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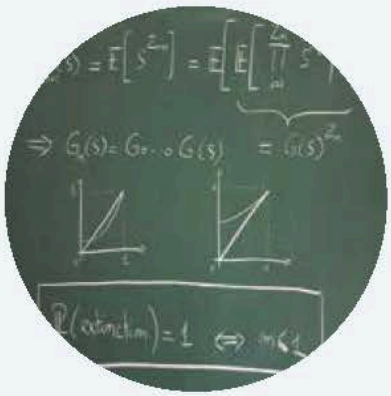

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Real-time Ride-sharing with Intermediate Locations

Kamel AISSAT · Ammar Oulamara

Abstract Ride-sharing is a travel mode that provides several benefits and solutions, such as reducing travel costs, traffic congestion and the provision of travel options.
In the classical ride-sharing approach, the driver makes a detour to the rider's origin in order to pick-up the rider, then drives him to his destination and finally goes to his own destination. This implies that the driver is incharge of the whole detour and may refuse such matchings if the detour is long. However, the matching could be accepted if the rider meets the driver at an intermediate location.
In this paper, we present a general ride-sharing approach in which a driver and a rider accept to meet each other in an intermediate starting location and to separate in another intermediate ending location. We propose exact and heuristic methods to compute meeting locations that minimize the total travel cost of the driver and the rider and, finally, we perform a numerical study using a real road network and real dataset. Our experimental analysis shows that our heuristics provide efficient performances within short CPU times and improves participants cost-savings and matching rate compared to the classical ride-sharing.

Keywords Ride-sharing · Shortest path problem · Heuristic · Lower bound · Geographical maps

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

A ride-sharing service brings together users with similar itineraries and time schedules. More precisely, a driver and a rider share a part of their common itinerary using the driver's vehicle as well as a part of the vehicle-related expenses. The ride-sharing service is growing with the development of ICT (Information and Communications Technology), the extension of the smartphone usage and the embedded geolocation device (Global Positioning System - GPS). A new and innovative solution in ride-sharing service has emerged. It consists in automatic and instant matching of riders through a network service by using smartphone as geolocation and communication device. This service is called real-time ride-sharing. The ability of a real-time ride-sharing to successfully provide an instant matching depends (i) on the characteristics of the environment in terms of geographic density of users, traffic patterns, the available roadway and transit infrastructure (ii) on the efficiency of implemented algorithms to tackle

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the underlying decision problems, such as optimal instant matching of drivers.
Several research has been reported recently in the fields of ride-sharing, see [5] for overview. In the ride-sharing problem, we distinguish four variants, namely, single-driver and single-rider [6], single-driver and multiple-riders [1], multiple-drivers and single-rider [4], and finally multiple-drivers and multiple-riders [7]. In each variant, the matching between riders and drivers depends on one or more objective functions, such as the maximization of the number of matchings, the maximization of the cost saving, the maximization of distance saving, etc. Most studies in the literature focuses on the classical ride-sharing problem in which the driver makes a detour to the rider's origin in order to pick-up the rider, then drives him to his destination and finally goes to his own destination. This approach is the most implemented in ride-sharing services. However, in order to get more opportunities of ride-sharing, a rider can accept two different intermediate locations one for pick-up and another for the drop-off. More precisely, the rider travels by his own to the first intermediate meeting location, where he will be picked up by the driver, travel together till a second intermediate location, in which the rider is propped off, then he will continue to his ending location.
To the best of our knowledge, the only work considering the intermediate locations is addressed in [2]. The authors propose optimal and heuristic methods to find the pick-up and drop-off locations in a multimodal transportation network, in which the objective is to minimize the total travel time for both the driver and the rider. However the time complexity of this heursitic $O(m \cdot n^2)$, where n is the number of nodes and m is the number of edges in the graph, prevents its use in real-time ride-sharing, and their model does not take into account the detour time constraint, i.e., the total time of detour should be less than given value fixed by the driver (rider) and detour cost constraint i.e., the incurred cost of the driver (rider) is more attractive than the incurred cost when they travel alone.
Our Contribution. In our model, we minimize the total travel cost in scenarios involving transportation modes with time-independent arc costs, while ensuring that their detour costs and times remain reasonable.

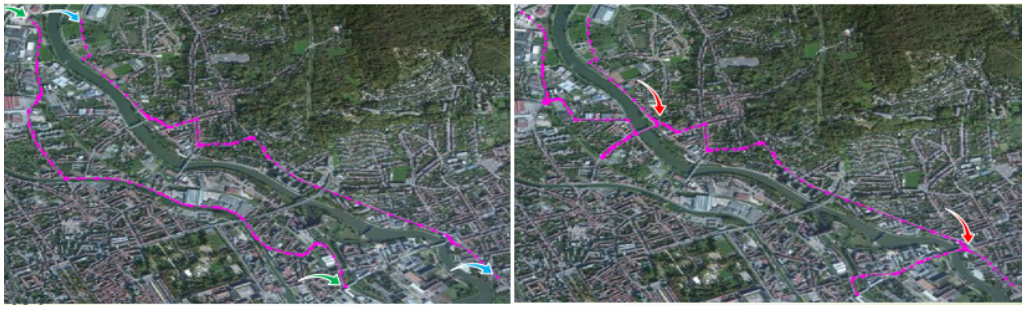

Fig. 1 The left-graph represents the shortest path of driver (green arrows) and the rider (blue arrows) before matching, while the graph on the right represents the new path of the driver and the rider after the matching with two new intermediate locations (two red arrows).

2 Description of our model

The road network is represented by a weighted graph $G = (V, E)$, where V is the set of nodes, E the set of edges. Nodes model intersections and edges depict street segments. Edge $(i, j) \in E$ associated two weights. In our model $w(i, j)$ represents the traveling cost and $\tau(i, j)$ the traveling time. A path or a route in a graph is defined as a vector $\mu = (x, \cdots, v)$ with an order list of nodes, for which holds that each pair of consecutive nodes is connected by an edge of E. The length of a path $w(P)$ is the sum its edge weights. A path with minimum cost between a source node x and a target node y is called a shortest path and denoted by $\langle x \to y \rangle$ with cost $w(\langle x \to y \rangle)$. We consider an offer and a demand of ride-sharing represented by $o = (s, t, [t_o^{\min}, t_o^{\max}], \Delta_o)$ and $d = (s', t', [t_d^{\min}, t_d^{\max}], \Delta_d)$, respectively, where s and s' (t and t') are the starting (ending) locations of the driver and the rider, respectively, $[t_o^{\min}, t_o^{\max}]$ ($[t_d^{\min}, t_d^{\max}]$) is the departure time window of the driver (rider), Δ_o (Δ_d) is the time detour for the driver (rider). An edge (i, j) has a nonnegative traveling cost $w_k(i, j)$ depending on the fact that the edge is used by driver ($k = o$) or by rider ($k = d$). In our model the rider can travel on his own to intermediate location r_1 with a cost $w_d(\langle s' \to r_1 \rangle)$, where he will be picked up by the driver and dropped off at another intermediate location r_2, the rider will continue his travel from r_2 to t' on his own. Thus, the driver and the rider will share $w_o(\langle r_1 \to r_2 \rangle)$. Therefore, the cost of rider is $w_d(\langle s' \to r_1 \rangle) + \varepsilon \cdot w_o(\langle r_1 \to r_2 \rangle) +$

$w_d(\langle r_2 \to t' \rangle)$, where ε is the cost sharing factor.
Let $\varepsilon > 0$, we say that an offer o and a demand d form a reasonable fit if and only if there exist two intermediate locations r_1 and r_2 ($r_2 \neq r_1$) such that: o and d form a time synchronization at location r_1 and

$$w_o(\langle s \to r_1 \rangle) + w_o(\langle r_1 \to r_2 \rangle) + w_o(\langle r_2 \to t \rangle) - w_o(\langle s \to t \rangle) \leq \varepsilon \cdot w_o(\langle r_1 \to r_2 \rangle) \quad (1)$$

$$w_d(\langle s' \to r_1 \rangle) + w_o(\langle r_1 \to r_2 \rangle) + w_d(\langle r_2 \to t' \rangle) - w_d(\langle s' \to t' \rangle) \leq (1 - \varepsilon) \cdot w_o(\langle r_1 \to r_2 \rangle) \quad (2)$$

$$\tau_o(\langle s \to r_1 \rangle) + \tau_o(\langle r_1 \to r_2 \rangle) + \tau_o(\langle r_2 \to t \rangle) \leq (\tau_o(\langle s \to t \rangle) + \Delta_o) \quad (3)$$

$$\tau_d(\langle s' \to r_1 \rangle) + \tau_o(\langle r_1 \to r_2 \rangle) + \tau_d(\langle r_2 \to t' \rangle) \leq (\tau_d(\langle s' \to t' \rangle) + \Delta_d) \quad (4)$$

Applying the ε on the shared path gives both the driver and the rider an incentive to meet in r_1 and to separate in r_2. The constraint (1) ensures that the driver's detour cost is less important than the reward that rider provides to driver ($\varepsilon \cdot w_o(\langle r_1 \to r_2 \rangle)$). However constaint (2) ensures that the saving cost by the rider in the shared path ($(1 - \varepsilon) \cdot w_o(\langle r_1 \to r_2 \rangle)$) is greater than the detour cost ($w_d(\langle s' \to r_1 \rangle) + w_o(\langle r_1 \to r_2 \rangle) + w_d(\langle r_2 \to t' \rangle) - w_d(\langle s' \to t' \rangle)$). The term ($\tau_o(\langle s \to t \rangle) + \Delta_o$) in (3) (resp. $\tau_d(\langle s' \to t' \rangle) + \Delta_d$ in (4)) allows to limit the amount of time that the driver (resp. rider) pass in traveling. In the following we use the term *global-path* $\langle s, s', r_1, r_2, t, t' \rangle$ to describe the concatenation of paths $\langle s \to r_1 \rangle$, $\langle s' \to r_1 \rangle$, $\langle r_1 \to r_2 \rangle$, $\langle r_2 \to t \rangle$ and $\langle r_2 \to t' \rangle$. i.e. $\langle s, s', r_1, r_2, t, t' \rangle = \langle \langle s \to r_1 \rangle \oplus \langle s' \to r_1 \rangle \oplus \langle r_1 \to r_2 \rangle \oplus \langle r_2 \to t \rangle \oplus \langle r_2 \to t' \rangle \rangle$. A shortest *global-path* between source nodes s, s' and target nodes t, t' is the *global-path* with minimal weight $w(\langle s, s', r_1, r_2, t, t' \rangle)$ of any *global-path* from s, s' to t, t', where

$$w(\langle s, s', r_1, r_2, t, t' \rangle) = w_o(\langle s \to r_1 \rangle) + w_d(\langle s' \to r_1 \rangle) + w_o(\langle r_1 \to r_2 \rangle) + w_o(\langle r_2 \to t \rangle) + w_d(\langle r_2 \to t' \rangle) \quad (5)$$

The objective is to determine the best intermediate locations r_1 and r_2 that minimize the shortest *global-path* such that constraints (1), (2), (3) and (4) are satisfied.

3 Solving approaches

To solve the problem of ride-sharing with intermediate locations, we proposed two approaches. The first approach is an enumerative method, the second approach is a heuristic one.
The enumerative method lists all pairs of possible intermediate locations (r_1, r_2) and select the pair of intermediate locations (r_1^*, r_2^*) with minimal *global-path* cost and satisfying constraints (1), (2), (3) and (4). Using Dijkstra Algorithm [3] to calculate minimal path, this enumerative method runs in $O(n(n \log n + m))$. The complexity of enumerative method prevents its using in the context of real-time ride-sharing. Thus, we develop heuristic methods with complexity in the range of $O(n \log n)$.
The idea of heuristics consist firstly in determining a set C of potential meeting locations using lower bounds on detour constraints. Secondly, using metrics based on travel cost, the set of potential meeting locations is separated into two disjoint subsets of nodes C_1 and C_2, where C_1 contains the potential of initial intermediate locations and C_2 contains the set of potential ending intermediate location. Then, for all nodes in $\mathcal{C}_2$, we compute the optimal global path having such node as intermediate ending location. Finally, once global paths have been enumerated, we select the *global-path* with minimal cost that satisfies constraints (1), (2), (3) and (4). These approaches were validated by experiments based on real data of ride-sharing provided by Covivo company. These data concern employees of Lorraine region traveling between their homes and their work places. The main advantages of this approach are increasing the opportunity of matching between riders and drivers and then a significant reduction of the total travel cost compared to the classical ride-sharing approach.

References

1. R. Baldacci, V. Maniezzo, and A. Mingozzi. An exact method for the car pooling problem based on lagrangean column generation. *Operations Research*, 52:422–439, 2004.
2. A. Bit-Monnot, C. Artigues, M.J. Huguet, and M.O. Killijian. Carpooling : the 2 synchronization points shortest paths problem. In *13th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS)*, volume 33, pages 88–99, 2013.
3. E.W. Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematik*, 1:269–271, 1959.
4. F. Drews and D. Luxen. *Multi-Hop Ride Sharing*, volume 52. 2013.
5. M. Furuhata, M. Dessouky, F. Ordonez, M.E. Brunet, X. Wang, and S. Koenig. ride-sharing: The state-of-the-art and future directions. *Transportation Research Part B: Methodological*, 57:28–46, 2012.
6. R. Geisberger, D. Luxen, S. Neubauer, P. Sanders, and L. Volker. Fast detour computation for ride sharing. In *10th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS10)*, 2010.
7. W. Herbawi and M. Weber. Modeling the multihop ridematching problem with time windows and solving it using genetic algorithms. In *Proceedings of the 2011 IEEE 24th International Conference on Tools with Artificial Intelligence*, 2012.

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Complexity of policy iteration for stochastic zero-sum games

Marianne Akian · Stéphane Gaubert

Abstract Recent results of Ye and Hansen, Miltersen and Zwick show that policy iteration for one or two player (perfect information) zero-sum stochastic games, restricted to instances with a fixed discount rate, is strongly polynomial. We show that policy iteration for mean-payoff zero-sum stochastic games is also strongly polynomial when restricted to instances with bounded first mean return time to a given state. The proof is based on methods of nonlinear Perron-Frobenius theory, allowing us to reduce the mean-payoff problem to a discounted problem with state dependent discount rate. Our analysis also shows that policy iteration remains strongly polynomial for discounted problems in which the discount rate can be state dependent (and even negative) at certain states, provided that the spectral radii of the nonnegative matrices associated to all strategies are bounded from above by a fixed constant strictly less than 1.

Keywords Stochastic games, policy iteration, mean return time, Doeblin state, cone spectral radius

Mathematics Subject Classification (2000) MSC 91A20 · MSC 47H09 · MSC 91A15 · MSC 91A43 · MSC 90C40

1 Motivation and earlier works

Policy iteration algorithm is a classical algorithm to solve discounted Markov decision problems (one player games) with finite state and actions spaces. A policy is a map from the set of states to the set of actions, representing a Markovian decision rule. The algorithm constructs a sequence of policies such that the associated sequence of values is strictly decreasing (assuming that the player minimizes her cost function). Hence, its number of iteration is bounded by the number of policies. The method carries over to discounted zero-sum games with perfect information, still with finite state and action spaces. It now makes external iterations in the space of policies of the first player, and at each step, solves an auxiliary Markov decision problem, making then internal iterations in the space of policies of the second player. Again, the first player never selects twice the same policy, which entails that the algorithm does terminate

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in a time which is bounded by the product of the numbers of policies of both players. This yields an exponential bound on the execution time, as the number of policies of one player can be exponential in the number of states. However, this general exponential bound does not capture the experimental efficiency of the algorithm on most applications.

Some recent results shed light on the behavior of policy iteration as a function of some particular parameters, such as the discount factor. Friedmann constructed in [5] an infinite family of 2-player discounted deterministic games with a discount factor tending to 1, showing that the number of policy iterations can indeed be exponential. Fearnley [3] and Andersson [2] extended his result to 1-player stochastic games. However, Ye showed in [10] that policy iteration solves 1-player discounted games with a *fixed* discount factor $\lambda < 1$ in *strongly polynomial* time (λ is not part of the input). Then, Hansen, Miltersen and Zwick extended this result in [6] to zero-sum 2-player discounted games with perfect information, and improved Ye's bound. They showed that the number of external iterations of the policy iteration algorithm for 2-player games with a fixed discount factor $\lambda < 1$ is bounded by:

$$(m+1)(1+\frac{\log(n^2/(1-\lambda))}{-\log(\lambda)}) = \mathcal{O}(\frac{m}{1-\lambda}\log\frac{n}{1-\lambda}), \tag{1}$$

where n is the number of states, and m is the *total number of actions of both players*, that is the number of triples (i,a,b) where i is a state, a is an action of first player, and b is an action of second player. Moreover, Feinberg and Huang [4] showed that the bound of Ye holds for a one-player game with mean-payoff and a distinguished state to which the probability to go in one step is lower bounded by $1-\gamma$. Finally, Scherrer [9] considered one-player games such that the set of transient states is independent of the policy, and in that case he found a bound independent of the discount factor, but depending on some constants that may be seen (and are equal when the discount factor tends to 1) as bounds on the expected first return time to recurrent states and on the expected exit time from transient states. This result was inspired by the stongly polynomial bound obtained by Post and Ye [8] for the simplex algorithm for deterministic 1-player games, and by the extension to stochastic one-player games that Scherrer gave also in [9].

