Incorporating Discrete Choice Models into Revenue Management Decisions

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Evolution of Revenue Management Models

Quantity-Based (80's)

Demand for a product is captured by an exogenous random variable
Control through capacity allocation decisions

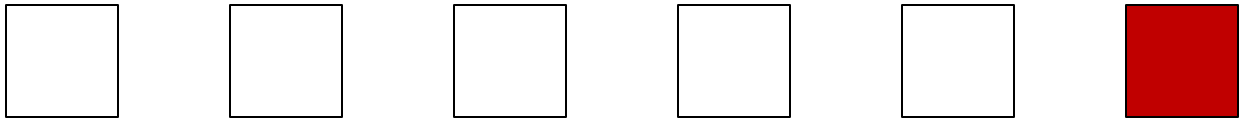
Price-Based (90's)

Price charged for a product shapes the demand for the product
Control through pricing decisions

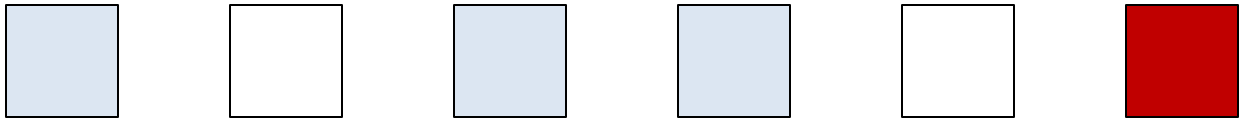
Choice-Based (00's)

Customers choose and substitute among offered products
Control through assortment decisions

Static Assortment Optimization



Static Assortment Optimization



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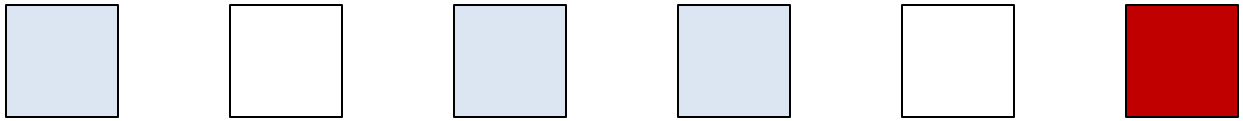
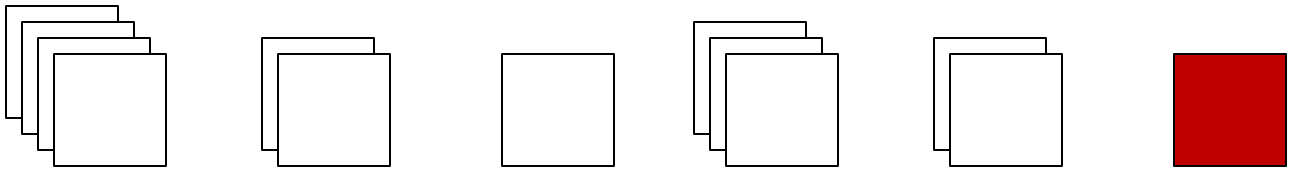


N : Set of products
r_i : Revenue of product i
$\phi_i(S)$: Choice probability of product i from assortment S

$$\max_{S \subseteq N} \left\{ \sum_{i \in N} \phi_i(S)\, r_i \right\}$$

Dynamic Assortment Optimization



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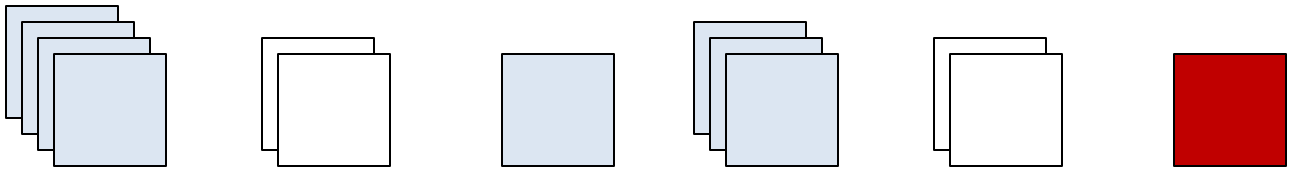

