \tag{1}$$

2 Conclusions

Model 1 produces offer stacks and a dispatch schedule (see figure 3) illustrating the redeclared schedule and the respective deviations.

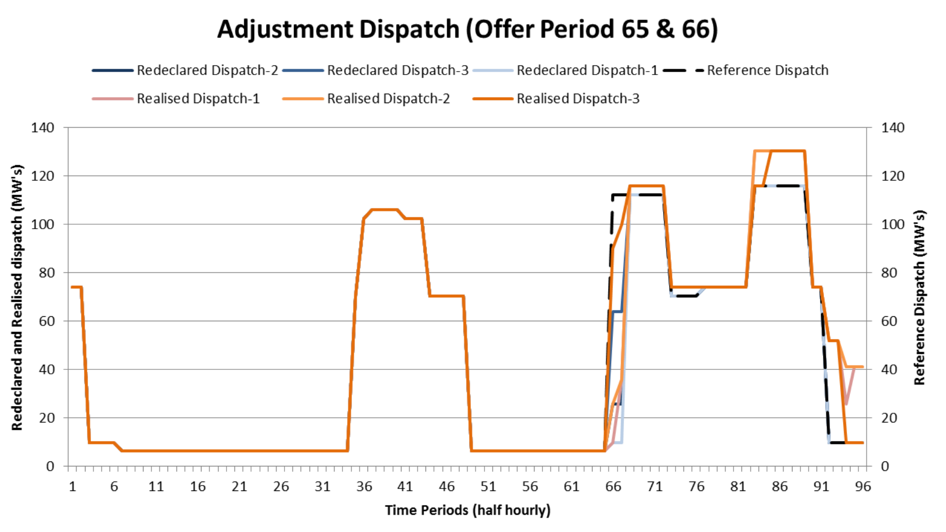

Fig. 3 Example schedule of a hydro valley optimised by HERBS

We apply a fusion of Dantzig-Wolfe and Stochastic Dual Dynamic Programming approach to reduce computational difficulties faced by the presence of these discrete variables, such as when $\mathcal{U}$ is a set of releases that must lie in a discrete set. We evaluate these methods with respect to their efficiency and quality of the solution and present the findings.

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SunHydrO: Pumped-storage for an improved renewable energy integration. Contributing to the spinning reserve

Ariel Waserhole · Francis Sourd · Davy Marchand-Maillet

Abstract We study the integration of renewable energy in the electricity market with the use of storage systems. We present an overview of the markets and the current models. We discuss the difficulty to tackle the spinning reserve market (reactive power market) while aggregating renewable intermittent energy.

Keywords Energy storage · Renewable energy aggregation · Stochastic optimization

1 The SunHydrO project

The ever-growing share of renewable energy of the electricity mix represents a new challenge. In energy markets, such as EPEX, an actor sells (or buys) a fixed quantity of energy to produce (or consume) during an one-hour long time period. This energy is supposed to be produced at a constant power. The network authority periodically controls that the produced (and consumed) electricity volumes corresponds to what has been sold (and bought). In France, the transmission system operator RTE controls this balance every half-hour and charges penalties for each imbalance.

Currently most of sun and wind energy is sold to the network at a guaranteed feed-in tariff. However, the feed-in tariff is intended to disappear and renewable energy producers will have to go to market to sell their production. Each producer will then be accountable for its own production forecast and will have to cope with the randomness aspect of his production. The left side of Figure 1 shows a typical daily photovoltaic (PV) production. One can observe that hourly variance in the production can be huge. Therefore, even with a good forecast, a production based on a single plant would imply penalties imbalance.

Sun'R Smart Energy [6] proposes to aggregate different renewable energy sources (producers) in order to sell the global production on the market. By considering different sites and types of energy, renewable energy production is smoothed. The right part of Figure 1 represents such aggregated PV production. The aggregator is then able to obtain better forecasts and to increase the value of the renewable energy.

To reduce even more imbalance penalties, Sun'R Smart Energy has elaborated a research and development project, named *SunHydrO*, to study the joint management of a renewable energy production

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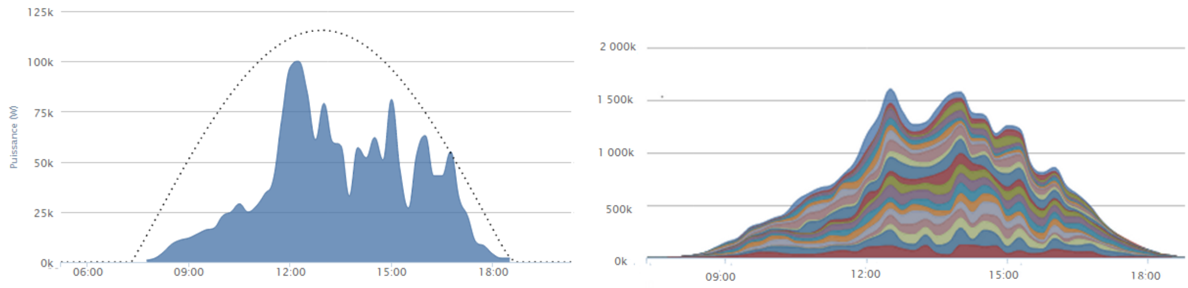

Fig. 1 Photovoltaic production for a single plant and for a group of aggregated plants. Screen shots captured from QOS Energy Quantum® [5] interface.

and storage systems. The storage enables to increase the value of renewable energy by delaying the sell instant after the production. Moreover, it offers the opportunity to contribute to the system services such as the secondary load-frequency control.