2 Main results

We show that policy iteration still has a strongly polynomial behavior for a class of mean payoff games, as well as for a more general class of discounted games.

Let us consider a *dynamic programming* or *Shapley operator* associated to a perfect information zero-sum game with a finite state space and finite action spaces, that is a self-map f of $\mathbb{R}^n$ given by:

$$[f(v)]_i := \min_{a\in A_i}\max_{b\in B_i}\left(\sum_{j\in[n]} M_{ij}^{ab}\, v_j \;+\; r_i^{ab}\right), \qquad \forall i\in[n],\ v\in\mathbb{R}^n, \tag{2}$$

with M_{ij}^{ab} nonnegative scalars, and A_i and B_i finite sets. Then, the sets of feedback policies $A_{\mathrm{M}} := \{\sigma : [n]\to A \mid \sigma(i)\in A_i\,\forall i\in[n]\}$ and $B_{\mathrm{M}} := \{\delta : [n]\times A\to B \mid \delta(i,a)\in B_i\,\forall i\in[n],\, a\in A_i\}$ are finite, and the map f can be written in the following form:

$$f(v) = \min_{\sigma\in A_{\mathrm{M}}} f^{(\sigma)}(v)\ , \tag{3}$$

with

$$f^{(\sigma)}(v) = \max_{b\in B_i}\left(\sum_{j\in[n]} M_{ij}^{\sigma_i,b}\, v_j \;+\; r_i^{\sigma_i,b}\right), \qquad \forall i\in[n],\ v\in\mathbb{R}^n,$$

and where in the above expression, the minimum is with respect to the partial order of $\mathbb{R}^n$. Moreover, the maps f and $f^{(\sigma)}$ are order preserving for this partial order.

If the game is with infinite horizon, the value of the game is a fixed point v of f. Moreover, the policy iteration algorithm for 2-player games consists in iterating the following steps: (i) given the policy σ^s, compute the fixed point v^s of $f^{(\sigma^s)}$; (ii) then improve the policy by choosing an optimal policy for v^s, that is $\sigma^{s+1} \in A_{\mathrm{M}}$ such that $f(v^s) = f^{(\sigma^{s+1})}(v^s)$, with $\sigma^{s+1} = \sigma^s$ as soon as this is possible. Moreover, step (i) is solved by the same algorithm for $f^{(\sigma^s)}$ instead of f.

The following bound improves the bound (1), in the original situation considered in [6]. It is obtained by adapting the technique of Ye and Hansen, Miltersen and Zwick to nonlinear maps which allows us in particular to replace m by m_1. Note that the bound (4) is linear in the size of the input, for a fixed λ.

Theorem 1 *Assume that the maps $f^{(\sigma)}$ are contracting in the sup-norm with the same contraction factor λ. Then, the policy iteration algorithm stops after at most $s_{\max}$ iterations, where*

$$s_{\max} := (m_1 - n)(1 + \lfloor \frac{\log(1-\lambda)}{\log(\lambda)} \rfloor) = \mathcal{O}(\frac{m_1 - n}{1-\lambda} \log \frac{1}{1-\lambda}), \tag{4}$$

and m_1 is the cardinality of $\mathsf{SA} := \{(i,a) \mid i \in [n],\ a \in A_i\}$.

Then, we consider games with state dependent discount factors, possibly greater than 1 locally.

Theorem 2 *Assume that the spectral radii of all the matrices*

$$M^{(\sigma\delta)} := (M_{ij}^{\sigma_i \delta_i})_{ij=1,\dots,n},$$

with $\sigma \in A_{\mathrm{M}}$, $\delta \in B_{\mathrm{M}}$, are strictly less than 1, so that $\bar{\omega} := \max_{\sigma \in A_{\mathrm{M}}, \delta \in B_{\mathrm{M}}} r(M^{(\sigma\delta)}) < 1$. Then the conclusion of Theorem 1 holds with $\lambda = \bar{\omega}$.

For the proof, we introduce a natural scaling transformation, which has the property of leaving invariant the combinatorial trace of the policy iteration algorithm. This scaling is obtained using techniques of non-linear Perron-Frobenius theory [7,1]. An advantage of the present bound is that it is invariant by scaling. For instance, with a state dependent discount factor < 1, it leads to a tighter bound than the one which may be derived from (1) or (4).

Finally, we derive a strongly polynomial bound for the subclass of mean payoff games such that there is a distinguished state to which the mean return time is bounded by a constant $K = 1/(1-\lambda)$, for every choice of policies. This condition implies that each transition matrix associated to a pair of policies of both players has a unique recurrence class, and that there is a state which is common to each of these classes.

References

1. M. Akian, S. Gaubert, and R. Nussbaum. A Collatz-Wielandt characterization of the spectral radius of order-preserving homogeneous maps on cones. 2011.
2. D. Andersson. Extending Friedmann's lower bound to the Hoffman-Karp algorithm. *preprint, June*, 2009.
3. John Fearnley. Exponential lower bounds for policy iteration. In *Automata, Languages and Programming*, pages 551–562, 2010.
4. E.A. Feinberg and J. Huang. Strong polynomiality of policy iterations for average-cost mdps modeling replacement and maintenance problems. *Operations Research Letters*, 41:249–251, 2013.
5. Oliver Friedmann. An exponential lower bound for the parity game strategy improvement algorithm as we know it. In *LICS*, pages 145–156. IEEE Computer Society, 2009.
6. T.D. Hansen, P.B. Miltersen, and U. Zwick. Strategy iteration is strongly polynomial for 2-player turn-based stochastic games with a constant discount factor. In *Innovations in Computer Science 2011*, pages 253–263. Tsinghua University Press, 2011.
7. R.D. Nussbaum. Convexity and log convexity for the spectral radius. *Linear Algebra Appl.*, 73:59–122, 1986.
8. I. Post and Y. Ye. The simplex method is strongly polynomial for deterministic markov decision processes. In *SODA*, pages 1465–1473, 2013. Also arXiv:1208.5083.
9. Bruno Scherrer. Improved and generalized upper bounds on the complexity of policy iteration, 2013. arXiv:1306.0386.
10. Y. Ye. The simplex and policy-iteration methods are strongly polynomial for the Markov decision problem with a fixed discount rate. *Math. Oper. Res.*, 36(4):593–603, 2011.

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A linear time natural gradient algorithm for black-box optimization in high dimension

Youhei Akimoto · Anne Auger · Nikolaus Hansen

Abstract The covariance matrix adaptation evolution strategy (CMA-ES) [1–3] is the state-of-the-art comparison-based randomized search algorithm for black-box continuous optimization. It samples candidate solutions from a multivariate normal distribution, evaluates the candidate solutions and updates the parameters of the distribution, the mean vector and the covariance matrix. Empirically, we observe that the mean vector converges towards the optimum of the objective and the covariance matrix adapts the inverse of the Hessian of the objective.

The CMA-ES is comparison-based. That is, it is invariant to any strictly increasing transformation of the objective function, which is essential for black-box optimization since we can assume no problem property such as convexity and continuity. Moreover, it is invariant to any linear transformation of the search space. However in consequence, its internal time and space complexity per function evaluation is quadratic. When optimizing a function in high dimension, quadratic scaling of the internal time and space complexity is often too time consuming and linear scaling is desired.

In this work, we propose a novel comparison-based randomized search algorithm with linear time and space complexity, named VD-CMA. It maintains a multivariate normal distribution with restricted covariance matrix with twice the dimension as the degree of freedom, representing an arbitrarily oriented long axis and additional axis-parallel scaling. We derive the components of the algorithm from the same design principle as the CMA-ES—so called natural gradient and cumulation—and show the internal time and space complexity per function evaluation is linear. We empirically show that the algorithm adapts the long axis and the axis-parallel scaling of the inverse Hessian of the objective. Compared with existing linear time comparison-based algorithms [4, 5], VD-CMA efficiently solves wider class of functions including non-separable functions. Moreover on functions covered by the internal model of VD-CMA and on the Rosenbrock functions, we find that VD-CMA outperforms CMA-ES not only in computational time but also in the number of function calls as the dimension increases.

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Keywords Black-box optimization · Linear time algorithm · Covariance matrix adaptation · Natural gradient · Information-geometric optimization

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References

1. Nikolaus Hansen and A Auger. Principled Design of Continuous Stochastic Search: From Theory to Practice. In Y Borenstein and A Moraglio, editors, *Theory and Principled Methods for the Design of Metaheuristics*. Springer, 2013.
2. Nikolaus Hansen, Sibylle D. Muller, and Petros Koumoutsakos. Reducing the time complexity of the derandomized evolution strategy with covariance matrix adaptation (CMA-ES). *Evolutionary Computation*, 11(1):1–18, 2003.
3. Nikolaus Hansen and Andreas Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation*, 9(2):159–195, 2001.
4. Raymond Ros and Nikolaus Hansen. A simple modification in CMA-ES achieving linear time and space complexity. In *Parallel Problem Solving from Nature - PPSN X*, pages 296–305. Springer, 2008.
5. Yi Sun, Faustino Gomez, Tom Schaul, and Jürgen Schmidhuber. A linear time natural evolution strategy for non-separable functions. In *GECCO'13 Companion, July 6–10, 2013, Amsterdam, The Netherlands.*, pages 61–62, 2013.

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The Tropical Shadow-Vertex Algorithm Solves Mean Payoff Games in Polynomial Time On Average*

Xavier Allamigeon · Pascal Benchimol · Stéphane Gaubert

Abstract We introduce an algorithm which solves mean payoff games in polynomial time on average, assuming the distribution of the games satisfies a flip invariance property on the set of actions associated with every state. The algorithm is a tropical analogue of the shadow-vertex simplex algorithm, which solves mean payoff games via linear feasibility problems over the tropical semiring $(\mathbb{R} \cup \{-\infty\}, \max, +)$. The key ingredient in our approach is that the shadow-vertex pivoting rule can be transferred to tropical polyhedra, and that its computation reduces to optimal assignment problems through Plücker relations.

Keywords Simplex algorithm · Shadow-vertex pivoting rule · Mean payoff games · Tropical polyhedra

Mathematics Subject Classification (2010) 14T05 · 90C49 · 91A46

1 Context of this work

A *mean payoff game* involves two opponents, "Max" and "Min", who alternatively move a pawn over the nodes of a weighted bipartite digraph. The latter consists of two classes of nodes, represented by squares and circles, and respectively indexed by $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, n\}$. The weight of the arc (i, j) (resp. (j, i)) is a real number denoted by A_{ij} (resp. B_{ij}). We set $A_{ij} := -\infty$ (resp. $B_{ij} := -\infty$) when there is no such arc. An example of game is given in Figure 1. When the pawn is placed over a square node i, Player Max selects an outgoing arc (i, j), and then moves the pawn to circle node j and receives the payment A_{ij} from Player Min. Conversely, when the pawn is located on a circle node j, Player Min chooses an arc (j, i'), moves the pawn to square node i', and Player Max pays her the amount $B_{i'j}$. The game starts from a circle node $j_0 = j$, and then the two players make infinitely many moves, visiting a sequence $j_0, i_1, j_1, i_2, \dots$ of nodes. The objective of Player Max is to maximize his mean payoff, defined as the liminf of the following ratio when $p \to +\infty$:

$$(-B_{i_1 j_0} + A_{i_1 j_1} - B_{i_2 j_1} + A_{i_2 j_2} + \cdots - B_{i_p j_{p-1}} + A_{i_p j_p})/p \; . \tag{1}$$

Symmetrically, Player Min aims at minimizing her mean loss, defined as the limsup of (1) when $p \to +\infty$.

Mean payoff games were first studied by Ehrenfeucht and Mycielski in [7], where they proved that these games have a value and positional optimal strategies. In more detail, for every initial state $j \in [n]$, there

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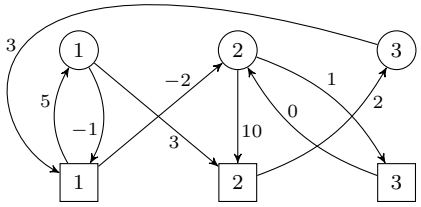


Fig. 1 An example of mean payoff game. Circle node 1 is a winning initial state for Player Max, while circle node 2 is losing.

exists a real χ_j and positional strategies σ and τ, such that: (i) Player Max is certain to win a mean payoff greater than or equal to χ_j with the strategy σ, (ii) Player Min is sure that her mean loss is less than or equal to χ_j with the strategy τ. The decision problem associated with mean payoff games consists in determining whether the initial state j is *winning* for Player Max, *i.e.* $\chi_j \geq 0$. The question of the existence of a polynomial time algorithm solving this problem was first raised by Gurvich, Karzanov and Khachiyan in [10]. This problem was shown to be in NP $\cap$ co-NP by Zwick and Paterson in [14]. While mean payoff games received an important attention over the past years [10,14,11,9,13,5,12,8,6], all the algorithms developed so far are superpolynomial, and the question raised by Gurvich *et al.* is still open.

The present work exploits the equivalence between mean payoff games and linear feasibility problems in tropical algebra. Indeed, it was shown in [2] that the initial state n is winning for Player Max in the game with payments matrices A, B if, and only if, there exists a solution $x \in (\mathbb{R} \cup \{-\infty\})^{n-1}$ to the following system of inequalities:

$$\forall i = 1, \dots, m, \qquad \max(A_{i1} + x_1, \dots, A_{i(n-1)} + x_{n-1}, A_{in}) \geq \max(B_{i1} + x_1, \dots, B_{i(n-1)} + x_{n-1}, B_{in}) \ . \tag{2}$$

The constraints of the form (2) correspond to affine inequalities over the tropical (max-plus) semiring $\mathbb{T} := \mathbb{R} \cup \{-\infty\}$ endowed with the operations $x \oplus y := \max(x, y)$ as addition, and $x \odot y := x + y$ as multiplication. The conjunction of finitely many such inequalities defines a *tropical polyhedron*. Solving a mean payoff game consequently amounts to determine whether a tropical polyhedron is empty, which can be thought of as the tropical analogue of the feasibility problem in linear programming. This is among the motivations leading to the development of a tropical simplex method in [3]. One of the properties of the tropical simplex method is that it traces the path followed by the classical simplex method. In particular, basic points and edges are in one-to-one correspondence between the classical and the tropical worlds. Then, complexity results known for the classical simplex algorithm can be potentially transferred to the tropical setting. However, the main obstacle is to "tropicalize" the pivoting rule involved, *i.e.* to define a tropical pivoting rule which is both compatible with the classical one, and computable, if possible, in a reasonable time complexity. So far [4], the only pivoting rules which have been tropicalized are *combinatorial*, *i.e.* they are defined in terms of the neighborhood of the current basic point in the vertex/edge graph of the polyhedron.

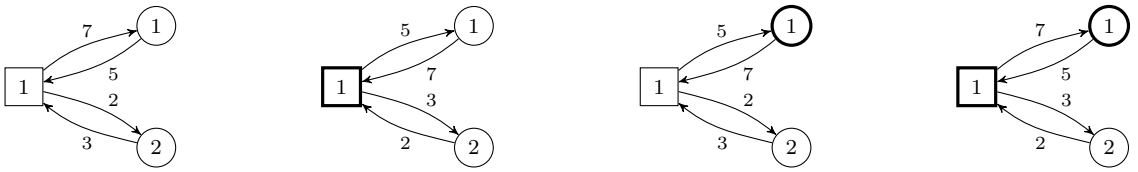
2 Contributions

The shadow-vertex simplex algorithm refers to the simplex algorithm equipped with the so-called *shadow-vertex pivoting rule*. Given two objective vectors $\boldsymbol{u}, \boldsymbol{v}$, it aims at solving the following parametric family of linear programs for increasing values of $\boldsymbol{\lambda} \geq 0$:

$$\begin{array}{rl} \text{minimize} & (\boldsymbol{u}^\intercal + \boldsymbol{\lambda}\boldsymbol{v}^\intercal)\boldsymbol{x} \\ \text{subject to} & \boldsymbol{x} \geq 0, \ \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b} \geq 0 \end{array} \tag{3}$$

When $\boldsymbol{\lambda}$ is continuously increased from 0, the basic points of $\boldsymbol{\mathcal{P}}$ minimizing the function $\boldsymbol{x} \mapsto (\boldsymbol{u}^\intercal + \boldsymbol{\lambda}\boldsymbol{v}^\intercal)\boldsymbol{x}$ form a sequence $\bar{\boldsymbol{x}}^0, \dots, \bar{\boldsymbol{x}}^p$. The shadow-vertex pivoting rule amounts to iterate over this sequence.