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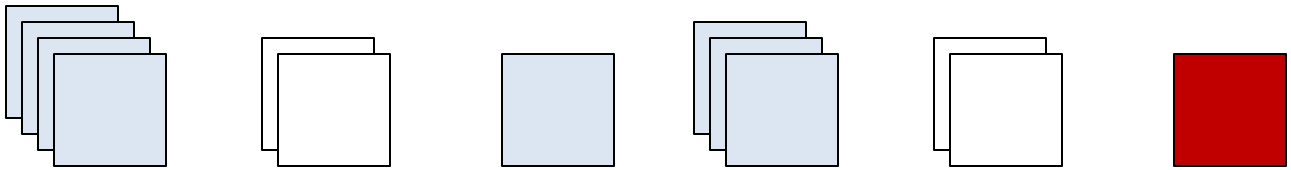

N : Set of products
r_i : Revenue of product i
$\phi_i(S)$: Choice probability of product i from assortment S
x_i : Remaining inventory of product i

$$V_t(x) = \max_{S \subseteq N(x)} \left\{ \sum_{i \in N} \phi_i(S) \left\{ r_i + V_{t+1}(x - e_i) \right\} + \left\{ 1 - \sum_{i \in N} \phi_i(S) \right\} V_{t+1}(x) \right\}$$

$$N(x) = \{ i \in N : x_i > 0 \}$$

Assortment Optimization

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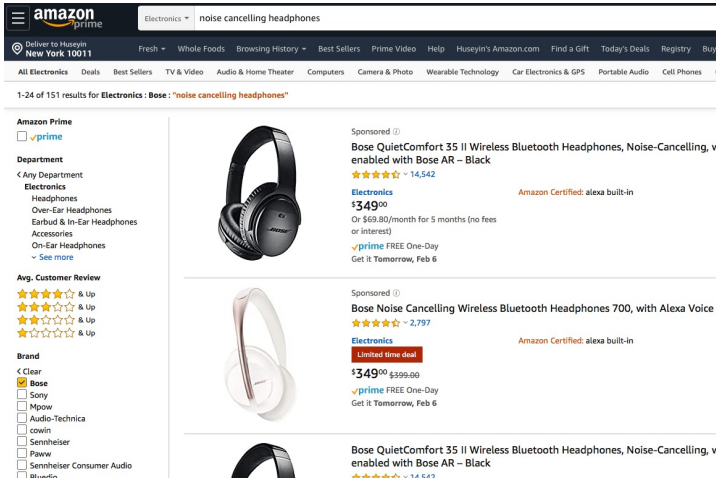

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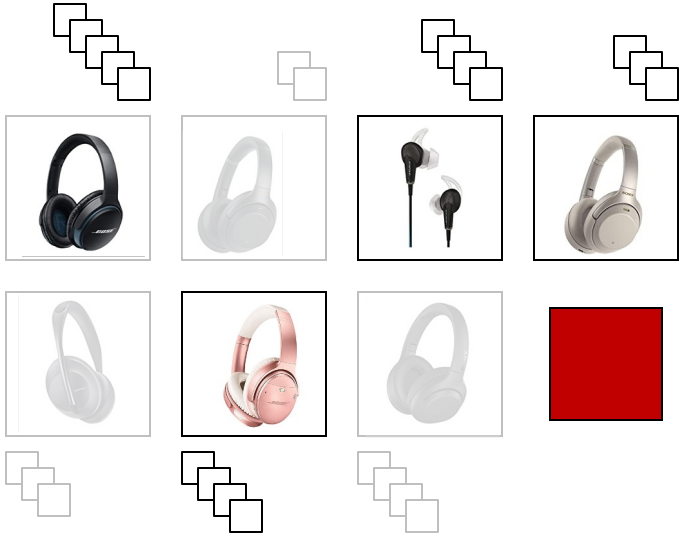

