Three examples of maths in applications

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Modelling of incentives for the production of renewable

Context and statement of the problem

Collaboration between :

- Researchers at EDF (Clémence Alasseur and Matteo Basei)
- Researchers at École polytechnique (Alekos Cecchin and CB)
- Originated from a long time partnership between EDF and École polytechnique (Chaire Finance et développement durable).

What we worked on :

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- In France, the government is issuing "primes" or allocations to create incentives to install individual capacity of production of renewable energies (solar panels for instance)

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What we worked on :

- In France, the government is issuing "primes" or allocations to create incentives to install individual capacity of production of renewable energies (solar panels for instance)
- Main question : How should the regulator divides its incentives between installation allocation and reduced price to achieve a desired capacity of production ?

The math involved

- Capacity of production K_t evolves according to

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- The equations are coupled. To solve this issue, we look instead for a solution U of

$$0 = -(r+\delta)U(K) + (\lambda U(K) - \alpha - \delta K)\partial_K U(K) + \left(\frac{p}{K+\epsilon} - c \right)^+$$

The math involved II

- The problem at interest is

$$\inf_{\alpha,c} \{\text{Target}[U_{\alpha,c}] + \text{Cost}(\alpha, c)\}$$

  Hence, it is an optimization problem.
- This type of problem belongs to the class of optimal control of partial differential equations
- We were able to provide explicit computations in certain cases and quite general numerical computations.

Strategic interactions on the oil markets

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Collaboration between :

- Mathematicians (Y Achdou, JM Lasry, PL Lions, CB)
- Economist (J Scheinkman)
- Industrial expert (A Rostand)
- Originated from a research initiative funded by Kayrros

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- The oil production market is shared between a cartel (OPEC) and a fringe of small producers

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- The oil production market is shared between a cartel (OPEC) and a fringe of small producers
- New data from Kayrros on the global level of oil storage
- Main question : Can we explain the strategic interactions between the OPEC and its competitors ?

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A complex model :

- Several types of agents : OPEC, small producers, storage arbitrageurs, consumers

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- Two states equations :

$$dz_t = \sqrt{ap_t - b}dt$$

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- OPEC determines its level of production q_t through stochastic optimal control, the price p_t is determined through a no arbitrage relation

The math involved II

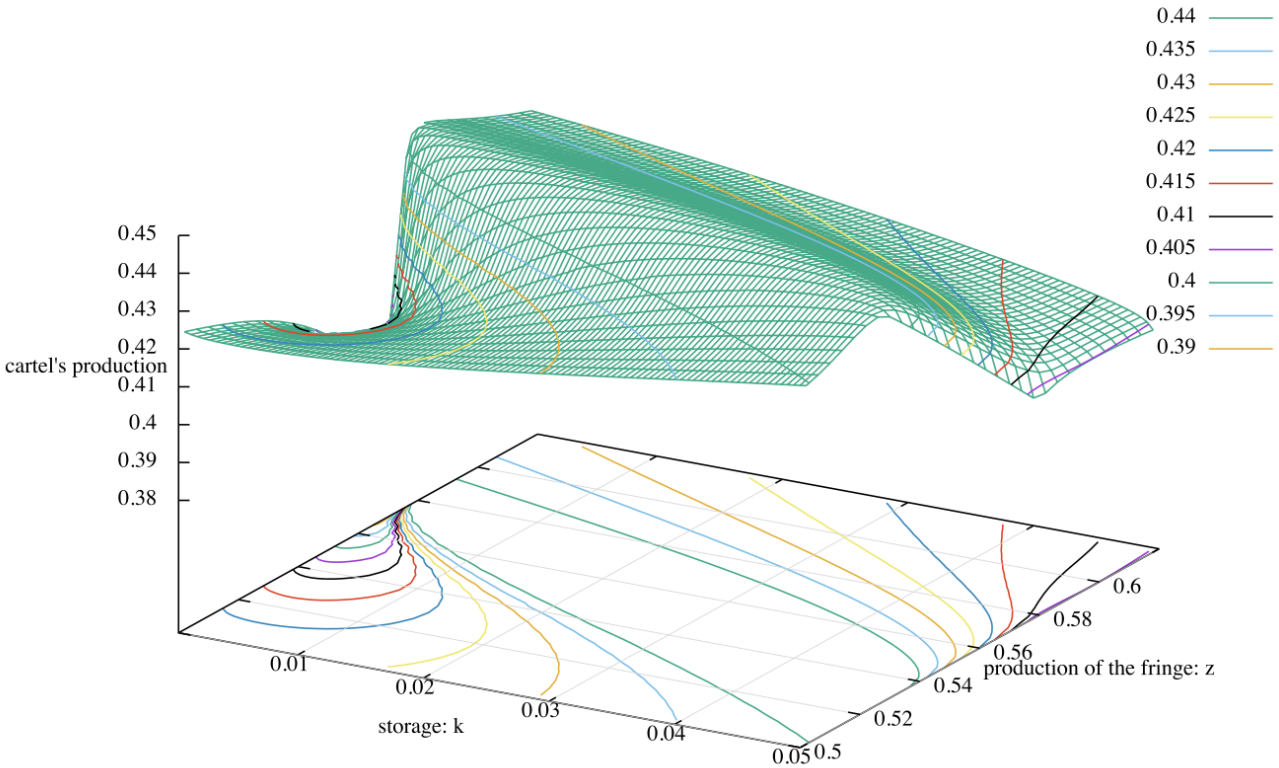
Solving the model reduces to solve a system of PDE