We prove that the shadow-vertex simplex algorithm can be tropicalized. The key difficulty in our approach is to show that the computation of the tropical shadow-vertex pivoting rule can be done in



$A = (7\ 2)$ $B = (5\ 3)$ | $A = (5\ 3)$ $B = (7\ 2)$ | $A = (5\ 2)$ $B = (7\ 3)$ | $A = (7\ 3)$ $B = (5\ 2)$

Fig. 2 A distribution of game satisfying the flip invariance property (with $m = 1$ and $n = 2$), together with the payment matrices. The four configurations are supposed to be equiprobable. The nodes on which the flip operations have been performed are depicted in bold.

polynomial time. To this end, we exploit the fact that the shadow-vertex rule is semi-algebraic, *i.e.* it is defined in terms of the signs of finitely many polynomials. Under some genericity conditions, we deduce that the tropical rule reduces to classical linear programs over some Newton polytopes, which are actually (Minkowski sums of) bipartite perfect matching polytopes:

Theorem 1 *On instances satisfying genericity conditions, the tropical shadow-vertex rule is compatible with the classical shadow-vertex rule, and the leaving variables returned by the former can be computed in time $O(n^4)$.*

Following the average-case analysis of the shadow-vertex algorithm due to Adler, Karp and Shamir [1], we establish a polynomial bound on the average-case complexity of mean payoff games. As far as we know, this is the first result of this nature on this class of games. Our bound holds when the distribution of games satisfies a *flip invariance* property, which requires that the distribution is left invariant by every transformation consisting, for an arbitrary node of the game, in flipping the orientation of all the arcs incident to this node. Equivalently, the probability distribution on the set of payment matrices A, B is invariant by every transformation consisting in swapping the ith row of A with the ith row of B, or the jth column of A with the jth column of B. Figure 2 provides the illustration of a discrete distribution of games satisfying the property. Our main result is given in the following theorem:

Theorem 2 *Assuming that the distribution of the games satisfies a flip invariance property, the tropical shadow-vertex algorithm solves mean payoff games in time $O(mn(m+n^2)\min(m^2,n^2))$ on average.*

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References

1. I. Adler, R. M. Karp, and R. Shamir. A simplex variant solving an $m \times d$ linear program in $O(\min(m^2, d^2))$ expected number of pivot steps. *Journal of Complexity*, 3(4):372 – 387, 1987.
2. M. Akian, S. Gaubert, and A. Guterman. Tropical polyhedra are equivalent to mean payoff games. *Int. J. Algebr. Comput.*, 22(1):125001, 2012.
3. X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig. Tropicalizing the simplex algorithm. Submitted. E-print arXiv:1308.0454, 2013.
4. X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig. Combinatorial simplex algorithms can solve mean payoff games. Submitted. E-print arXiv:1309.5925, 2014.
5. H. Björklund and S. Vorobyov. A combinatorial strongly subexponential strategy improvement algorithm for mean payoff games. *Discrete Appl. Math.*, 155:210–229, 2007.
6. L. Brim, J. Chaloupka, L. Doyen, R. Gentilini, and J.F. Raskin. Faster algorithms for mean-payoff games. *Formal Methods in System Design*, 38(2):97–118, 2011.
7. A. Ehrenfeucht and J. Mycielski. Positional strategies for mean payoff games. *International Journal of Game Theory*, 8(2):109–113, 1979.
8. O. Friedmann. An exponential lower bound for the parity game strategy improvement algorithm as we know it. In *LICS*, pages 145–156. IEEE, August 2009.
9. S. Gaubert and J. Gunawardena. The duality theorem for min-max functions. *C. R. Acad. Sci. Paris.*, 326, Série I:43–48, 1998.
10. V. A. Gurvich, A. V. Karzanov, and L. G. Khachivan. Cyclic games and an algorithm to find minimax cycle means in directed graphs. *USSR Computational Mathematics and Mathematical Physics*, 28(5):85–91, 1988.

11. M. Jurdziński. Deciding the winner in parity games is in UP ∩ co-UP. *Information Processing Letters*, 68(3):119–124, 1998.
12. M. Jurdziński, M. Paterson, and U. Zwick. A deterministic subexponential algorithm for solving parity games. *SIAM Journal on Computing*, 38(4):1519–1532, 2008.
13. J. Vöge and M. Jurdziński. A discrete strategy improvement algorithm for solving parity games. In E. A. Emerson and A. P. Sistla, editors, *Computer Aided Verification*, volume 1855 of *LNCS*, pages 202–215. Springer Berlin Heidelberg, 2000.
14. U. Zwick and M. Paterson. The complexity of mean payoff games on graphs. *Theoretical Computer Science*, 158(1-2):343–359, 1996.

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Long and Winding Central Paths

Xavier Allamigeon · Pascal Benchimol · Stéphane Gaubert · Michael Joswig

Abstract We disprove a continuous analog of the Hirsch conjecture proposed by Deza, Terlaky and Zinchenko, by constructing a family of linear programs with $3r+4$ inequalities in dimension $2r+2$ where the central path has a total curvature in $\Omega(2^r/r)$. Our method is to tropicalize the central path in linear programming. The tropical central path is the piecewise-linear limit of the central paths of parameterized families of classical linear programs viewed through logarithmic glasses. We show in particular that the tropical analogue of the analytic center is nothing but the tropical barycenter, *i.e.*, the maximum of a tropical polyhedron. It follows that unlike in the classical case, the tropical central path may lie on the boundary of the tropicalization of the feasible set, and may even coincide with a path of the tropical simplex method. Finally, our counter-example is obtained as a deformation of a family of tropical linear programs introduced by Bezem, Nieuwenhuis and Rodríguez-Carbonell.

Keywords Interior point methods · Tropical geometry

Mathematics Subject Classification (2010) MSC 90C51 · MSC 14T05

1 Contributions

Since Karmarkar's seminal work [14], interior-point methods have become indispensable in mathematical optimization. They provide algorithms with a polynomial complexity in the bit model for linear programming. Moreover, interior points method are also useful for more general convex optimization problems such as semi-definite programming. Path-following interior point methods are driven to an optimal solution along a trajectory called the *central path*. Consider a linear program of the form:

$$\begin{array}{ll} \text{minimize} & c^\top x \\ \text{subject to} & Ax \leq b,\ x \geq 0,\ x \in \mathbb{R}^n\,, \end{array} \qquad \mathrm{LP}(A,b,c)$$

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where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$. Under natural assumptions, the following *barrier problem*:

$$\begin{array}{ll} \text{minimize} & \dfrac{c^\top x}{\mu} - \displaystyle\sum_{j=1}^{n} \log(x_j) - \sum_{i=1}^{m} \log(w_i) \\ \text{subject to} & Ax + w = b,\ x > 0, w > 0\ x \in \mathbb{R}^n, w \in \mathbb{R}^m\ . \end{array}$$

admits a unique solution (x^μ, w^μ) for any positive real number μ. The central path is the curve $\mathcal{C} : \mu \mapsto (x^\mu, w^\mu)$. As $\mu > 0$ tends to 0, the central path tends to an optimal solution of the linear program $\mathrm{LP}(A, b, c)$. Path-following interior points methods approximately follow the central path so as to reach this optimal solution. Thus the performance of an interior point method is tightly linked to the shape of its central path. The purpose of this paper is to apply tools from tropical geometry to study the central path.

Tropical geometry can be seen as the (algebraic) geometry on the semiring $(\mathbb{T}; \oplus, \odot)$ where the set $\mathbb{T} = \mathbb{R} \cup \{-\infty\}$ is endowed with the operations $a \oplus b = \max(a, b)$ and $a \odot b = a + b$. A tropical variety can be obtained as the limit at infinity of a sequence of classical algebraic varieties depending on one real parameter t and drawn on logarithmic paper, with t as the logarithmic base. This process is known as Maslov's *dequantization* [15], or Viro's method [16]. It can be traced back to the work of Bergman [5]. In a way, dequantization yields a piece-wise linear shadow of classical algebraic geometry. Tropical geometry has a strong combinatorial flavor, and yet it retains a lot of information about the classical objects [13].

The tropical semiring can also be thought of as the image of a non-archimedean field under its valuation map. We adopt the approach of Alessandrini [1] who suggested to study tropicalizations of real semi-algebraic sets via a Hardy field, $\mathbb{K}$, of germs of real-valued functions. The functions $\boldsymbol{f} \in \mathbb{K}$ are definable in some o-minimal structure, which ensures a tame topology. In particular, the limit $\lim_{t\to\infty} \log(\boldsymbol{f}(t))/\log(t)$ always exists, and this defines a valuation map $\mathrm{val} : \mathbb{K} \to \mathbb{R} \cup \{-\infty\}$. Furthermore, Alessandrini's framework is flexible enough to include all power functions into $\mathbb{K}$; this makes the valuation map surjective onto $\mathbb{R} \cup \{-\infty\}$.

As $\mathbb{K}$ is an ordered field, the basic results of linear programming (Farkas' lemma, strong duality, *etc*) still hold true on $\mathbb{K}$. We consider linear programs on the Hardy field $\mathbb{K}$ defined by:

$$\begin{array}{ll} \text{minimize} & \boldsymbol{c}^\top \boldsymbol{x} \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b},\ \boldsymbol{x} \geq 0,\ \boldsymbol{x} \in \mathbb{K}^n\ . \end{array} \qquad \mathrm{LP}(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c})$$

for some $\boldsymbol{A} \in \mathbb{K}^{m\times n}$ and $\boldsymbol{b} \in \mathbb{K}^m$. The elements of $\mathbb{K}$ are real-valued functions. As a result, a linear program over $\mathbb{K}$ encodes the family of linear programs $\mathrm{LP}(\boldsymbol{A}(t), \boldsymbol{b}(t), \boldsymbol{c}(t))$ over $\mathbb{R}$, for $t \in \mathbb{R}$ large enough. Also, since $\mathbb{K}$ is real closed, the central path of $\mathrm{LP}(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c})$ is well-defined, and it describes the central paths of the family of real linear programs $(\mathrm{LP}(\boldsymbol{A}(t), \boldsymbol{b}(t), \boldsymbol{c}(t)))_t$. The tropical central path is then defined as the image under the valuation map. Thus, the tropical central path is a logarithmic limit of a family of classical central paths. We establish that this convergence is uniform on closed intervals.

The tropical central path has a purely geometric characterization. Applying the valuation map to the feasible region yields a tropical polyhedron. We show that the tropical analytic center is the greatest element of this tropical polyhedron, the tropical equivalent of a barycenter. Thus, the tropical analytic center does not depend on the external representation of the feasible set. Similarly, any point on the tropical central path is the tropical barycenter of the tropical polyhedron obtained by intersecting the values of the feasible region with a tropical sublevel set induced by the objective function. More precisely, we prove the following theorem.

Theorem 1 *Let $(\boldsymbol{x^\mu}, \boldsymbol{w^\mu})$ be the point on the central path of the Hardy linear program $\mathrm{LP}(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c})$ at $\boldsymbol{\mu} \in \mathbb{K}$ with $\boldsymbol{\mu} > 0$, and let $\boldsymbol{\nu}$ be that LP's optimal value. Then $\mathrm{val}(\boldsymbol{x^\mu}, \boldsymbol{w^\mu})$ is the tropical barycenter of $\mathrm{val}(\boldsymbol{\mathcal{P}^\mu})$ where*

$$\boldsymbol{\mathcal{P}^\mu} := \{(\boldsymbol{x}, \boldsymbol{w}) \in \mathbb{K}^{n+m} \mid \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w} = \boldsymbol{b},\ \boldsymbol{c}\boldsymbol{x} \leq \boldsymbol{\nu} + (n+m)\boldsymbol{\mu},\ \boldsymbol{x} \geq 0,\ \boldsymbol{w} \geq 0\}\ .$$

This is in stark contrast with the classical case, where the central path depends on the halfspace description of the feasible set. In this way, Deza, Nematollahi, Peyghami and Terlaky [10] bent the central path of the Klee-Minty cube by adding superfluous halfspaces in its representation, so that it visits a neighborhood of every vertex of the cube.

A maybe surprising feature is that the tropical central path can degenerate to a path taken by the tropical simplex method introduced in [3,2]. We can even provide a quite general sufficient condition under which the tropical central path coincides with the image of a path of the classical simplex method under the valuation map. Consequently, the tropical central path may have the same worst-case behavior as the simplex method.

A main contribution of this work comes from studying the total curvature of the real central paths arising from lifting tropical linear programs to the Hardy field $\mathbb{K}$. The curvature measures how far a path differs from a straight line. Intuitively, a central path with high curvature should be harder to approximate with line segments, and thus this suggests more iterations of the interior point methods. The total curvature has been studied by Dedieu, Malajovich and Shub [9] via the multihomogeneous Bézout Theorem and by De Loera, Sturmfels and Vinzant [8] using matroid theory. These two papers provide an upper bound of $O(n)$ on the total curvature averaged over all regions of an arrangement of hyperplanes in dimension n. The redundant Klee-Minty cube of [12] and the "snake" in [11] are instances which show that that the total curvature can be in $\Omega(m)$ for a polytope described by m inequalities. By analogy with the classical Hirsch conjecture, Deza, Terlaky and Zichencko [11] conjectured that $O(m)$ is also an upper bound for the total curvature. We disprove their conjecture by constructing a family of linear programs with $3r+4$ inequalities in dimension $2r+2$ where the central path has a total curvature in $\Omega(2^r/r)$. This family arises by lifting tropical linear programs introduced by Bezem, Nieuwenhuis and Rodríguez-Carbonell [6] to show that an algorithm of Butkovič and Zimmermann [7] has exponential running time. The tropical central path shows a fractal-like pattern, which looks like a staircase shape with $\Omega(2^r)$ steps.

More details on the present work can be found in [4].

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References

1. D. Alessandrini. Logarithmic limit sets of real semi-algebraic sets. *Adv. Geom*, 13:155–190, 2013.
2. X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig. Combinatorial simplex algorithms can solve mean payoff games. Submitted. E-print arXiv:1309.5925, 2013.
3. X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig. Tropicalizing the simplex algorithm. Submitted. E-print arXiv:1308.0454, 2013.
4. X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig. Long and winding central paths. E-print arXiv:1405.4161, 2014.
5. G.M. Bergman. The logarithmic limit-set of an algebraic variety. *Transactions of the American Mathematical Society*, 157:459–469, 1971.
6. M. Bezem, R. Nieuwenhuis, and E. Rodríguez-Carbonell. Exponential behaviour of the Butkovič-Zimmermann algorithm for solving two-sided linear systems in max-algebra. *Discrete Appl. Math.*, 156(18):3506–3509, 2008.
7. P. Butkovič and K. Zimmermann. A strongly polynomial algorithm for solving two-sided linear systems in max-algebra. *Discrete Applied Mathematics*, 154(3):437–446, March 2006.
8. J.A. De Loera, B. Sturmfels, and C. Vinzant. The central curve in linear programming. *Foundations of Computational Mathematics*, 12(4):509–540, 2012.
9. J.-P. Dedieu, G. Malajovich, and M. Shub. On the curvature of the central path of linear programming theory. *Foundations of Computational Mathematics*, 5(2):145–171, 2005.
10. A. Deza, E. Nematollahi, R. Peyghami, and T. Terlaky. The central path visits all the vertices of the klee–minty cube. *Optimisation Methods and Software*, 21(5):851–865, 2006.
11. A. Deza, T. Terlaky, and Y. Zinchenko. Polytopes and arrangements: diameter and curvature. *Operations Research Letters*, 36(2):215–222, 2008.
12. A. Deza, T. Terlaky, and Y. Zinchenko. Central path curvature and iteration-complexity for redundant Klee-Minty cubes. In *Advances in applied mathematics and global optimization*, volume 17 of *Adv. Mech. Math.*, pages 223–256. Springer, New York, 2009.

13. I. Itenberg, G. Mikhalkin, and E.I. Shustin. *Tropical algebraic geometry*, volume 35. Springer, 2009.
14. N. Karmarkar. A new polynomial-time algorithm for linear programming. *Combinatorica*, 4(4):373–395, 1984.
15. G.L. Litvinov. Maslov dequantization, idempotent and tropical mathematics: a brief introduction. *Journal of Mathematical Sciences*, 140(3):426–444, 2007.
16. O. Viro. Dequantization of real algebraic geometry on logarithmic paper. In *European Congress of Mathematics*, pages 135–146. Springer, 2001.

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Recent Advances in Continuous Randomized Black-Box Optimization: an Overview

Anne Auger

Abstract Numerical optimization problems are often naturally framed as black-box problems, where the objective function is seen as a black-box that returns f-values only (but no gradient). To tackle difficult black-box numerical problems, stochastic or randomized algorithms are generally the methods of choice, in particular when difficulties arise from non-convexity, multi-modality, noise, etc. In addition, state-of-the-art stochastic algorithms are comparison-based, that is updates are based on f-comparison of solutions and not on the intrinsic f values.

Since the first comparison-based stochastic algorithms, the pure random search, introduced more than fifty years ago, much progress has been done. The objective of this presentation introducing the invited session on *Recent Advances on Continuous Randomized black-box optimization* is to give an introduction and overview of the field of continuous comparison-based randomized algorithms. Important features of the methods like step-size adaptation and covariance matrix adaptation are reviewed. The link between step-size adaptation and linear convergence is explained. Comparisons with classical *derivative-free-algorithms* are given. In addition, a recent connexion between comparison-based stochastic adaptive algorithms and optimization by means of gradients on statistical manifolds is outlined.