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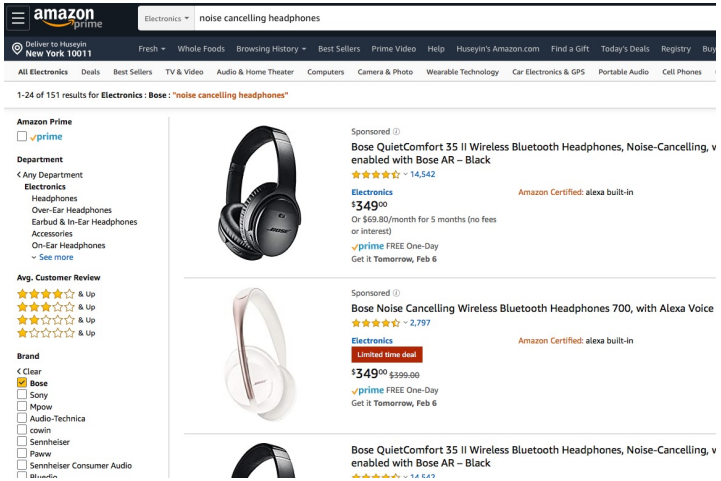


Outline

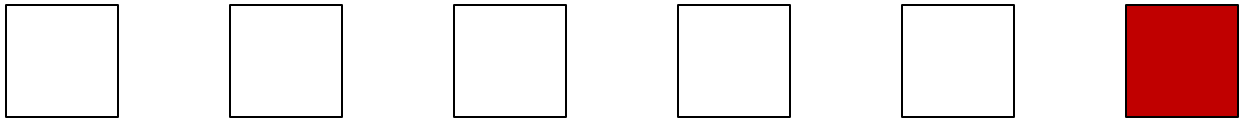
Static Assortment Optimization

Multinomial logit, Markov chain choice models

Dynamic Assortment Optimization

Linear programming-based policies

Multinomial Logit Model



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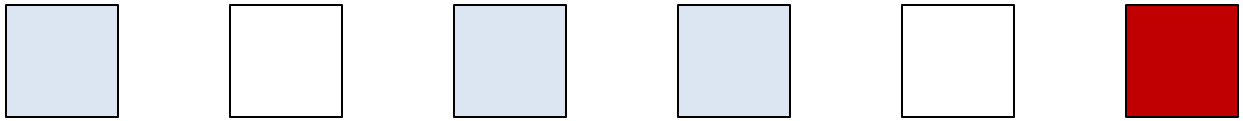

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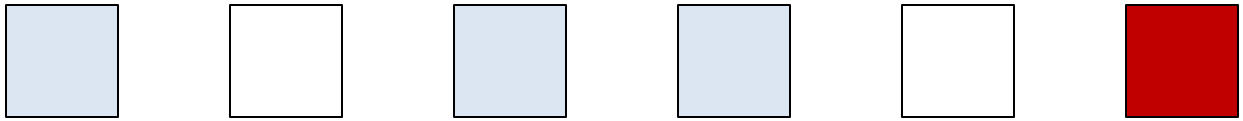
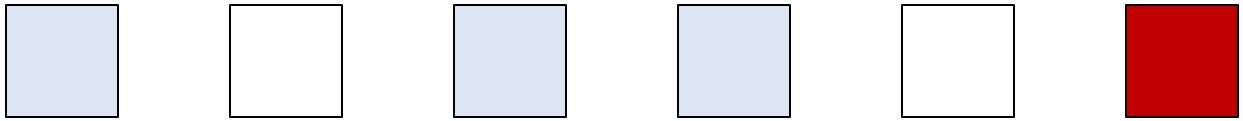


N : Set of products
r_i : Revenue of product i
v_i : Preference weight of product i