$$g(k) = -rp(z,k) + (q(z,k) + z - D(1 - \epsilon p(z,k)))\partial_k p(z,k) + (\sqrt{ap(z,k) - b} - cz)\partial_z p(z,k) + \frac{\sigma^2}{2}\partial_{kk} p(z,k)$$

$$0 = -rU(z,k) + H(z, p(z,k), \partial_k U(z,k)) + \frac{\sigma^2}{2}\partial_{kk} U(z,k) + (\sqrt{ap(z,k) - b} - cz)\partial_z U(z,k),$$

with $\epsilon = 4.10^{-4}$!

Numerical results



Interest rate models in decentralized finance

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Collaboration between :

- Mathematicians (O Guéant, CB)
- Economist (L Bertucci)
- Engineers (M Gontier Delaunay, M Lesbre)
- Technical and academic collaboration with Morpho Labs

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What we worked on :

- Decentralized finance aims at creating a (new, free, transparent,...) financial system based on the technology of Blockchains
- Morpho Labs is a french start-up specialized in lending protocols.
- Main question : can we design a lending protocol which is completely decentralized / onchain ?

The model

- Lenders deposit cryptocurrencies on a lending pool
- Borrowers are allowed to take the deposit against a collateral, but they have to pay an interest rate
- Lenders shares the total earned interest rate.
- How do we choose the interest rate ?
- We can use a model

$$dL_t = \lambda L_t \left(\left(\frac{B_t}{L_t} r_t - \bar{r} \right) dt + \sigma dW_t \right),$$
$$dB_t = \mu B_t \left((r_0 - r_t)\, dt + \sigma dW'_t \right).$$

under the constraint $B_t \leq L_t$.

How to attack this problem ?

- Theoretically, it is a constrained stochastic optimal control problem

$$\inf_{(\alpha_t)_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-rt} C(\alpha_t, L_t, B_t) dt\right]$$

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- However, the previous is impossible to use in practice, and we ended up with a quite simple model of the form

$$r'(t) = \psi(u(t), r(t), u'(t)),$$

  where ψ is a given (simple) function and $u(t) = B_t/L_t$.

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- The project with EDF lead to completely open mathematical questions.Those questions are the subject of different large research projects at the moment and remain at the heart of several discussions between us.
- The project with Kayrros was not continued, but the mathematical tools developed are now used to model the European carbon market, namely in CACIB. Some mathematical questions remain opened.
- The interest rate model was implemented and a huge success, the equivalent of 5B$ are managed by it ! No mathematical question remains.

Thanks to the jury, FMJH and the donators !

Thank you for your attention