Keywords Numerical optimization · continuous black-box optimization · comparison-based algorithms

1 Comparison-based stochastic black-box optimization

We consider continuous or numerical optimization where the aim is to minimize $f : \Omega \subset \mathbb{R}^n \mapsto \mathbb{R}$ with Ω being the search domain. For many numerical optimization problems, the function is available to the algorithm only as a black-box, that is the algorithm can query some f-values at different search points $\mathbf{x}$ but no other information about the problem is available. A typical example is a problem where the objective function is accessible only via an executable code that can only return function values of a queried points. In this case, the optimizer will not have access to the source code. In other settings, the optimizer has access to the source code or to an analytical expression of the function, but it is not helpful to extract meaningful informations to be used within the algorithm.

In such contexts, continuous black-box optimization algorithms come into play. We focus in this invited session on *stochastic or randomized continuous black-box algorithms* where the update of the state variables includes random components. Stochastic algorithms are generally more robust when dealing with

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difficulties like noise on the objective function (i.e. two independent calls of the black-box from the same point $\mathbf{x}$ give two different outcomes) or multi-modality (the function can have several local optima).

An additional component of the algorithms of interest here is that while the black-box provides them f-values, they use only the outcome of *comparisons* of candidate solutions for the updates of their state variables. The methods are hence *comparison-based* or *function-value-free*. This comparison-based property provides on the one hand *robustness* as errors on f-values (due to noise for instance) have an impact only if it changes the f-ranking of candidate solutions, and very large or a very small f-values have only a limited impact and on the other hand *generalization* as optimizing *any* $g \circ f$ where $g : \mathbb{R} \to \mathbb{R}$ is strictly increasing is the same in terms of sequence of search points followed by the algorithm as optimizing f.

2 Comparison-based stochastic optimizers

Stochastic continuous algorithms can be seen as algorithms where a probability distribution—the one used to sample candidate solutions—is iteratively updated. Often this probability distribution is Gaussian, parametrized by a mean vector $\mathbf{m}$ and covariance matrix. This latter being often parametrized as a scaling parameter σ^2 and a matrix $\mathbf{C}$ such that the resulting covariance matrix equals $\sigma^2\mathbf{C}$ and σ is called the step-size. Given a family of probability distributions p_θ parametrized by θ (in the case of Gaussian distribution, the parameters θ equal $(\mathbf{m}, \sigma, \mathbf{C})$), a general template for describing stochastic continuous algorithms is the following:
While unhappy, loop over the following steps:

1. **Sample** $\lambda \in \mathbb{N}$ candidate solutions according to p_θ: $\mathbf{x}_1, \ldots, \mathbf{x}_\lambda$
2. **Evaluate** on the black-box f the candidate solutions, i.e. compute $f(\mathbf{x}_i)_{1 \leq i \leq \lambda}$.
3. **Update** p_θ by updating the parameter θ: $\theta \leftarrow F(\theta, \mathbf{x}_1, \ldots, \mathbf{x}_\lambda, f(\mathbf{x}_1), \ldots, f(\mathbf{x}_\lambda))$

The update function F is algorithm dependent. For comparison-based algorithms, the update only depends on the ranking of candidate solutions, more precisely the following template will hold:
While unhappy, loop over the following steps:

1. **Sample** $\lambda \in \mathbb{N}$ candidate solutions according to p_θ: $\mathbf{x}_1, \ldots, \mathbf{x}_\lambda$
2. **Evaluate** on the black-box f the candidate solutions, i.e. compute $f(\mathbf{x}_i)_{1 \leq i \leq \lambda}$. **Rank** solutions according to f, this defines a permutation $\mathfrak{S}$ such that $f(\mathbf{x}_{\mathfrak{S}(1)}) \leq \ldots \leq f(\mathbf{x}_{\mathfrak{S}(\lambda)})$
3. **Update** p_θ by updating the parameter θ: $\theta \leftarrow F_{\mathrm{cb}}(\theta, \mathbf{x}_1, \ldots, \mathbf{x}_\lambda, \mathfrak{S})$

State-of-the-art comparison-based randomized algorithms use Gaussian distributions and update the mean vector $\mathbf{m}$, the step-size σ and the covariance matrix $\mathbf{C}$ using different mechanisms. Nowadays, the covariance-matrix-adaptation evolution strategy (CMA-ES) [6] is recognized as the leading algorithms and various variants are among the top algorithms on benchmarking challenges (see BBOB workshops `coco.gforge.inria.fr/` and CEC challenges on Real-Parameter Single Objective Optimization `http://www.ntu.edu.sg/home/EPNSugan/index_files/CEC2013/CEC2013.htm`).

The mean vector represents the favorite solutions or best estimate at a given iteration of the optimum. The step-size scales the sampling distribution, its adaptation is connected to linear convergence (see below). The covariance matrix determines the geometrical shape of the distribution (as for a Gaussian vectors with covariance matrix A and mean vector $\mathbf{m}$, lines of equal densities are given by $\{\mathbf{x} \in \mathbb{R}^n | (\mathbf{x} - \mathbf{m})^T A^{-1} (\mathbf{x} - \mathbf{m}) = \mathit{constant}\}$) and should learn second order information (see below).

2.1 Step-size adaptation and linear convergence

The step-size by scaling the overall sampling distribution controls the speed of convergence. Ideally linear convergence should be achieved. It is observed on a relatively wide class of functions and proven for some

step-size adaptive comparison-based stochastic algorithms on certain convex-quadratic functions [7] and more recently on composite of positively homogeneous functions by strictly increasing functions, that include all $\mathbf{x} \mapsto g(\|\mathbf{x}\|)$ where $\|\ \|$ is an arbitrary norm and $g : \mathbb{R} \mapsto \mathbb{R}$ is strictly increasing [2]. More generally linear convergence of step-size adaptive algorithms can be proven by studying the stability of Markov chains underlying the algorithms [4].

2.2 Covariance matrix adaptation and learning second order information

The covariance matrix adaptation determines the shape of the sampling distribution or defines a metric in the search space. It is the key mechanism to solve *ill-conditioned*, *non-separable* problems. For a convex-quadratic functions, the covariance matrix should become proportional to the inverse Hessian of the problem, i.e. second order information should be learned. This is what is in effect observed for CMA-ES or xNES [5]. Interestingly the performance of second order comparison-based algorithms on convex-quadratic functions is close from the performance of derivative-free-optimization algorithms using convex-quadratic models, i.e. BFGS or NEWUOA [9] as shown in [3].

2.3 Information Geometric Optimization [8]

Recently, a nice connection between certain stochastic comparison-based algorithms and optimization on manifolds was achieved. Considering a family of probability distributions defining a statistical manifold, the original problem of optimizing f can be associated to a joint optimization problem defined on the statistical manifold. For instance, consider $J(\theta) = \int_{\mathbb{R}^n} f(\mathbf{x}) p_\theta(\mathbf{x}) d\mathbf{x}$, then J is minimal if θ corresponds to a distribution concentrated on the global minimum of f . A natural algorithm to minimize J consists in a gradient descent algorithm on the statistical manifold. The right metric to be considered for that being the Fisher information metric. The gradient of J is however expressed as an integral, so to obtain a tractable algorithm, its Monte-Carlo approximation is considered. While the so-obtained algorithms would not be comparison-based, an alternative joint optimization problem (instead of J) can be defined to enforce the comparison-based property. For different instances of family of probability distribution, the resulting Information Geometric Optimization algorithms recover well-known algorithms and in particular the CMA-ES with rank-μ update [8, 1].

References

1. Y. Akimoto, Y. Nagata, I. Ono, and S. Kobayashi. Bidirectional relation between CMA evolution strategies and natural evolution strategies. volume 6238 of *Lecture Notes in Computer Science*, pages 154–163. Springer Verlag, 2010.
2. A. Auger and N. Hansen. Linear convergence on positively homogeneous functions of a comparison based step-size adaptive randomized search: the (1+1) ES with generalized one-fifth success rule, 2013. ArXiv eprint.
3. A. Auger, N. Hansen, J. M. Perez Zerpa, R. Ros, and M. Schoenauer. Empirical comparisons of several derivative free optimization algorithms. In *Acte du 9ime colloque national en calcul des structures*, volume 1, pages 481–486, 2009.
4. Anne Auger and Nikolaus Hansen. On Proving Linear Convergence of Comparison-based Step-size Adaptive Randomized Search on Scaling-Invariant Functions via Stability of Markov Chains, 2013. ArXiv eprint.
5. T. Glasmachers, T. Schaul, Y. Sun, D. Wierstra, and J. Schmidhuber. Exponential natural evolution strategies. In *Genetic and Evolutionary Computation Conference (GECCO 2010)*, pages 393–400. ACM Press, 2010.
6. N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation*, 9(2):159–195, 2001.
7. Jens Jägersküpper. Rigorous runtime analysis of the (1+1)-ES: 1/5-rule and ellipsoidal fitness landscapes. In LNCS, editor, *Foundations of Genetic Algorithms: 8th International Workshop, FoGA 2005*, volume 3469, pages 260–281, 2005.
8. Y. Ollivier, L. Arnold, A. Auger, and N. Hansen. Information-geometric optimization algorithms: A unifying picture via invariance principles. *ArXiv e-prints*, June 2013.
9. Michael J.D. Powell. Developments of newuoa for unconstrained minimization without derivatives. Technical Report DAMTP 2007/NA05, CMS, University of Cambridge, Cambridge CB3 0WA, UK, June 2007.

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About variational reformulations of electricity market models

D. Aussel · M. Cervinka · R. Gupta · A. Mehra · M. Marechal

Keywords Electricity market model · Variational inequality

Mathematics Subject Classification (2000) MSC 58F15 · MSC 58F17 · MSC 53C35.

A multi-leaders-one-follower game formulation has been used by many authors in recent litterature to model deregulated electricity markets.

Our aim in this talk is twofold. First we focus on models in which transmission losses and production bounds are considered, a situation for which we bring to the fore that models have to be defined with caution. Then we provide, and justify, an MPCC reformulation of the proposed model for which, thanks to coderivative calculus, we discuss necessary conditions and associated qualification conditions to solve the MPCC problem.

In a second part of the talk, we investigate the properties of a time-dependent model for an electricity market. In this case, strategies of the players/producers are element of $L^2([0,T])$ expressing thus that each producer propose a bid curve not only for a fixed spot time but for a period of time. A variational reformulation of this model is proposed and existence of a Generalized Nash equilibrum is proved assuming that the profit functions of each agent are possibly non differentiable. Additionally the classical hypothesis of concavity of those profit functions, which is very often not satisfied in nonlinear electricity market model, is weakened to semistrict quasiconcavity.

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References

1. Aussel D., Correa R., Marechal M., Spot electricity market with transmission losses, J. Industrial Manag. Optim 9 (2013), 275-290.
2. D. Aussel, M. Cervinka & M. Marechal, *Day-ahead electricity market with production bounds*, preprint (2014), 11 pp.
3. D. Aussel, R. Gupta & A. Mehra, *Evolutionary Variational Inequality Formulation of Generalized Nash Equilibrium Problem*, preprint (2014), 14 pp.

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Toward an Efficient Exploration of Fitness Landscapes

Matthieu Basseur · Adrien Goëffon

Abstract Within local search algorithms, descent methods are rarely studied experimentally. However, these search techniques are the basis of many modern metaheuristics and have an influence on the ability of an algorithm to achieve good solutions of a fitness landscape. Through a large empirical study of classic runs, we show that certain ideas about descents methods are false. These results indicate that it is possible to make a descent 'intelligent' and lead to better solutions, regardless of the problem addressed.

Keywords fitness landscape · local search

Mathematics Subject Classification (2000) MSC 68T20 · MSC 90C27

1 Introduction

Solving combinatorial optimization problems using local search techniques consists in transforming an initial configuration by application of local moves chosen from a neighborhood structure. During the last decades, a large panel of neighborhood searches have been designed, like hill-climbing, tabu search, simulated annealing or iterated local search. These metaheuristics mainly differ in the move selection heuristic employed. Hill-climbings are simple local search techniques and are widely used as basic elements of more sophisticated metaheuristics.

While most studies in the metaheuristics field focus on proposing advanced diversification techniques, we choose to investigate the determinant factors allowing to guide the search directly towards highest local optima. In preliminary works [1], we compared the ability of classical hill-climbing techniques to avoid low local optima. Obtained results lead us to believe that designing alternative selection mecanisms could allow to improve signicantly the search efficiencies. This is the main purpose of the PGMO project entitled "Towards design of efficient local searches based on fitness landscape analysis". We choose to focus here on a particular investigation about this project, whose positive results reinforce the interest of such a project.

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2 Worst Improvement based Hill-Climbings

Two commonly-used climbing move strategies, first and best improvement, constitute the widely used pivoting rules. These rules define how to select a better neighbor from a not locally optimal configuration [4,2]. More precisely, the *best improvement* strategy consists in selecting, at each iteration, a neighbor which achieves the best fitness. This implies to generate and evaluate the whole neighborhood at each step of the search. The *first improvement* strategy accepts the first evaluated neighbor which satisfies the moving condition and avoids the systematic generation of the entire neighborhood. The *worst improvement* strategy, can easily be considered but is never envisaged for the design of local search algorithms. We focus here on determining the actual efficiency of this underestimated move strategy.

In [1], we showed that first improvement tends to outperform best improvement on many landscapes derived from several combinatorial problems. An analysis of landscapes was also provided, indicating that first improvement is mostly efficient to explore most landscapes, exepting smooth ones. Intuitively, since best improvement systematically choose the highest neighbor, it should conduce the search towards the nearest peak (local optima) of the landscape. On the contrary, first improvement often performs reduced improvements, which tends to drive progressively the search toward higher areas, where the potential local optima are also higher. Then, it seems interesting to determine if choosing the worst improving neighbor at each step of the search increases the possibility to reach higher areas of landscapes by avoiding the climbing of small steepest peaks.

3 Experiments on NK landscapes

The NK family of landscapes has been proposed in [3] in order to generate artificial combinatorial landscapes with tunable shape properties: size and ruggedness. NK landscapes use a basic search space, with binary strings as configurations and bit-flip as neighborhood. Characteristics of an NK landscape are determined by two parameters N and K. N refers to the length of binary string configurations. $K \in \{0, \ldots, N-1\}$ specifies the level of variables interdependency, which directly affects the ruggedness of the landscape. By increasing the value of K from 0 to $N-1$, NK landscapes can be tuned from smooth to rugged.

Experimental process aim at comparing 3 hill-climbing versions (best, first and worst improvement), as well as several approximated worst improvements, which consist in selecting the worst neighbor among the k first improving ones. These 7 climber variants will be respectively denoted as B, F, W, W_k ($k \in \{2, 4, 8, 16\}$). In our experiments, we have considered many NK landscapes parameterizations. Results for $N = 1024$ and $K \in \{4, 6, 8, 10, 12\}$are given in table 1 (in a previous study, we reported that first improvement performs better than best improvement on rugged NK landscape instances, where $K \geqslant 4$).

100 random configurations were generated for each NK landscape, which will be used as starting points for hill-climbings. For each NK landscape, 100 executions of the 7 hill-climbing versions were performed. Searches are stopped when a local optimum is reached. Since we focus on the quality of the local optima reached by hill-climbings variants, we report the average fitness of the 1,000 local optima resulting from the corresponding searches. Comparison between best, first and worst improvement show that worst improvement is clearly more efficient that first improvement each time first improvement outperforms best improvement.

However, the worst improvement strategy is time-consuming for two reasons: the number of steps needed to reach a local optimum is increase, and the whole neighborhood has to be generated for ensuring the selection of the worst neighbor. This leads us to consider alterative hill-climbing variants which approximate worst improvement. Comparative results of the 7 hill-climbings shows that the quality of the local optima obtained is negatively correlated with the average quality of the selected improving solutions. Globally, $W \succ W_{16} \succ W_8 \succ W_4 \succ W_2 \succ F \succ B$. Worst improvement approximation

N_K	B	F	W_2	W_4	W_8	W_{16}	W
1024_4	.7232	.7238	.7253	.7270	.7286	.7291	**.7298**
	302k	12k	25k	54k	112k	223k	2336k
1024_6	.7223	.7250	.7280	.7310	.7330	.7343	**.7353**
	251k	13k	29k	65k	145k	311k	4151k
1024_8	.7176	.7215	.7251	.7285	.7306	.7316	**.7330**
	214k	13k	31k	74k	172k	382k	5837k
1024_10	.7121	.7165	.7204	.7235	.7255	.7265	**.7270**
	187k	14k	33k	80k	189k	439k	7325k
1024_12	.7064	.7107	.7150	.7178	.7197	.7206	**.7210**
	166k	14k	34k	84k	206k	487k	8544k

Table 1 Comparison of best, first, worst based pivoting rules (average fitness of local optima reached from 1,000 hill-climbings distributed on 10 random instances).

(W_k) is particularly interesting since it clearly requires less solution evaluations than complete worst improvement. Setting k consists in determining the best compromise between hill-climbing efficiency and computational costs. This is emphasized in figure 1, which report the average fitness evolution w.r.t. the number of solutions evaluated on a 1024_6 instance, focusing on the first 400000 evaluations, which is enough to terminate all local searches, expect the (complete) worst improvement strategy. To summarize this work, experiment showed that choosing the worst improving neighbor often leads to attain better local optima. Morevoer by slightly modifying the worst improvement strategy, one can design efficient hill-climbings which outperform first and best improvement in terms of tradeoff between quality and computational effort.