If we offer assortment S, then a customer chooses product i with probability

$$\phi_i(S) = \frac{v_i}{1 + \sum_{\ell \in S} v_\ell}$$

Multinomial Logit Model



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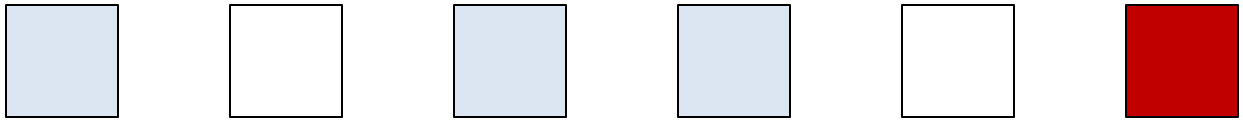

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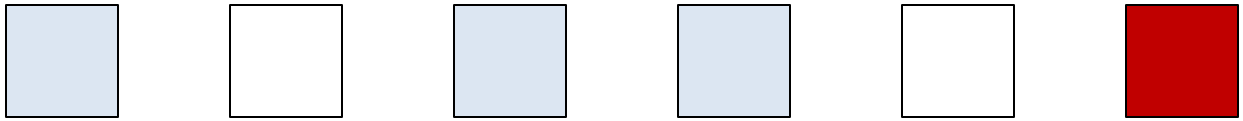

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$$\max_{S \subseteq N} \left\{ \frac{\sum_{i \in S} r_i \, v_i}{1 + \sum_{i \in S} v_i} \right\}$$

Optimal Assortment Through Dual Formulation

$$z^* = \max_{S \subseteq N} \left\{ \frac{\sum_{i \in S} r_i \, v_i}{1 + \sum_{i \in S} v_i} \right\}$$

$$\begin{aligned} \min \quad & z \\ \text{st} \quad & z \geq \frac{\sum_{i \in S} r_i \, v_i}{1 + \sum_{i \in S} v_i} \quad \forall \, S \subseteq N \end{aligned}$$

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$$\begin{aligned} \min \quad & z \\ \text{st} \quad & z \geq \sum_{i \in N} v_i \left[r_i - z \right]^+ \end{aligned}$$

Mixture of Multinomial Logit Models

M : Set of customer types
N : Set of products
r_i : Revenue of product i
θ_j : Market share of customer type j
v_{ij} : Preference weight a customer of type j attaches to product i

If we offer assortment S, then a customer of type j chooses product i with probability

$$\phi_{ij}(S) = \frac{v_{ij}}{1 + \sum_{\ell \in S} v_{\ell j}}$$

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NP-hard to approximate within a factor of $1/m$

mixture

$$\max_{S \subseteq N} \left\{ \sum_{j \in M} \theta_j \times \text{Rev}_j(S) \right\}$$

NP-hard to approximate within a factor of 1/m

robust

$$\max_{S \subseteq N} \left\{ \min_{j \in M} \text{Rev}_j(S) \right\}$$

Can be solved in polynomial time

customization

$$\max_{\substack{S \subseteq N : \\ |S| \leq C}} \left\{ \sum_{j \in M} \theta_j \times \max_{S_j \subseteq S} \text{Rev}_j(S_j) \right\}$$

Constant-factor approximation

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Incorporating constraints

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Preference list-based choice models

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Markov Chain Choice Model

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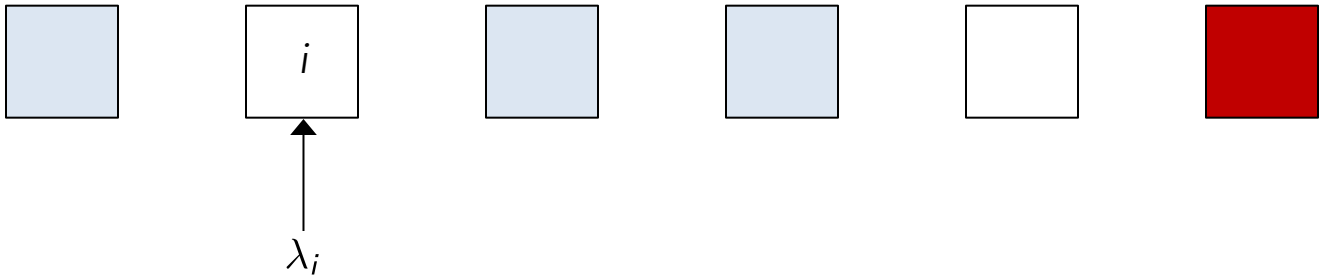