The generators of renewable energy and the storage system are viewed as a Virtual Power Plant (VPP). When the sun and wind production deviates from the forecast, the storage system can be activated to satisfy the market commitment of the VPP. Figure 2 schemes a week production plan where an aggregated PV production is optimized jointly with a storage system.

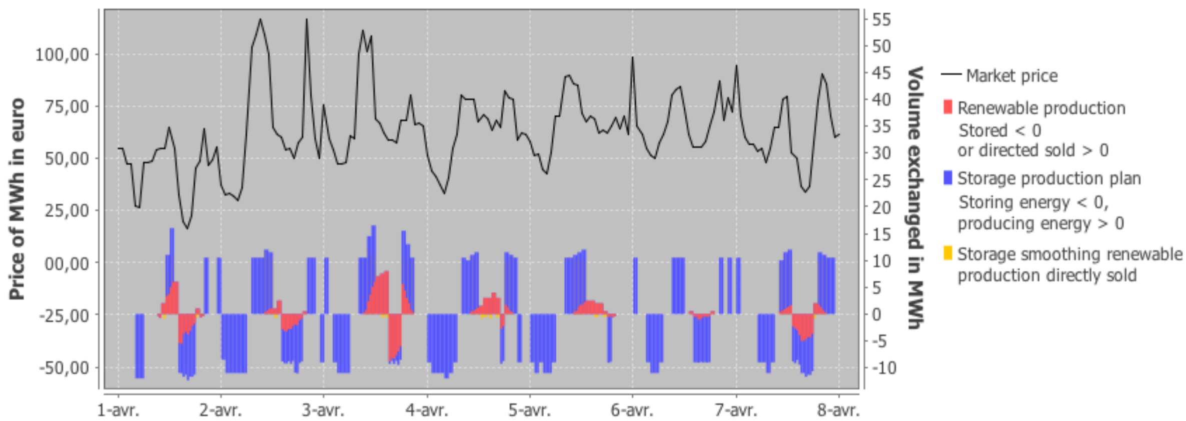

Fig. 2 An optimized production plan of a VPP composed of a storage system and an aggregated renewable energy production.

2 Joint optimization of storage and intermittent renewable energy

The energy is traded in two markets. First in the *Day-Ahead market* that is a spot market where offers are submitted at 12h00 for the whole next day. Secondly in the *Intraday market* that is a mutual agreement continuous market open from 16h00 the day before till 45 minutes before delivery.

Regarding the production at a given H-Hour of the D-Day (more precisely during the hour that follows the H-Hour), the decision process is hence the following:

1. Offers (bids) on the Day-Ahead market at 12h00 D-1 with a response at 12h40.
2. Offers in continuous-time on the Intraday market between 16h00 D-1 to H-45min.
3. At H minus 45 minutes, no more bids can be proposed. The sum of the volumes of accepted bids gives the amount of energy the VPP is committed to produce.

4. During the production hour H, we consider that the renewable energy production is variable, that is uncontrolled. Hence, storing and generating of the storage system are the only leverage on VPP production left. This storage control is made in real-time in order to maximize the (total) expected benefit (using the most efficient modes of the storage system, minimizing penalties. . .).

If one discretizes the time horizon of the (continuous) Intraday market, the problem can be modeled as a multi-stages stochastic program. In the literature, Jiang and Powell [1] have considered the problem of hour-ahead bidding in an Intraday market. However, currently in France the Intraday market is not liquid enough to plan to sell all the production in such a short time before the delivery. Therefore our approach is to favor strategies that trade most of the energy on Day-Ahead market and use the Intraday market as a recourse.

In the literature, Löhndorf and Minner [2] have introduced a model of the optimal bidding strategy for a VPP composed of renewable power generation and energy storage. Later, Löhndorf *et al.* [3] have proposed a model to tackle the short-term intraday market with day-ahead decision. However this model does not consider renewable energy.

3 Contributing to the spinning reserve

Storage is useful to renewable energy producers to satisfy their commitment. However, the main problem of the storage is its high infrastructure cost. To increase the benefits generated by the storage, in addition to trading on the energy market, it appears necessary to participate to all (existing and future) energy and power markets. Some markets can be easily tackled, such as the one associated to the replacement reserve (RTE's tertiary reserve). Since it is able to quickly increase (or decrease) the power of its storage unit, the VPP is also able to contribute to the spinning reserve (RTE's secondary reserve).

From the operational point of view, contributing to the spinning reserve implies a real-time monitoring of the distributed renewable energy production in order to control the VPP power every 10 seconds. Without the stochasticity introduced by renewable energy production, Puglia *et al.* [4] have been able to model the spinning reserve market. They have tackled a bidding problem to obtain optimal quantities to bid for the day-ahead market, while reserving the remaining production to the reactive power market.

To the best of our knowledge, no studies have been done yet on tackling the spinning reserve market with a VPP composed of renewable energy and a storage system. We propose a stochastic model to estimate the cost of such a contribution (even if the spinning reserve market does not exist yet). Our stochastic model, corroborated with simulation, shows the difficulty of entering into the spinning reserve market. In particular, we exhibit a lower bound on the cost for smoothing the VPP power in order to be eligible to contribute to the current spinning reserve.

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