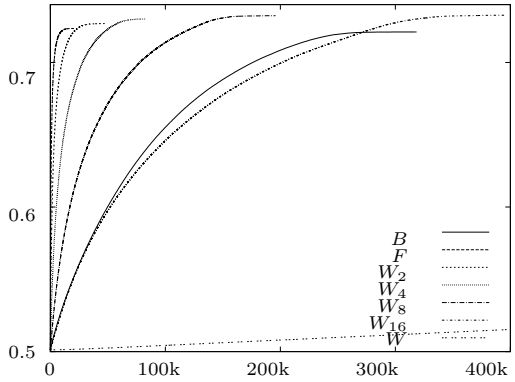

Fig. 1 Average fitness evolution on a 1024_6 NK instance, w.r.t. the number of evaluations (focus on the first 400,000 evaluations).

References

1. Matthieu Basseur and Adrien Goëffon. Hill-climbing strategies on various landscapes: an empirical comparison. In *Genetic and Evolutionary Computation Conference*, pages 479–486. ACM, 2013.
2. Holger Hoos and Thomas Stützle. Stochastic Local Search: Foundations & Applications. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 2004.
3. Stuart A. Kauffman. *The Origins of Order: Self-Organization and Selection in Evolution.* Oxford University Press, USA, 1 edition, 06 1993.
4. Mihalis Yannakakis. The analysis of local search problems and their heuristics. In *Proceedings of the seventh annual symposium on Theoretical aspects of computer science*, STACS 90, pages 298–311, New York, NY, USA, 1990. Springer-Verlag New York, Inc.

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Optimization of strain selection in a chemostat with n species

Térence Bayen · Francis Mairet · Pierre Martinon

Abstract In this work, we study a minimal time control problem governed by a chemostat model with n species ($n \geq 2$) and one limiting substrate. We consider a target where the concentration of the species of interest is significantly larger than the other ones. We show that for any value of $n \geq 2$, the substrate concentration s is constant along any singular arc and depends on the initial condition when $n > 2$. The characterization of optimal controls is refined by numerical simulations via a direct method. This allows to obtain an optimal control as a concatenation of a bang arc and a singular arc.

Keywords Optimization · Bioprocesses · Minimal time · Pontryagin Maximum Principle

Mathematics Subject Classification (2000) 49J15, 49K15, 49N25

1 Introduction

In this work, we study an extension of [3] to the case where n species are considered in the reactor. The main objective is to optimize the selection of a species of interest for bioprocess applications (see e.g. [6]). We consider a chemostat model with one limited resource and n species (with adimensioned yield coefficients) in the attractive invariant manifold (see e.g. [8]):

$$\dot{x}_i = [\mu_i(s) - u]x_i, \;\; 1 \leq i \leq n, \tag{1}$$

with $s := s_{in} - \sum_{1 \leq j \leq n} x_j$. Here, x_i, $i = 1, \ldots, n$ is the concentration of the species i in the reactor, s is the concentration of substrate, s_{in} is the input substrate concentration, and u is the dilution rate. The specific growth rates of the species μ_i, $i = 1, \ldots, n$ are nonnegative $\mathcal{C}^1$ increasing functions with $\mu_i(0) = 0$ (typically of Monod type [8]). When the dilution rate u is kept constant, the dynamical system (1) satisfies the *competitive exclusion principle* [8]: generically, only one species $i \in \{1, \ldots, n\}$ survives (depending on the choice of u), and the concentrations of the other ones go asymptotically to zero. As

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a consequence, this principle provides a way to select one species of interest by choosing an adequate value of u and waiting for the asymptotic convergence of the state.

In this work, we consider an alternative way to this principle (in the same spirit as in [2]) by optimizing the selection of the species using the dilution rate u which can be taken as a control variable within the set $\mathcal{U} := \{u : [0, +\infty] \to [0, u_{max}] \ ; \ u \text{ meas.}\}$. Here u_{max} denotes the maximum dilution rate (that can be normalized by 1 in the following). Hence, we consider a target $\mathcal{T}$ defined by:

$$\mathcal{T} := \Big\{(x_1, \ldots, x_n) \in \mathbb{R}_+^* \times \ldots \times \mathbb{R}_+^* \ ; \ x_1 \geq (1-\varepsilon) \sum_{1 \leq j \leq n} x_j \Big\}, \tag{2}$$

considering the selection of the first species. Here $\varepsilon \ll 1$ is a small parameter. The optimal control problem (OCP) can be formulated as follows:

$$\mathbf{v}(x_0) := \min_{u \in \mathcal{U}} \ t(u) \ \text{ s.t. } x(t(u), x_0) \in \mathcal{T}, \tag{OCP}$$

where $x(\cdot, x^0)$ is the unique solution of (1) starting at $x^0 \in \mathbb{R}_+^* \times \ldots \times \mathbb{R}_+^*$, $t(u)$ is the time to steer (1) to the target $\mathcal{T}$ and $\mathbf{v}$ is the value function of the problem. Notice that the reachability of the target $\mathcal{T}$ is a consequence of the competitive exclusion principle. By standard arguments, we deduce the existence of an optimal control of (OCP). Our objective is to find an optimal feedback control of the problem using the Pontryagin Maximum Principle (PMP) [5] and numerical simulations. When $n = 2$ (see [3]), the problem can be studied using standard arguments of planar affine systems with one input [5]. Finding an optimal synthesis in the more general case $n \geq 3$ is often intricate in view of the non-linearity of the system.

2 Characterization of the singular arcs

In this part, we give our main result on the characterization of the singular arcs of the problem.

Theorem 1 *Let $n \geq 2$ be given and γ a singular extremal trajectory on a time interval $[t_1, t_2]$. Then, there exists $s^* \in (0, s_{in})$ such that for all $t \in [t_1, t_2]$, we have $s(t) = s^*$.*

Proof We only give an overview of the proof without going into details. From the PMP, the trajectory γ is such that $\phi(t) = 0$ for any $t \in [t_1, t_2]$ where $\phi := -\sum_{1 \leq j \leq n} \lambda_j x_j$ is the switching function and $\lambda(\cdot)$ denotes the adjoint state. Hence, for $t \in [t_1, t_2]$, the vector $\lambda(t)$ must be orthogonal to the vector $u_0(t) := (x_1(t), ..., x_n(t))$. Suppose now that s is non-constant over $[t_1, t_2]$. By differentiating ϕ k-times ($1 \leq k \leq n-1$) w.r.t. to t, we find that $\lambda(t)$ must be orthogonal to the vectors $u_k(t) := (x_1(t)\mu_1^{(k)}(t), ..., x_n(t)\mu^{(k)}(t))$, $1 \leq k \leq n-1$ for any $t \in [t_1, t_2]$. We can show that $\text{span}(u_0(t), ..., u_{n-1}(t)) = \mathbb{R}^n$ for a.e. $t \in [t_1, t_2]$, which implies that $\lambda = 0$ and gives a contradiction. Hence, s must be constant on $[t_1, t_2]$.

Remark 1 The transversality condition on the adjoint state $\lambda(t(u)) \in N_{\mathcal{T}}(x(t(u)))$ implies that ϕ is vanishing at the terminal time $t(u)$ (here $N_{\mathcal{T}}(z)$ denotes the normal cone to $\mathcal{T}$ at point z, see [9]). Hence, either the trajectory has a switching point at the terminal time, or it contains a singular arc $s(t) = s^*$ in a neighborhood of $t(u)$. In the latter, we can show that the following condition should be fulfilled :

$$\sum_{1 \leq j \leq n} x_j(t(u))(\mu_j'(s^*) - \mu_1'(s^*)) = 0. \tag{3}$$

The previous equality is useful in order to determine the value of s^* along a singular arc.

3 Numerical simulations for $n = 3$

Numerical simulations have been performed using the software `bocop` [4]. The direct method uses a time discretization to transform the infinite-dimensional optimal control problem into a nonlinear optimization problem, solved here by interior point techniques. Software `bocop` typically uses a discretization by a Lobatto IIIC formula (6th order) with 100 time steps per day, a constant initialization, and a tolerance for NLP solver set at 10^{-10}. Numerical simulations indicate that optimal trajectories are of the form $B_+ - S$ or $B_- - S$ where B_- (resp. B_+) denotes a bang arc with $u = 0$ (resp. $u = 1$), and S a singular arc (eventually reduced to zero). Fig. 1 depicts several optimal trajectories computed for different initial points x_θ and shows that the value of s^* depends on x_θ. From (3), the value of s^* must be such that $s^* \in (1; 2.02)$, which is verified numerically.

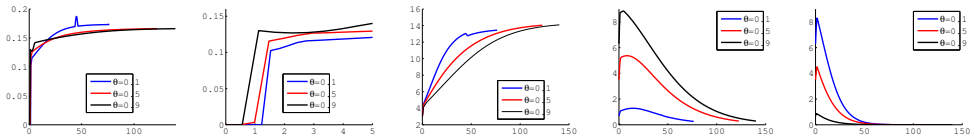

Fig. 1 First and second picture: control u and a zoom of u. Next pictures: plot of the concentrations x_1, x_2, and x_3. The different colors correspond to different initial conditions $x_\theta = (4.5, 10\theta, 10(1-\theta))$, $\theta = 0.1, 0.5, 0.9$.

4 Conclusions

Numerical simulations together with a characterization of singular arcs seem to indicate that optimal trajectories are of the form $B_\pm - S$ (that is a most rapid approach toward a singular arc in the same spirit of [1]). Such a feedback control furnishes to the practitioner an adequate operating mode for selecting a species of interest. The question of the optimality of this feedback law and the determination of the substrate concentration (as a function of the initial condition) along the singular arc remains difficult and will deserve further studies. In particular, we intend to exploit the cooperative property of the adjoint system. Developing `bocop` in order to automate the optimization w.r.t. the initial condition will be also considered. We also intend to combine the direct approach method with a relaxation method of the problem based on semi-definite programming [7]. This approach would allow to obtain a certificate of validity of the feedback control.

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References

1. T. Bayen, P. Gajardo, F. Mairet, *Optimal synthesis for the minimal time control problems of fed-batch processes for growth functions with two maxima*, Journal of Optim. Theory and Applications, vol. 158, 2, pp 521-553, 2013.
2. T. Bayen, A. Rapaport, M. Sebbah, *Minimal time control of the two tanks gradostat model under a cascade inputs constraint*, submitted 2013, `http://hal.archives-ouvertes.fr/hal-00798651`.
3. T. Bayen, F. Mairet, *Optimization of the separation of two species in a chemostat*, Automatica (2014), http://dx.doi.org/10.1016/j.automatica.2014.02.024.
4. J.-F. Bonnans, V. Grelard, P. Martinon, Bocop, the optimal control solver, Open source toolbox for optimal control problems, `http://bocop.org` 2011.
5. U. Boscain, B. Piccoli, *Optimal Syntheses for Control Systems on 2-D Manifolds*, Springer SMAI, vol. 43, 2004.
6. W. Harder, J.G. Kuenen, A. Matin, *A review. Microbial selection in continuous culture*, J. Appl. Bacteriol, Vol. 43, 1, pp. 2827–2856, 1977.
7. J.-B. Lasserre *Moments, Positive Poynomials and Their Applications*, Imperial college Press, 2010.
8. H.L. Smith and P. Waltman, *The theory of the chemostat*, Dynamics of microbial competition, Cambridge University Press, 1995.
9. R. Vinter, *Optimal Control*, Systems and Control: Foundations and Applications, Birkhauser, 2000.

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Linking Prefixes and Suffixes for Constraints Encoded Using Automata with Accumulators*

Nicolas Beldiceanu · Mats Carlsson · Pierre Flener · María Andreína Francisco Rodríguez · Justin Pearson

Abstract Keywords Learning models on a sequence · Automaton · Necessary condition

1 Introduction

In the context of learning models from production data [2] we get a conjunction of constraints that has the following special structure: each constraint restricts a specific characteristic of a same sequence that remains unchanged when the sequence is reversed. Since these characteristics do not vary independently, it is crucial to come up in a systematic way with strong necessary conditions. Taking advantage of the fact that our constraints can be encoded via an automaton augmented with accumulators [3] we come up with a necessary condition linking the result on the full sequence with the results on any partition of the sequence. This necessary condition can be summarized by a so called glue matrix, a matrix indexed by the states of the automaton providing formula for each pair of states. We show how to use this glue matrix, both in the context of constraint propagation, and in the context of local search.

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References

1. N. Beldiceanu, M. Carlsson, P. Flener, M. A. F. Rodríguez, and J. Pearson. Linking Prefixes and Suffixes for Constraints Encoded Using Automata with Accumulators. In B. O'Sullivan, editor, *Principles and Practice of Constraint Programming (CP'2014)*, volume 8656 of *LNCS*, pages 142–157, Lyon, France, 2014. Springer-Verlag.

* A full paper corresponding to this abstract can be found at [1].

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2. Nicolas Beldiceanu, Georgiana Ifrim, Arnaud Lenoir, and Helmut Simonis. Describing and generating solutions for the EDF unit commitment problem with the ModelSeeker. In Christian Schulte, editor, *CP 2013*, volume 8124 of *LNCS*, pages 733–748. Springer, 2013.
3. Mehryar Mohri. Weighted automata algorithms. In Manfred Droste, Werner Kuich, and Heiko Vogler, editors, *Handbook of Weighted Automata*, Monographs in Theoretical Computer Science, pages 213–254. Springer, 2009.

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A mean Field Game approach to technological transition

Imen Ben Tahar · René Aïd

Abstract We develop a model to assess the diffusion of a new-technology among a population of potential adopters. Our main objective is to analyze the effect of the strategic interaction of the firms which supply this new technology in a context where production costs decline with cumulated production (*learning by doing effect*), and where a firm's learning or experience benefits its rivals (*learning spillover effect*). To produce a tractable model in a dynamic setting, we adopt a mean-field-game approach.

Keywords diffusion of a new technology · spill over effect · strategic interaction · mean field game approach

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

In response to critical environmental challenges, it is widely admitted that a deep and large-scale transformation of current technological systems is necessary. Complex issues are entangled in the transition from a reference technology to more environmentally safe yet no-mature one. Such a transition usually calls for important public policies involvement, for instance via subsidies or tax-measures, to encourage the demand for the new technology and bring it to some critical level insuring a sufficient moment for its development and diffusion. In order to design an efficient policy it is important to understand, not only the demand side, but also the supply side dynamics.

In this paper we develop a quantitative model for the diffusion of a new technology with the objective to asses the effect of the strategic interaction of its supplying firms. Our model is in lines with the model of Stoneman and Ireland [2]:

– We consider a Davies [1] probit type model for the demand side: a potential user actually adopts the new technology if his *willingness to pay*, w, exceeds the current price p_t. The criterion w is assumed

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to be distributed according to a cumulative distribution function $F(\cdot)$, which allows to to deduce the inverse demand function

$$p_t = G\left(\frac{1}{n}d_t\right) \quad \text{where} \quad G:\ x \mapsto F^{-1}(1-x)\ .$$

Here n represents the size of the potential adopters.

– We assume that N firms produce this new technology. We denote by x_t^i the cumulative production volume of firm i up to time t and $x_t^{j\neq i} := (x_t^1, \cdots, x_t^{i-1}, x_t^{i+1}, \cdots, x_t^N)$. Each firm i maximizes its inter temporal profit

$$\max_{x^i} \int_0^T e^{-rt}\left(p_t \dot{x}_t^i - C^i(t, \dot{x}_t^i, x_t^i, x_t^{j\neq i})\right) dt \tag{1}$$

Here $C^i(t, \dot{x}_t^i, x_t^i, x_t^{j\neq i})$ represents the cost of producing at time t the volume $q_t^i = \dot{x}_t^i$, given the cumulative production level x_t^i and $x^{j\neq i}$.

As in [**?**], we consider that production costs decline with cumulated production, this is the *learning by doing effect*. Our main contribution is that we explicitly take into account the *learning spillover effect*, that is the fact that a firm's learning or experience benefits its rivals. To produce a tractable model, we consider a cost function of the form:

$$C^i(t, q, x^i, x^{j\neq i}) \ = \ \Psi_1(x_t^i, \bar{\mathbf{x}}_t)q + \Psi_2(x_t^i, \bar{\mathbf{x}}_t)q^2 \ \text{ where } \bar{\mathbf{x}}_\mathbf{t} = \frac{1}{N}\sum_j x_t^j \tag{2}$$

and adopt a mean-field game approach to analyze the strategic interaction of the firms.

References

1. S. Davies. *The diffusion of process innovations*. Cambridge University Press, Cambridge, 1979.
2. P. Stoneman and N.J. Ireland. The role of supply factors in the diffusion of new process technology. *The Economic Journal*, 93:66–78, 1983.

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Intra-Day Problem and Group Sparsity

Agnes Bialecki, Laurent El Ghaoui, Riadh Zorgati

Abstract In energy management, the so-called "Intra-Day Problem" (IDP) is the hourly adjustment process of the initial daily planning of production given the uncertainties which affect the demand and the offer. It can be defined as a very large, stochastic optimization problem, where the current solution needs to be quickly updated when some parameters change, under complex constraints and challenging computing time limits. This paper deals with a group sparsity approach for solving this problem.