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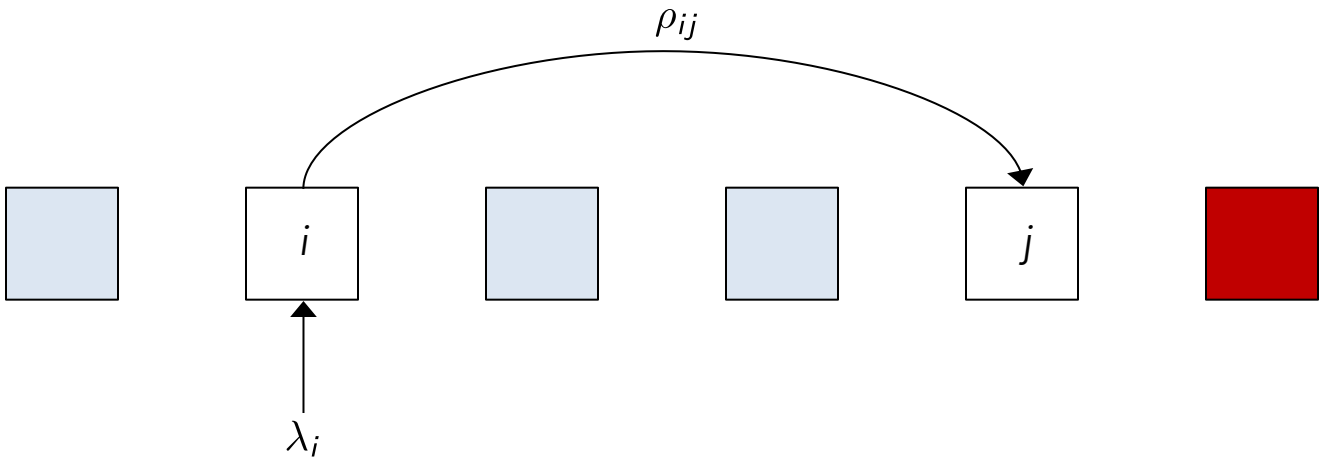

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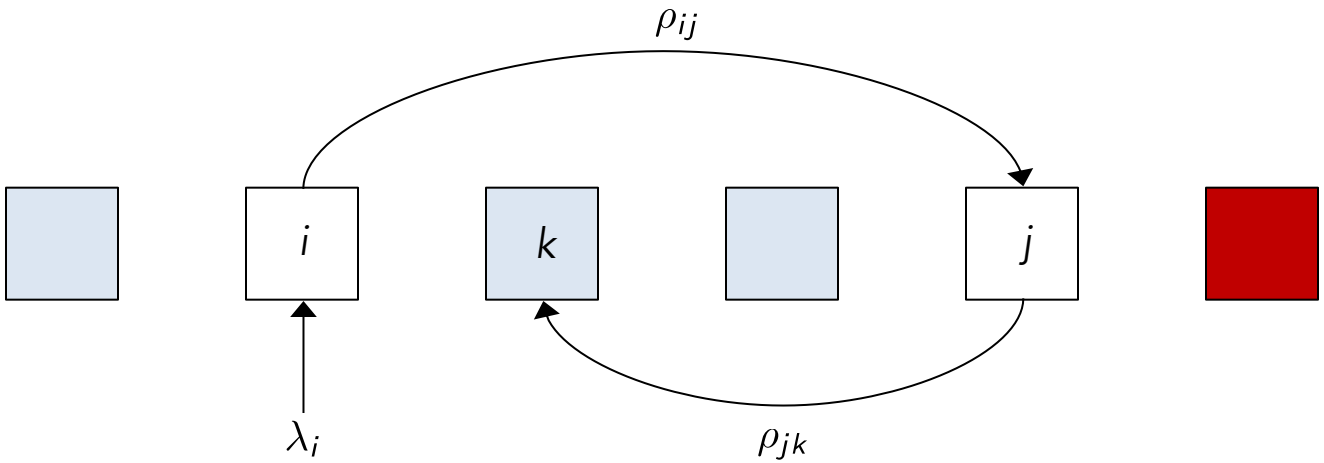

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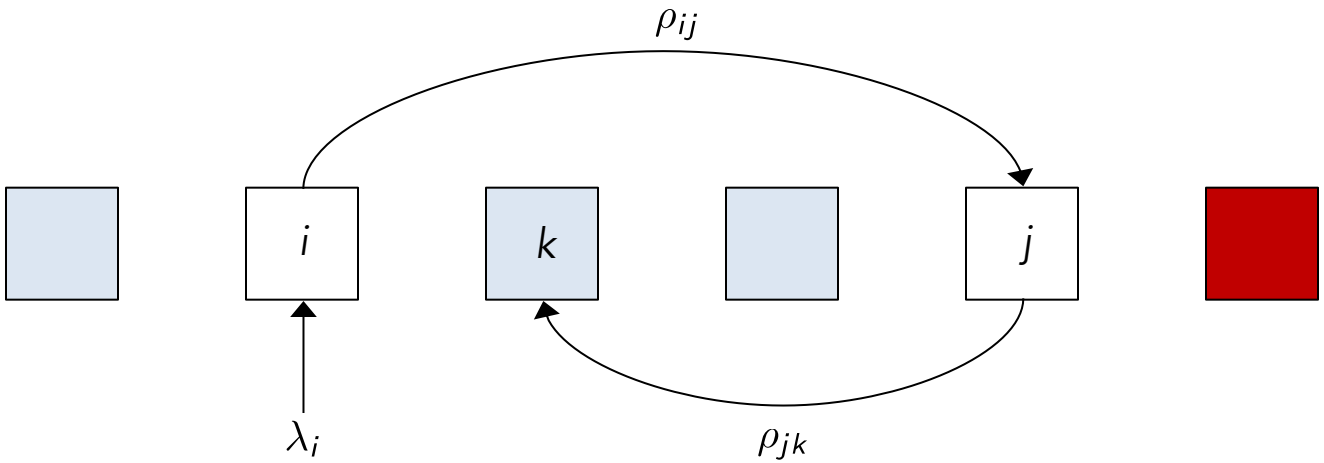
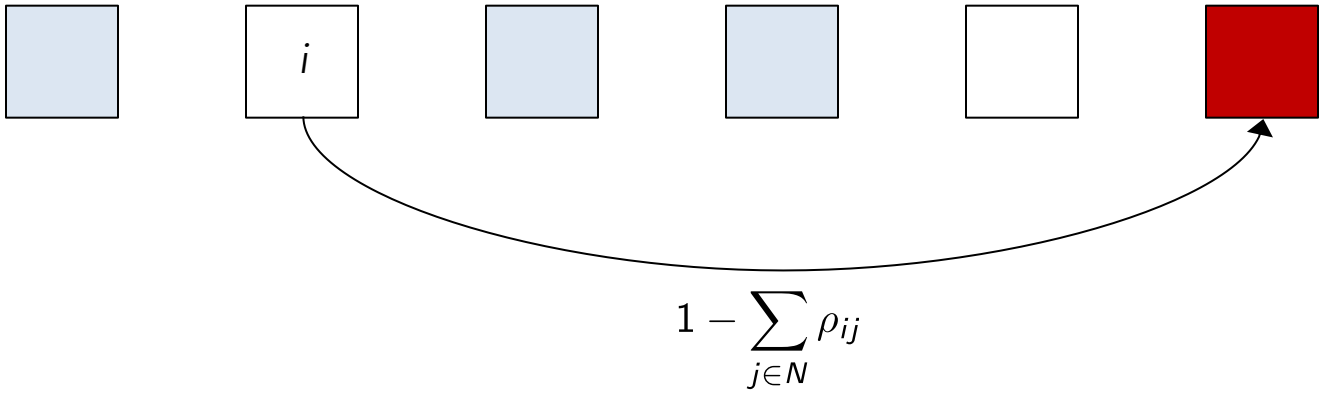