Keywords Sparse optimization · Energy management

1 Problem setting and existing solutions

EDF, the main French Electricity board, daily plans its production for the next day by solving the Unit-Commitment Problem (UCP) and must submit the obtained generation schedule to the grid operator. The UCP consists of defining the minimal-cost power generation schedule for a given set of power plants satisfying at each time step a supply-demand balance and physical constraints. This short-term problem, solved in a deterministic setting, is a large-scale, non-convex, non-linear optimization problem, due to many thermal and hydro power-plants constraints, which lead to discontinuous operation domains and non-convex dynamic constraints.

Many uncertainties could affect the demand and the production so that the initial program becomes nonoptimal or nonfeasible. In France, the load curve and the hydraulic inflows are very climate sensitive. The availability of power plants is subject to random failure. Wind generation, electricity and gas market prices are also subject to strong uncertainties. In order to guarantee network security and adapt the planning to uncertainties, adjustments of the initial production schedule are needed. The network manager allows one adjustment of the initial program every hour during the day, provided that these modifications apply to a given limited number of power plants. The computation of these hourly adjustments is the so-called intra-day planning problem. At the present time, this problem is solved by human experts. The process is not optimal but, currently, given the size of the French network, solving this intra-day planning problem with an optimization approach, based on Boolean formulation of the intra-day constraint, requires one to several hours of CPU time.

Two approaches have been considered to propose a resolution technique that would solve this problem with CPU time compatible with the online operations and with little loss in optimality. The first one

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is based on a learning approach. Machine learning techniques have been used to learn the behavior of solutions for the large-scale, mixed combinatorial UCP problem, based on observations of input-output pairs for a set of problem instances [1]. The second approach is a two stage formulation of an extended unit-commitment problem, which includes the intra-day problem. This approach is based on primal-dual decomposition [2].

2 Proposed approach and first results

In our work, we investigate an alternate approach inspired by El Ghaoui, based on a group sparsity technique [3]. Group sparsity refers to requirements on the decision variable of an optimization problem, which enforces some groups of variables to be zero. When the decision variable is a matrix, a group sparsity requirement consists in finding a matrix with a given number of columns of the matrix to be zero. In this context, the Intra-Day constraint, which consists in modifying only a limited number of individual unit schedules, can be expressed as a group sparsity requirement. Indeed, observing that a generation schedule could be represented as a matrix where each column is the production planning of a given unit, the hourly adjustment process of the initial day planning of production regarding the uncertainties can be translated as a requirement on the matrix of the difference between the adjusted planning and the previous planning to be "column-sparse": most columns of the adjusted planning should remain the same as the corresponding one in the previous planning.

We apply this approach to a toy Intra-Day Problem. Our goal is to evaluate the suitability of the technique for the real intra-day problem. Given the very early results obtained on a toy problem, the group sparsity approach appears to be a very promising alternative to classical Boolean models of Intra-day problem. Our preliminary studies indicate that it can deliver similar results in a significant reduction of computing time.

References

[1] E. Rachelson, A. Ben-Abbes, S. Diemer. Combining Mixed Integer Programming and Supervised Learning for Fast Re-planning. *ICTAI, 22nd IEEE International Conference on Tools with Artificial Intelligence, Vol. 2, pp. 63-70, 2010.*

[2] W. van Ackooij, J. Malick. Decomposition algorithm for large-scale two-stage unit-commitment. Draft Submitted 2014.

[3] L. El Ghaoui, V. Viallon, T. Rabbani. Safe feature elimination for the lasso and sparse supervised learning problems. Pacific Journal of Optimization, 8(4):667698, January 2012, Special Issue on Conic Optimization.

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Narrowing the difficulty gap for the Celis-Dennis-Tapia problem

Immanuel M. Bomze · Michael L. Overton

Abstract We study the *Celis-Dennis-Tapia (CDT) problem*: minimize a non-convex quadratic function over the intersection of two ellipsoids. In contrast to the well-studied trust region problem where the feasible set is just one ellipsoid, the CDT problem is not yet fully understood. Our main objective here is to narrow the difficulty gap that occurs when the Hessian of the Lagrangian is not positive-semidefinite at all Karush-Kuhn-Tucker points. We prove new sufficient and necessary conditions both for local and global optimality, based on copositivity, giving a complete characterization in the degenerate case.

Keywords copositive matrices · global optimality conditions · non-convex optimization · polynomial optimization · trust region problem

1 Introduction

We study the *Celis-Dennis-Tapia (CDT) problem* [2]: minimize a non-convex quadratic function over the intersection of two ellipsoids. This problem is a natural extension of the well-studied *trust region problem* [3] in which there is just one ellipsoidal constraint. Such problems arise quite naturally in iterative non-linear optimization procedures where in one iteration step, the objective and the constraints are approximated by quadratic models. However, while any trust region problem can be solved both in theory and in practice quite efficiently, the additional constraint makes the CDT problem substantially more challenging. Many articles have treated the analysis of this and related problems, for references see [1].

After scaling the constraints and an affine transformation, we can reduce any CDT problem to the following form:

$$\left.\begin{aligned} z^* &:= \min\{f(\mathsf{x}) : r(\mathsf{x}) \le 0 \ \text{ and } \ s(\mathsf{x}) \le 0\}\ , \ \text{with} \\ f(\mathsf{x}) &:= \tfrac{1}{2}\,\mathsf{x}^\top \mathsf{Q}\mathsf{x} + \mathsf{q}^\top \mathsf{x} \\ r(\mathsf{x}) &:= \tfrac{1}{2}(\mathsf{x}^\top \mathsf{x} - 1) \le 0 \quad \text{and} \\ s(\mathsf{x}) &:= \tfrac{1}{2}(\mathsf{x}^\top \mathsf{A}\mathsf{A}^\top \mathsf{x} - 2\mathsf{a}^\top \mathsf{A}^\top \mathsf{x} + \|\mathsf{a}\|^2 - 1) \le 0\,. \end{aligned}\right\} \tag{1}$$

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where Q is a real symmetric $n \times n$ matrix which is not positive-semidefinite (psd), A is an $n \times m$ matrix with full row rank n while $\mathsf{q} \in \mathbb{R}^n$ and $\mathsf{a} \in \mathbb{R}^m$.

The gradients at a point $\bar{\mathsf{x}}$ feasible to the CDT-problem (1) read

$$\bar{\mathsf{g}} := \nabla f(\bar{\mathsf{x}}) = \mathsf{Q}\bar{\mathsf{x}} + \mathsf{q}\,, \quad \bar{\mathsf{x}} = \nabla r(\bar{\mathsf{x}})\,, \quad \text{and} \quad \bar{\mathsf{y}} := \nabla s(\bar{\mathsf{x}}) = \mathsf{A}\mathsf{A}^\top\bar{\mathsf{x}} - \mathsf{A}\mathsf{a}\,.$$

To avoid cases where the feasible set is empty or consists only of a single point, we assume Slater's condition: there exists $\hat{\mathsf{x}} \in \mathbb{R}^n$ such that $\max\left\{\|\hat{\mathsf{x}}\|, \|\mathsf{A}^\top\hat{\mathsf{x}} - \mathsf{a}\|\right\} < 1$. This can be checked by solving a convex trust region problem.

Consider the following two trust region problems:

$$\min\left\{f(\mathsf{x}) : \|\mathsf{x}\| \le 1\right\} \quad \text{and} \quad \min\left\{f(\mathsf{x}) : \|\mathsf{A}^\top\mathsf{x} - \mathsf{a}\| \le 1\right\}. \tag{2}$$

Any *global solution* to either of the trust region problems (2) that is also feasible for the other one constitutes a global solution to the CDT problem (1). Moreover, any *local solution* $\bar{\mathsf{x}}$ to (1) where at most one of the constraints is binding, i.e. which satisfies $\min\left\{\|\bar{\mathsf{x}}\|, \|\mathsf{A}^\top\bar{\mathsf{x}} - \mathsf{a}\|\right\} < 1$, is necessarily a local solution to one of the trust region problems (2), and we know that there can be at most one local, non-global solution to a trust region problem [4].

This leaves us with only one problematic region of the feasible set, namely

$$B := \left\{\mathsf{x} \in \mathbb{R}^n : \|\mathsf{x}\| = 1 \text{ and } \|\mathsf{A}^\top\mathsf{x} - \mathsf{a}\| = 1\right\} = \{\mathsf{x} \in \mathbb{R}^n : r(\mathsf{x}) = s(\mathsf{x}) = 0\}$$

where both constraints are binding. We focus on this case in what follows.

Our main objective in this study is to narrow the so-called difficulty gap. As long as the Hessian $\mathsf{H}(\bar{u}, \bar{v}) := \mathsf{Q} + \bar{u}\,\mathsf{I}_n + \bar{v}\,\mathsf{A}\mathsf{A}^\top$ of the Lagrangian is psd at some Karush-Kuhn-Tucker (KKT) point $\bar{\mathsf{x}}$ with multipliers $(\bar{u}, \bar{v})$, trust region problem methods can be employed, so these cases are considered easy. However, it may happen that the Hessian of the Lagrangian is not psd at all KKT points [5], and this phenomenon is usually called "difficulty gap".

2 Optimality conditions and copositivity

To discuss local and global optimality conditions, we first need the *linearized tangent cone* at a (1)-feasible $\bar{\mathsf{x}}$, i.e.

$$\mathit{\Gamma}(\bar{\mathsf{x}}) := \left\{\begin{array}{ll} \left\{\mathsf{d} \in \mathbb{R}^n : \bar{\mathsf{x}}^\top\mathsf{d} \le 0 \text{ and } \bar{\mathsf{y}}^\top\mathsf{d} \le 0\right\} & \text{if } \bar{\mathsf{x}} \in B \\ \left\{\mathsf{d} \in \mathbb{R}^n : \bar{\mathsf{x}}^\top\mathsf{d} \le 0\right\} & \text{if } s(\bar{\mathsf{x}}) < r(\bar{\mathsf{x}}) = 0 \\ \left\{\mathsf{d} \in \mathbb{R}^n : \bar{\mathsf{y}}^\top\mathsf{d} \le 0\right\} & \text{if } r(\bar{\mathsf{x}}) < s(\bar{\mathsf{x}}) = 0 \\ \mathbb{R}^n & \text{if } \max\left\{r(\mathsf{x}), s(\bar{\mathsf{x}})\right\} < 0 \end{array}\right\}.$$

If $\bar{\mathsf{x}}$ is locally optimal for the CDT problem (1), Slater's condition implies the local first-order condition

$$\bar{\mathsf{g}}^\top\mathsf{d} \ge 0 \quad \text{for all } \mathsf{d} \in \mathit{\Gamma}(\bar{\mathsf{x}})$$

which is equivalent to $\bar{\mathsf{x}}$ being a KKT point, i.e., a feasible point satisfying the KKT conditions

$$\bar{\mathsf{g}} + \bar{u}\bar{\mathsf{x}} + \bar{v}\bar{\mathsf{y}} = \mathsf{o} \quad \text{and} \quad \bar{u}r(\bar{\mathsf{x}}) = \bar{v}s(\bar{\mathsf{x}}) = 0 \tag{3}$$

for some (not necessarily unique) multiplier pair $(\bar{u}, \bar{v}) \in \mathbb{R}^2_+$. We refer to $(\bar{\mathsf{x}}; \bar{u}, \bar{v})$ as a KKT triple. Clearly, the second condition in (3) holds automatically when $\bar{\mathsf{x}} \in B$.

A KKT point $\bar{\mathsf{x}} \in B$ is *nondegenerate* if the constraint gradients are linearly independent and therefore the multiplier pair is unique. In the degenerate case where $\bar{\mathsf{y}} = \alpha\bar{\mathsf{x}}$ for some $\alpha > 0$, we have $\bar{\mathsf{g}} = -(\bar{u} + \alpha\bar{v})\bar{\mathsf{x}}$.

Then $(\tilde{u}, 0) := (\|\bar{\mathsf{g}}\|, 0)$ and $(0, \tilde{v}) := (0, \frac{\|\bar{\mathsf{g}}\|}{\alpha})$ are both KKT multiplier pairs for $\bar{\mathsf{x}}$, as are all pairs in their convex hull, which is a line segment in $\mathbb{R}^2_+$ of the form

$$(\bar{u}(t), \bar{v}(t)) := (t\tilde{u}, (1-t)\tilde{v}) : \quad t \in [0,1]. \tag{4}$$

Because of the nonnegativity condition, no other multiplier pairs for $\bar{\mathsf{x}}$ exist. Interestingly enough, the degenerate case allows for no difficulty gap, at least for some $t \in [0,1]$; see below.

Next, we need to introduce the *reduced* polyhedral tangent cone comprising all feasible directions along which no first-order change in the objective is possible:

$$\mathit{\Gamma}_{\rm red}(\bar{\mathsf{x}}) := \left\{ \mathsf{d} \in \mathit{\Gamma}(\bar{\mathsf{x}}) : \bar{\mathsf{g}}^\top \mathsf{d} = 0 \right\} .$$

An important property of symmetric matrices is that of *copositivity*. For a given cone $\mathit{\Gamma} \subset \mathbb{R}^n$, recall that a symmetric $n \times n$ matrix S is said to be *$\mathit{\Gamma}$–copositive* if and only if

$$\mathsf{d}^\top \mathsf{S} \mathsf{d} \geq 0 \quad \text{for all } \mathsf{d} \in \mathit{\Gamma} ,$$

i.e., if S generates a quadratic form taking no negative values over the cone $\mathit{\Gamma}$. Therefore, any psd matrix S is $\mathit{\Gamma}$-copositive, regardless of $\mathit{\Gamma}$, but not conversely. A matrix S is said to be *strictly $\mathit{\Gamma}$–copositive* if and only if

$$\mathsf{d}^\top \mathsf{S} \mathsf{d} > 0 \quad \text{for all } \mathsf{d} \in \mathit{\Gamma} \setminus \{\mathsf{o}\} .$$

Any positive-definite matrix is strictly $\mathit{\Gamma}$-copositive, but again, not conversely.

To formulate a hierarchy of global and local optimality conditions, it is convenient to denote by $\psi(\mathsf{M})$ the number of negative eigenvalues of a symmetric matrix M, counting their multiplicities. Let $(\bar{\mathsf{x}}; \bar{u}, \bar{v})$ be a nondegenerate KKT triple for (1). Then the following implications hold (all proofs can be found in [1, Section 2]):

	$\mathsf{H}(\bar{u}, \bar{v})$ is positive-semidefinite
$\Rightarrow$	$\mathsf{H}(\bar{u}, \bar{v})$ is $\mathit{\Gamma}(\bar{\mathsf{x}})$-copositive
$\Rightarrow$	$\bar{\mathsf{x}}$ solves CDT globally and $\psi(\mathsf{H}(\bar{u}, \bar{v})) \leq 1$;
	$\mathsf{H}(\bar{u}, \bar{v})$ is strictly $\mathit{\Gamma}_{\rm red}(\bar{\mathsf{x}})$-copositive
$\Rightarrow$	$\bar{\mathsf{x}}$ solves CDT locally
$\Rightarrow$	$\mathsf{H}(\bar{u}, \bar{v})$ is $\mathit{\Gamma}_{\rm red}(\bar{\mathsf{x}})$-copositive
$\Rightarrow$	$\psi(\mathsf{H}(\bar{u}, \bar{v})) \leq 2$.

In general, checking $\mathit{\Gamma}$-copositivity of a matrix H is NP-hard. However, for $\mathit{\Gamma} = \mathit{\Gamma}(\bar{\mathsf{x}})$ here, this question can be solved in polynomial time even if $\mathsf{H}(\bar{u}, \bar{v})$ fails to be psd [1, Section 3]. Therefore the difficulty gap is narrowed.

Still stronger results hold in the degenerate case. Let $\bar{\mathsf{x}}$ be a degenerate KKT point for (1), with the line segment of multiplier pairs in $\mathbb{R}^2_+$ given in (4). Then the following equivalence and implications hold (again, all proofs are in [1, Section 2]):

	$\mathsf{H}(\bar{u}(t), \bar{v}(t))$ is positive-semidefinite for some $t \in [0,1]$
$\Leftrightarrow$	$\bar{\mathsf{x}}$ solves CDT globally;
	$\mathsf{H}(\bar{u}(t), \bar{v}(t))$ is strictly $\mathit{\Gamma}_{\rm red}(\bar{\mathsf{x}})$-copositive for some $t \in [0,1]$
$\Rightarrow$	$\bar{\mathsf{x}}$ solves CDT locally
$\Rightarrow$	$\mathsf{H}(\bar{u}(t), \bar{v}(t))$ is $\mathit{\Gamma}_{\rm red}(\bar{\mathsf{x}})$-copositive for some $t \in [0,1]$
$\Rightarrow$	$\psi(\mathsf{H}(\bar{u}(t), \bar{v}(t))) \leq 1$ for some $t \in [0,1]$.

# binding	2	2	2	1	1	0
condition	psd	Γ-copos	$\Gamma_{\rm red}$-copos	psd	$\Gamma_{\rm red}$-copos	psd
$n = 2$	2591	56	215	6455	488	194
$n = 3$	3618	50	448	5572	296	16
$n = 4$	4214	39	418	5151	178	0
$n = 5$	4396	40	409	5043	112	0
$n = 6$	4582	26	361	4954	77	0
$n = 7$	4646	18	291	4985	60	0
$n = 8$	4688	14	244	5007	47	0

Table 1 Number of times the psd and copositivity conditions on $\mathsf{H}(\bar{u}, \bar{v})$ occur at computed minimizers $\bar{\mathsf{x}}$ of 10,000 randomly generated instances of feasible CDT problems for each n from 2 to 8, categorized by the number of binding constraints at $\bar{\mathsf{x}}$. By randomness, no degeneracy occurred.

3 Experiments

We conducted some numerical experiments to observe how often the various cases occurred on randomly generated CDT problems. The entries of Q, A, q and a were independently generated from the normal distribution, and Q was replaced by its real symmetric part; then a vector $\tilde{\mathsf{x}}$ was generated in the same way, normalized to have length one, and then A and a were scaled by $1/\|\mathsf{A}^\top\tilde{\mathsf{x}} - \mathsf{a}\|$, guaranteeing the existence of at least one feasible point and therefore, generically, that the Slater condition holds. The vector $\tilde{\mathsf{x}}$ was then discarded and a candidate $\bar{\mathsf{x}}$ for the global solution of each problem obtained by using BFGS to minimize the exact penalty function $p(\mathsf{x}) = f(\mathsf{x}) + \rho\max(r(\mathsf{x}), 0) + \rho\max(s(\mathsf{x}), 0)$, for some $\rho > 0$ that was increased as needed to ensure feasibility, in a (tenfold) multistart fashion. In by far the majority of cases, global optimality was confirmed, and in all except one of 70,000 tests at least local optimality was confirmed. Details are given in Table 1.

4 Conclusion

We provide new copositivity-based optimality conditions for the CDT-problem, thereby reducing the difficulty gap. Table 1 shows that by far the most common scenario is that $\mathsf{H}(\bar{u}, \bar{v})$ is psd, but with positive probability it is $\Gamma(\bar{\mathsf{x}})$-copositive but not psd. The second most likely scenario with two binding constraints is that neither condition holds, indicating that there is still scope for further work to close the difficulty gap in characterizing global optimality.

References

[1] Immanuel M. Bomze and Michael L. Overton. Narrowing the difficulty gap for the Celis-Dennis-Tapia problem. Isaac-Newton-Institute Preprint Series NI13063-POP, University of Cambridge, submitted (2013).

[2] Maria Rosa Celis, John E. Dennis Jr., and Richard A. Tapia. A trust region strategy for nonlinear equality constrained optimization. In *Numerical Optimization, 1984 (Boulder, Colo., 1984)*, pages 71–82. SIAM, Philadelphia, PA, 1985.

[3] Andrew R. Conn, Nicholas I. M. Gould, and Philippe L. Toint. *Trust-region Methods*. MPS/SIAM Series on Optimization. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2000.

[4] José Mario Martínez. Local minimizers of quadratic functions on Euclidean balls and spheres. *SIAM J. Optim.*, 4(1):159–176, 1994.

[5] Ya-Xiang Yuan. On a subproblem of trust region algorithms for constrained optimization. *Math. Program.*, 47(1, (Ser. A)):53–63, 1990.

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Second order necesssary optimality conditions in Pontryagin form for stochastic optimal control problems

J. Frédéric Bonnans

Abstract We discuss stochastic optimal control problems whose volatility does not depend on the control, and which have finitely many equality and inequality constraints on the expected value of function of the final state, as well as control constraints. The main result is a proof of necessity of some second order optimality conditions involving Pontryagin multipliers.

Keywords Stochastic control, Pontryagin's principle, second order optimality conditions

Mathematics Subject Classification (2000) MSC 49K15 · MSC 60G99 · MSC 90C15

1 Introduction

In this paper we consider stochastic optimal control problems whose volatility does not depend on the control, and having two types of constraints: (i) bound constraints on the control, and (ii) finitely many equality and inequality constraints on the expected value of function of the final state, and control on the final state. Such problems can be studied by the Hamilton-Jacobi-Bellman (or dynamic programming) approach, using the notion of viscosity solution, see [10,15,18]. This approach has the advantage to give characterizations of global optimality in some cases. However, it is not easy to apply in the presence of final state constraints. In this paper we will rely on the variational approach, which consists in obtaining necessary or sufficient optimality conditions by analyzing small perturbations of an optimal trajectory.

For deterministic control problems, a major result along this approach is Pontryagin maximum principle, or PMP, which essentially says that with the solution of a deterministic optimal control problem, are associated some multipliers such that the optimal control minimizes the Hamiltonian of the problem. This has been extended to stochastic control problems, first by Kushner [14,13], Bensoussan [2] Bismut [3,4], and Haussmann [11,12]. A major advance, due to Peng [17], was the extension of such results to the case when the volatility depends of the control. See also Cadenillas and Karatzas [8] and Yong and Zhou [18].

On the other hand, it is classical for deterministic optimal control problems to derive second order necessary conditions. These conditions typically say that the curvature of the Lagrangian of the problem is nonnegative over a set of critical directions (for some multiplier that may depend on the direction). The only extension we know of such results for stochastic control problems is Bonnans and Silva [7].

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Finally, in some deterministic optimal control problems it is possible to obtain second order necessary conditions in Pontryagin form, i.e., where the involved multipliers satisfy the PMP, see [16]. A result of this form, for problems with state constraints and mixed state and control constraints, was recently obtained in BDP1 [6]. Corresponding sufficient conditions were obtained in BDP2 [5]. The aim of this paper is to obtain second order necessary conditions in Pontryagin form for stochastic control problems. We have to make the important restrictive hypothesis that the volatility does not depend on the control. Note, however, that in the second order optimality conditions obtained in [7], there were already imprtant restrictions on the dependence of the volatility w.r.t. the control.

As in [6] the analysis will be based on an auxiliary problem called "finite relaxation", that makes use of the notion of relaxed control. There is an important literature concerning the extension of this notions to the stochastic case, see the early reference Becker and Mandrekar [1], and El Karoui, Nguyen and Jeanblanc-Picqué [9].

The analysis is simplified here for two reasons: (i) we use only finite relaxations, which can be viewed as classical controls for the auxiliary problem, and (ii) the volatility does not depend on the control. This simplifies, in particular, the construction of classical controls approximating relaxed ones.

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References

1. H. Becker and V. Mandrekar. On the existence of optimal random controls. *J. Math. Mech.*, 18:1151–1166, 1968/1969.
2. A. Bensoussan. Stochastic maximum principle for distributed parameter systems. *J. Franklin Inst.*, 315(5-6):387–406, 1983.
3. Jean-Michel Bismut. Conjugate convex functions in optimal stochastic control. *J. Math. Anal. Appl.*, 44:384–404, 1973.
4. Jean-Michel Bismut. An introductory approach to duality in optimal stochastic control. *SIAM Rev.*, 20(1):62–78, 1978.
5. J. F. Bonnans, Xavier Dupuis, and Laurent Pfeiffer. Second-order sufficient conditions for strong solutions to optimal control problems. *ESAIM-COCV*, page 24, March 2013. Online First.
6. J.F. Bonnans, Xavier Dupuis, and Laurent Pfeiffer. Second-order necessary conditions in Pontryagin form for optimal control problems. Rapport de recherche RR-8306, INRIA, May 2013.
7. J.F. Bonnans and F.J. Silva. First and second order necessary conditions for stochastic optimal control problems. *Appl. Math. Optim.*, 65(3):403–439, 2012.
8. Abel Cadenillas and Ioannis Karatzas. The stochastic maximum principle for linear convex optimal control with random coefficients. *SIAM J. Control Optim.*, 33(2):590–624, 1995.
9. Nicole El Karoui, Du' Hùù Nguyen, and Monique Jeanblanc-Picqué. Compactification methods in the control of degenerate diffusions: existence of an optimal control. *Stochastics*, 20(3):169–219, 1987.
10. Wendell H. Fleming and H. Mete Soner. *Controlled Markov processes and viscosity solutions*, volume 25 of *Stochastic Modelling and Applied Probability*. Springer, New York, second edition, 2006.
11. U. G. Haussmann. General necessary conditions for optimal control of stochastic systems. *Math. Programming Stud.*, 6:30–48, 1976. Stochastic systems: modeling, identification and optimization, II (Proc. Sympos., Univ. Kentucky, Lexington, Ky., 1975).
12. U. G. Haussmann. Some examples of optimal stochastic controls or: the stochastic maximum principle at work. *SIAM Rev.*, 23(3):292–307, 1981.
13. H. J. Kushner. Necessary conditions for continuous parameter stochastic optimization problems. *SIAM J. Control*, 10:550–565, 1972.
14. Harold J. Kushner and Fred C. Schweppe. A maximum principle for stochastic control systems. *J. Math. Anal. Appl.*, 8:287–302, 1964.
15. P.-L. Lions. Optimal control of diffusion processes and Hamilton-Jacobi-Bellman equations. I. The dynamic programming principle and applications. *Comm. Partial Differential Equations*, 8(10):1101–1174, 1983.
16. A.A. Milyutin and N. P. Osmolovskiĭ. *Calculus of Variations and Optimal Control*. American Mathematical Society, Providence, 1998.
17. Shi Ge Peng. A general stochastic maximum principle for optimal control problems. *SIAM J. Control Optim.*, 28(4):966–979, 1990.
18. J. Yong and X.Y. Zhou. *Stochastic controls, Hamiltonian systems and HJB equations*. Springer-Verlag, New York, Berlin, 1999.

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Covariance Matrix Adaptation in Multiobjective Optimization

Dimo Brockhoff

Abstract Multiobjective optimization problems occur frequently in practice. One successful multiobjective optimization algorithm for the case of continuous variables is the so called MO-CMA-ES, a multiobjective version of the state-of-the-art single-objective optimizer Covariance-Matrix-Adaptation Evolution-Strategy (CMA-ES). Since the proposal of the original MO-CMA-ES by Igel, Hansen, and Roth in 2005–07, several improved variants have been introduced by various authors. In this talk of the invited session "Recent Advances on Continuous Randomized Black-box Optimization", I will introduce the basic concepts of multiobjective optimization as well as the concepts underlying the original MO-CMA-ES, give an overview of the main MO-CMA-ES variants, and briefly discuss their performance differences. The main purpose of this talk is to make the algorithm and its variants known to the wider public and in particular to practitioners from industry.

Keywords Multiobjective Optimization · Vector Optimization · Covariance Matrix Adaptation

1 Multiobjective Optimization

Optimization problems in which two or more objective functions need to be optimized simultaneously appear frequently in practical applications, e.g. when a new technical device or machine has to be designed and both the performance and the cost of it are taken into account. In such a multiobjective or vector optimization problem, no single optimal solution does exist anymore if the objective functions are conflicting; instead, it is typically worth to know a *set* of good solutions showing the trade-offs among the different criteria—even if later on in the decision process, only a single solution is to be implemented. This fact of finding a set of solutions is related to the concept of Pareto optimality or Pareto efficiency: we say that a solution x is (Pareto-)dominated by another solution y if for no objective function, y is better than x and if for at least one objective function, x is strictly better than y. The set of solutions, not dominated by any other feasible solution is then called the Pareto set or the efficient set and in practice, a good approximation of this set is sought.

The quality of a given solution set is thereby typically measured with respect to a *quality indicator*. Two popular indicators are the hypervolume and the ε-indicator as they do not contradict the above introduced Pareto-dominance relation when generalized to solution sets. The hypervolume of a solution set A, given a reference point, is the Lebesgue measure of the space that is dominated by solutions in A

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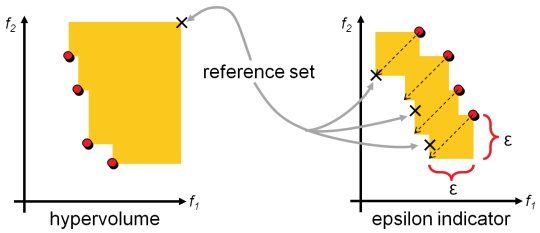

Fig. 1 Illustration of the hypervolume (left) and ϵ-indicator (right). The current set of four solution is depicted by circles, the reference point/set by crosses. Both objective functions are to be minimized.

and at the same time not dominated by the reference point. The ϵ-indicator of a set A, given a reference set R, measures how much the objective values of the solutions in A have to be translated in order to dominate all solutions in the reference set R. Figure 1 gives an illustration of the two indicators.

While typical classical algorithms for multiobjective optimization solve a multiobjective optimization problem by means of several independent scalarized versions of the original problem [9], evolutionary multiobjective optimization algorithms aim at finding good solution sets in one algorithm run [1]. Probably the most popular evolutionary multiobjective optimization algorithm is the second version of the Nondominated Sorting Genetic Algorithm, NSGA-II for short [2]—most likely because of its simplicity to be implemented and its good performance for problems with two and three objective functions. However, this algorithm was proposed more than a decade ago and further improvements in the field of evolutionary (multiobjective) optimization resulted in new algorithm concepts that outperform the classical NSGA-II algorithm—both on standard benchmark functions and on real-world optimization. Some of them possess additional invariances with respect to certain problem transformations (translation, rotation, etc.) such that their performances on a given function generalize to a whole class of problems (the class induced by the transformation). One example of a "modern" evolutionary multiobjective optimization algorithm tailored towards problems with continuous variables and the additional invariance against rotations of the search space is the so-called multiobjective version of the Covariance Matrix Adaptation Evolution Strategy (MO-CMA-ES) and the main topic of this talk.

2 The Original MO-CMA-ES

Building on the success of the state-of-the-art single-objective optimizer Covariance Matrix Adaptation Evolution Strategy (CMA-ES, [4,3]), Igel, Hansen, and Roth developed the multiobjective version in the mid-2000s [5,6], abbreviated as MO-CMA-ES, and showed that it outperforms NSGA-II on various test problems with respect to both the hypervolume and the ϵ-indicator.

The basic building block of the MO-CMA-ES is a single-objective (1+1)-CMA-ES, that keeps a single solution in its memory, together with parameters of a multivariate Gaussian distribution that are used to sample a new solution per iteration and that are updated according to the success of the new sample. In the single-objective case, a success is given if the new sample has a better function value than the old one and in the multiobjective case, the definition of success gives rise to multiple versions of the original MO-CMA-ES that are briefly discussed in the next section.

The MO-CMA-ES employs μ instances of a (1+1)-CMA-ES simultaneously, which means that in each iteration, μ new solutions are sampled. Under the assumption that one wants to keep the number of solutions fixed (at μ), the algorithm has to decide which μ solutions among the μ new and the μ old solutions to keep (and in addition, which of the sampling distribution's parameters). All current MO-CMA-ES variants use a combination of non-dominated sorting and the hypervolume contribution of a point to make these decisions. In a first step, the current points are sorted according to their non-dominance level: all solutions that are not dominated by any other are ranked first. Those solutions are removed from the consideration and the next set of non-dominated solutions (among the remaining) get ranked second and so forth. The solutions are then deleted based on their ranks—starting from the worst

rank and as long as all equally ranked solutions do not decrease the overall number of remaining points below μ. If in the end only a subset among equally ranked solutions have to be deleted to arrive at a set of size μ, the difference between the overall hypervolume (with all current solutions of equal rank) and the set of all equally ranked solutions except solution a is used to assign a quality to every solution a. Until the number of current solutions reaches again μ, the solution with the smallest hypervolume contribution is deleted iteratively from the ones of equal rank and all hypervolume contributions are recalculated each time. For more details about the algorithm, including the update of the sampling distribution's parameters, see [6].

3 Variants of the MO-CMA-ES

The success for the update of the sampling distributions' parameters within the single-objective (1+1)-CMA-ESs of MO-CMA-ES is defined in various ways. In the original algorithm, a success happens if the newly sampled solution dominates the old one [6]. In a later publication [11], an improved success definition is introduced where a success appears if the newly sampled solution "survives" the iterative selection step, based on the hypervolume contributions as described above. Further variants of MO-CMA-ES, improving the algorithm's applicability in practice, exist and I will detail them during this talk. Examples are fast approximations of the hypervolume contributions [10] or the use of surrogates when the objective functions are expensive [8,7].

4 Conclusions

The MO-CMA-ES is one of the state-of-the-art multiobjective optimization algorithms which got recent attention in the research community while in practice, often older and less powerful techniques such as NSGA-II and SPEA2 are still employed. It is the main purpose of this talk to make the MO-CMA-ES and its many variants more known to the wider public with the hope that it will be adopted by practitioners when the variables are continuous and the number of objective functions reasonable.