Parameters of the Markov chain choice model are

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} \qquad \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \vdots & & & \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{bmatrix}$$

Markov Chain Choice Model



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V_i : Optimal expected revenue from a customer arriving with the intention to purchase product i

$$V_i = \max \left\{ r_i \, , \; \sum_{j \in N} \rho_{ij} \, V_j \right\}$$

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optimal assortment

$$S^* = \left\{ i \in N : r_i \geq \sum_{j \in N} \rho_{ij} \, V_j \right\}$$

optimal expected revenue

$$\sum_{i \in N} \lambda_i \, V_i$$

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$$\begin{aligned} \min \quad & \sum_{i \in N} \lambda_i \, V_i \\ \text{st} \quad & V_i \; \geq \; r_i \qquad \forall \, i \in N \\ & V_i \; \geq \; \sum_{j \in N} \rho_{ij} \, V_j \qquad \forall \, i \in N \end{aligned}$$

References

Markov chain choice model

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Other choice models

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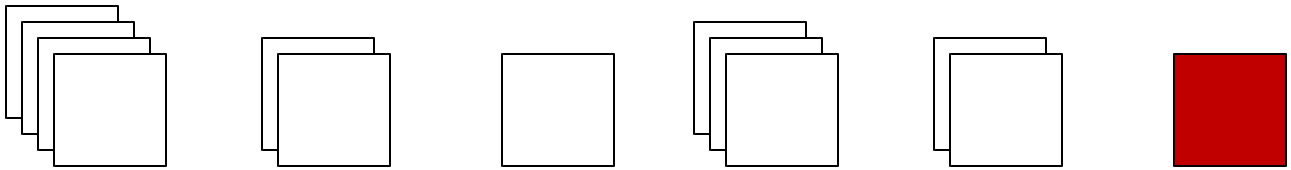

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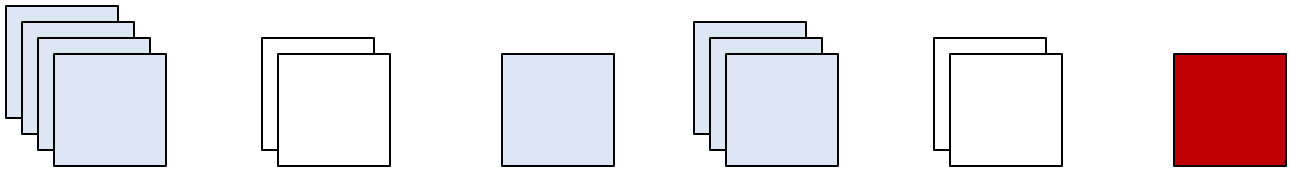

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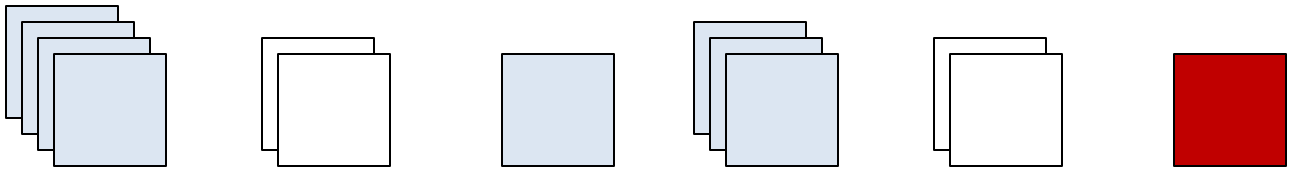
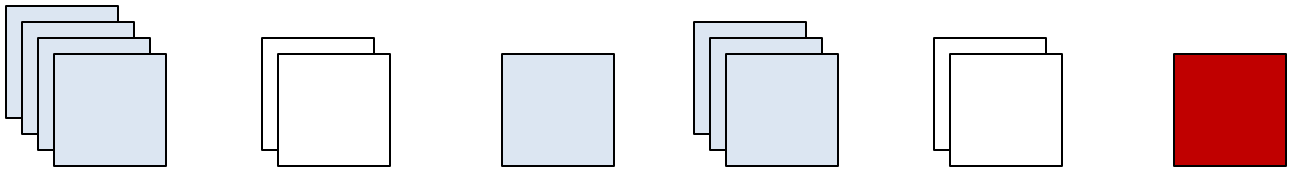


N : Set of products
T : Number of time periods in the selling horizon
r_i : Revenue of product i
c_i : Initial inventory of product i
$\phi_i(S)$: Choice probability of product i from assortment S
x_i : Remaining inventory of product i

$$V_t(x) = \max_{S \subseteq N(x)} \left\{ \sum_{i \in N} \phi_i(S) \left\{ r_i + V_{t+1}(x - e_i) \right\} + \left\{ 1 - \sum_{i \in N} \phi_i(S) \right\} V_{t+1}(x) \right\}$$

$$N(x) = \{ i \in N : x_i > 0 \}$$

Dynamic Assortment Optimization



N : Set of products
T : Number of time periods in the selling horizon
r_i : Revenue of product i
c_i : Initial inventory of product i
$\phi_i(S)$: Choice probability of product i from assortment S
x_i : Remaining inventory of product i

$$V_t(x) = \max_{S \subseteq N(x)} \left\{ \sum_{i \in N} \phi_i(S) \left\{ r_i + V_{t+1}(x - e_i) \right\} + \left\{ 1 - \sum_{i \in N} \phi_i(S) \right\} V_{t+1}(x) \right\}$$

$$\text{opt} = V_1(c)$$

$$N(x) = \{ i \in N : x_i > 0 \}$$

$z(S)$: Total expected number of times assortment S is offered

$$\begin{aligned} \max \quad & \sum_{S \subseteq N} \sum_{i \in S} r_i \, \phi_i(S) \, z(S) \\ \text{st} \quad & \sum_{S \subseteq N} \phi_i(S) \, z(S) \;\leq\; c_i \qquad \forall \, i \in N \\ & \sum_{S \subseteq N} z(S) = T \end{aligned}$$

$z(S)$: Total expected number of times assortment S is offered

$$\begin{aligned} \max \quad & \sum_{S \subseteq N} \sum_{i \in S} r_i \, \phi_i(S) \, z(S) \\ \text{st} \quad & \sum_{S \subseteq N} \phi_i(S) \, z(S) \;\leq\; c_i \qquad \forall \, i \in N \\ & \sum_{S \subseteq N} z(S) = T \end{aligned}$$

primal policy

At each time period in the selling horizon, offer assortment S with probability $z^*(S)/T$

$z(S)$: Total expected number of times assortment S is offered

$$\begin{aligned} \max \quad & \sum_{S \subseteq N} \sum_{i \in S} r_i \, \phi_i(S) \, z(S) \\ \text{st} \quad & \sum_{S \subseteq N} \phi_i(S) \, z(S) \;\leq\; c_i \qquad \forall \, i \in N \\ & \sum_{S \subseteq N} z(S) = T \end{aligned}$$

primal policy

At each time period in the selling horizon, offer assortment S with probability $z^*(S)/T$

Total expected revenue of primal policy is at least

$$\max \left\{ \frac{1}{4} \, , \; 1 - \sqrt{\frac{\log c_{\min}}{c_{\min}}} - \frac{1}{c_{\min}} \right\} \text{opt}$$

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Assortment optimization under logit variants
Markov chain choice model
Dynamic assortment optimization