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References

1. K. Deb. *Multi-Objective Optimization Using Evolutionary Algorithms*. Wiley, Chichester, UK, 2001.
2. K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.
3. J. Handl and J. Knowles. Evolutionary Multiobjective Clustering. In X. Yao et al., editors, *Conference on Parallel Problem Solving from Nature (PPSN VIII)*, volume 3242 of *LNCS*, pages 1081–1091. Springer, 2004.
4. N. Hansen and A. Ostermeier. Completely Derandomized Self-Adaptation in Evolution Strategies. *Evolutionary Computation*, 9(2):159–195, 2001.
5. C. Igel, N. Hansen, and S. Roth. The Multi-objective Variable Metric Evolution Strategy, Part I. Technical Report IRINI 2005-04, Institut für Neuroinformatik, Ruhr-Universität Bochum, 44780 Bochum, 2005.
6. C. Igel, N. Hansen, and S. Roth. Covariance Matrix Adaptation for Multi-objective Optimization. *Evolutionary Computation*, 15(1):1–28, 2007.
7. I. Loshchilov, M. Schoenauer, and M. Sebag. A mono surrogate for multiobjective optimization. In *Genetic and Evolutionary Computation Conference (GECCO 2010)*, pages 471–478. ACM, 2010.
8. I. Loshchilov, M. Schoenauer, and M. Sebag. Dominance-Based Pareto-Surrogate for Multi-Objective Optimization. In *Simulated Evolution and Learning (SEAL 2010)*, volume 6457 of *LNCS*, pages 230–239. Springer, 2010.
9. K. Miettinen. *Nonlinear Multiobjective Optimization*. Kluwer, Boston, MA, USA, 1999.
10. T. Voß, T. Friedrich, K. Bringmann, and C. Igel. Scaling Up Indicator-based MOEAs by Approximating the Least Hypervolume Contributor: A Preliminary Study. In *GECCO workshop on Theoretical Aspects of Evolutionary Multiobjective Optimization*, pages 1975–1978, New York, NY, USA, 2010. ACM.
11. T. Voß, N. Hansen, and C. Igel. Improved Step Size Adaptation for the MO-CMA-ES. In J. Branke et al., editors, *Genetic and Evolutionary Computation Conference (GECCO 2010)*, pages 487–494. ACM, 2010.

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Water storage optimisation

Marc Brunet, Nadia Oudjane

Abstract : Optimizing water storage represents an important issue for EDF. Mathematically, this problem can be stated as a stochastic optimization model and is known to induce substantial numerical difficulties.
In recent years, many advanced approaches have been proposed in that domain and water storage optimization could, in principle, take benefit of these developments. However, in the first part of the talk, we emphasize the reasons why in practice those refined algorithms are not always able to improve the real operational process.
In the second part of the presentation, we focus on a specific heuristic developed by practitioners to take into account some risk constraints while optimizing the dam. We question the theoretical foundations of this approach.

Keywords : hydro power generation, risk constraints, stochastic dynamic programming

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Dealing with complex errors in daily used optimization models for large size industrial problems

Marc Brunet de la Charie · Thomas TRIBOULET

Abstract Whatever

Unit-commitment, hydro-plants scheduling, data management

Abstract
The object of this presentation is to focus on data difficulties in daily used optimization problems. In industrial IT systems, optimization model inputs are refreshed all day long with thousands of data, from diverse origins : exchanges with other information systems, manually or automatically modified values. Furthermore, in case of optimization software, some of the most complex rules on data expected by models to run can be difficult to anticipate by simple rules verification.

Complementary approaches are developed by industries to tackle these thorny issues:

- Cautious process about data validation: cross-validation, automatic and human data checking.
- Adaptation of the optimization models to be robust to some of data requirement violations. For example, by developing optimization model use-case dedicated to check some of the data requirements.

Unit commitment at EDF, and especially hydro valleys optimization, is a good example of these difficulties and will illustrate the problem during the presentation.

Keywords : Unit-commitment, hydro-plants scheduling, data complexity.

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Using Topology for Nuclear Fuel Reloading Pattern Optimization

Roman Čada

Abstract We deal with optimization of reloading patterns for nuclear reactors. We present a method based on a combination of different approaches used in mathematical optimization. The particular combination of all these methods is guided by a topological structure of the solution space. We will present real-life examples of use of this method for VVER type reactors. The method is implemented in a software package Enyo.

Keywords nucler reactor · nucler fuel · reloading pattern optimization

Mathematics Subject Classification (2000) MSC 05C90 · MSC 49M37 · MSC 90C90

1 Introduction

A nuclear reactor operates in cycles. During a reactor (mainly scheduled) outage the most burned-up fuel is replaced by fresh one and this set of fuel assemblies is placed newly in the reactor. This way we obtain a reload pattern. However not every pattern (or loading) is suitable for operating the reactor as it has to meet many criteria, especially those related to safety. Next view on a design of such patterns is economical. The aim is to operate the reactor in an economical way, to increase production of energy by a minimal cost of fresh fuel. Because the design of fuel assemblies is fixed and one can ask just for several types of fuel assemblies offered by a producer, the next aim is to design operating cycles so that one can operate the reactor for many consecutive cycles. So it is necessary to perform so called feasibility studies that under a given scenario of outages one can design many (e.g. ten or twenty) consecutive cycles with the fuel assemblies types available.

In this extended abstract only one-cycle optimization is mentioned. The setting is the following. We are given a set of fuel assemblies from the previous cycle. We need to design a loading satisfying the given interval for cycle length (it corresponds to the number of days the loading is able to produce energy). It is necessary to meet safety criteria, the most important are power peaking factor and burnup of fuel assemblies (or more precisely of fuel rods, the fuel assembly is composed from). Typically the corresponding values must be under a given (strict) limit. It is necessary to optimize the number of replaced fuel assemblies (i.e. the number of fresh fuel assemblies for the cycle under design) and types of

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these fresh assemblies (they differ by enrichment with uranium) and for that number to maximize the cycle length within the given interval.

In general several criteria (limits) are to be met and several parameters should be optimized (maximized ro minimized). Both criteria and parameters are by nature sometimes discrete (number of fresh fuel assemblies) or continuous (cycle length).

The problem is that to obtain correct values of parameters one must solve a huge number of differential equations which are highly nonlinear. Typically a calculation of one loading takes several seconds to tens of minutes depending of accuracy of calculations.

So from mathematical point of view we have a mixed multicriteria nonlinear optimization problem.

Several methods for nuclear fuel loading optimization were proposed in the literature. Among those most important approaches we mention linear programming [1], heuristics methods (simulated annealing [2], tabu search [3], neural networks and genetic algorithms [4]), nonlinear optimization [5] and cellular automata [6].

On principle we can solve the problem by considering a suitable approximation model of the neutron behaviour in the core which is then simple enough that one can apply directly methods from mathematical optimization. The main disadvantage of this approach is that due to high nonlinearity of the problem the model might be still nonlinear as well. But even then the approximate values obtained may differ from correct ones by tens of percent which is unfeasible as it usually leads to an improper loading. These models are solved by gradient-like methods.

The other possibility is to calculate the values correctly (for a given precision) and use a black-box optimization methods. Possibly use it together with an approximation model as a guide for these methods. The macrocode for solving neutronic calculations is based on a solution of the following equation for neutron flux

$$-\mathrm{div}\,(D(x)\nabla u(x)) + \Sigma(x)u(x) = \lambda\sigma(x)u(x)\ x \in \Omega, \tag{1}$$

$$u(x) = 0\ x \in \partial\Omega \tag{2}$$

Ω corresponds to a reactor core area, u neutron flux, λ eigenvalue, D diffusion coefficient, Σ absorption cross section and σ fission cross section. Note that operator "div" means the standard divergence in vector analysis.

We use an iterative method which can be described in several steps.

- Define a system of neighborhoods (call it first topology) in the solution space.
- Identify basins of attraction in the solution space with respect to a particular topology
- Identify neighborhood structure (call it second topology) of basins obtained in the previous step
- In a particular basin find good parameters of an approximation model of (1)
- Solve approximation model to find estimates of most important parameters
- If the estimates seem to be good, solve the problem using black box optimization with exact values (during that we increase accuracy and also possibly change the model of the reactor core from 2D to 3D). Otherwise move to next basin guided by second topology
- Repeat moves using second topology untill a solution is found. Otherwise there is no solution with very high probability.

Note that it is a kind of hierarchical approach to the problem. The avantage is that many aspects are related to the geometrical structure of the reactor core and as such they can be precalculated and then used in every optimization process without recalculation. Also the probability of an error that the software finds no solution satisfying all criterions provided there is one seems to be rather small (according to some very exhaustive long time searches less than units of percents). Results of calculations are usually depicted as in Figure 1.

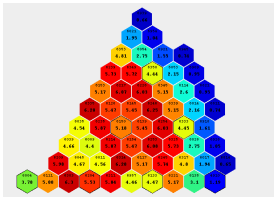

Fig. 1 An example of a power distribution in a reactor core during optimization

2 Conclusions

We described basics of a method for optimizing reload patterns for nuclear reactors. We have so far succesfully tested the proposed method for VVER type reactors but we believe that it is applicable also to other types of reactors as well. It is also possible to consider multi-cycle optimization, which has been already thoroughly tested, however some refinements are still under considerations.

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References

1. Kunsch, P.L., Teghem, J. Jr., Nuclear fuel cycle optimization using multi-objective stochastic linear programming, European Journal of Operational Research, Elsevier, vol. 31(2), pages 240-249, August 1987.
2. D. J. Kropaczek and P. J. Turinsky, "In-Core Nuclear Fuel Management Optimization for PWRs Utilizing Simulated Annealing," Nucl. Tech., 95, 9, 1991.
3. A. Castillo, G. Alonso, L. B. Morales, C. Martn del Campo, J. L. Franois and E. del Valle, BWR fuel reloads design using a Tabu search technique, Annals of Nuclear Energy Volume 31, Issue 2, January 2004, Pages 151-161
4. Adem Erdogan, Melih Geckinli, A PWR reload optimisation code (XCore) using artificial neural networks and genetic algorithms Annals of Nuclear Energy, Volume 30, Issue 1, January 2003, Pages 35-53
5. A. J. Quist, K. Roos, T. Terlaky, R. Van Geemert and E. Hoogenboom, Reloading Nuclear Reactor Fuel Using Mixed-Integer Nonlinear Optimization, Optimization and Engineering, Volume 2, Number 3, September, 2001, Pages 251-276
6. A. H. Fadaei, S. Setayeshi, A new optimization method based on cellular automata for VVER-1000 nuclear reactor loading pattern, Annals of Nuclear Energy Volume 36, Issue 5, 15 May 2009, Pages 659-667

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Solving inverse thermal problems with optimisation algorithms

R. Camy · D. Xu · J.P. Argaud

Abstract The validation of conjugate heat transfer predicted by the coupling of a Computational Fluid Dynamics (CFD) code and a thermal conduction code requires the exploitation of measurement data. Most of the time, these measurements are done by means of thermocouple sensors which provide data only for very local positions in the fluid and the solid. Here, optimisation algorithms proved to be able to find back the heat fluxes at the walls which cause the obtained measures. In this paper, a specific strategy based on common optimisation algorithms is presented.

Keywords Inverse problems · Heat transfer

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

As an industrial electricity provider, EDF operates many production processes based on various energy sources. In most of them, heat transfers (whether intended or not) impact the performances of the considered systems.

In this framework, EDF R&D develops numerical tools notably to:

- predict temperature evolutions in solids, SYRTHES (open source, http://researchers.edf.com/software/syrthes.html);
- predict flow fields in fluid domains, *Code_Saturne* (open source, http://code-saturne.org);
- perform optimisation studies, ADAO via SALOME (open source, http://www.salome-platform.org).

This paper focuses on the development of a strategy and its associated computer tools to estimate the heat flux at the walls of a solid, given measures at specific locations in the solid. The final goal is to contribute to the validation of predicted convective heat transfer in complex flow configurations.

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2 Description of an optimisation strategy

Finding heat fluxes at the walls of a solid with temperature records at a limited number of positions is essentially an ill-posed problem. It conceptually corresponds to an estimate of an infinite size vector of unknowns X (fluxes) using a finite size vector of data Y (measured temperatures). Hence, the need for methods or hypothesis to regularise the problem.

In the present strategy this regularisation comes from an interpolation in the physical space (x, y, z, t). In practice, the heat flux applied to the surfaces of the considered solid is taken piecewise (quadri-) linear to reduce size of the vector X to a finite value n. If n is "small enough" and with "relevant and detailed enough" measurements Y^m, the inverse problem can be solved.

Mathematically, it corresponds to the search of the minimum of a cost function J which can classically be defined as:

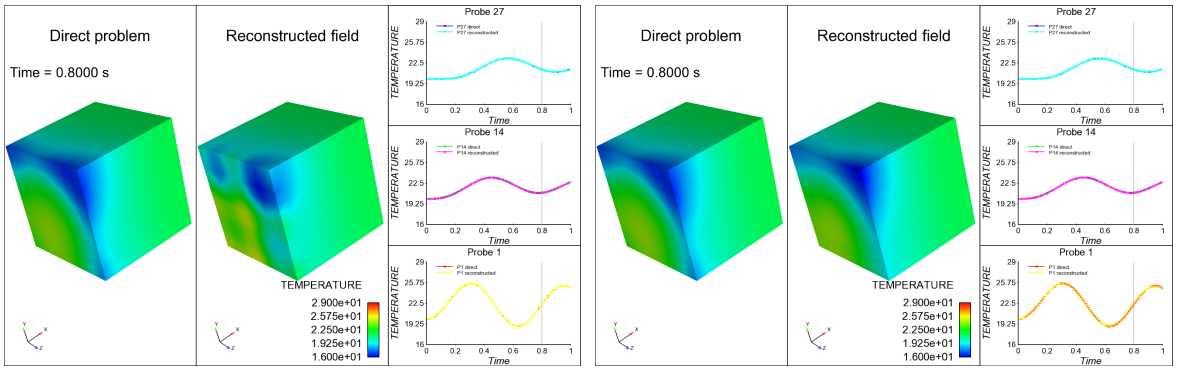
$$J(X) = (Y^m - H(X))^T \mathbf{R^{-1}} (Y^m - H(X)),$$

where H is the operator linking X to Y (here, this operator include the thermal code SYRTHES) and $\mathbf{R}$ is the measurement error covariances matrix.

Intuitively, the finer the discretisation (i.e. the bigger is n), the "better" the estimate of real field of fluxes. Yet, in 4 dimensions (space and time), the size of X can easily take large numbers which makes the solving difficult. To mitigate this phenomenon, the present strategy starts with solving a low dimension inverse problem (n^0 small) and then loops over ever higher dimension inverse problems ($n^{i+1} > n^i$) using each time the X^{*i} of the former solving as an initial point for the research of X^{*i+1}.

3 Results on a toy case

Results were obtained with the platform "SALOME" and its optimisation module "ADAO" which uses an algorithm "3D-VAR". The discretisation in physical space adresses the problem. The incremental increase of the unknowns vector's size improves results over a direct search in an high dimension space. It shows the need for a correct initial research point of the minimum.



(a) Direct search in high dimension

(b) Incremental size increase

4 Conclusions

The strategy has proved to be valid and it is expected to give objective and transferable results. A computer tool based on the SALOME platform has been developed and will be applied to validation of conjugate heat tranfer simulations.

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Frequency Constrained Unit Commitment

C. Cardozo · L. Capely · P. Dessante

Abstract The Unit Commitment (UC) problem deals with the short-term schedule of the electrical generation to meet the power demand. The main objective is to minimize production cost, while respecting technical and security constraints. In addition to the system load, a specific amount of spare capacity is committed to cope with uncertainties, such as forecasting errors and unit outages; this is called reserve and it has been traditionally specified following a static reliability criterion. In a system with a conventional generation mix, this security constraint allow achieving UC solutions that naturally provide an acceptable transient response. However, the increasing penetration of Variable Generation (VG) sources, such as wind and solar, can lead to UC solutions that no longer ensure system security. Thus, new UC models have been proposed to consider the power system dynamics when optimizing the day-ahead generation schedule. Some published works are focused on the formulation of these constraints in a Mixed-Integer Linear Programming (MILP) structure to apply classic optimization techniques. Nevertheless, power system dynamics is a non-linear problem, and, to the author's knowledge, the limits of these linear approximations have not been discussed in literature. This work examines the ability of different UC models to produce secure schedules when facing unit outages, through the implementation of a set of primary reserve & energy co-optimization models. These models are built based on linear approximations of dynamic constraints that are available in recent literature. Then, dynamic simulations are performed for every conceivable outage to observe the transient response of the system and to quantify the risk of Under Frequency Load Curtailment (UFLC). Depending on the energy mix and the dynamic parameters of the available production park, the system dynamic response can be improved at reasonable cost through slight changes in schedule and dispatch using adapted UC models. Further work will include net demand forecasting errors to determine the expected activation of UFLC.

Keywords Unit Commitment · Primary Reserve Optimization · Under Frequency Load Curtailment · Mixed-Integer Linear Programming · Variable Generation

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

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1 Introduction

The operational environment of power systems has not ceased to evolve in the last decades. On the one hand, even though the responsibility of power system security is held by system operators, following the deregulation of the energy industry, they no longer own the resources needed to ensure it. These resources are called ancillary services and include among others frequency control. In a competitive energy market, these services are becoming a product that needs to be also optimized. On the other hand, following environmental policies, Variable Generation (VG) sources, such as wind and solar, are prioritized in dispatch, even though in many cases they do not participate in the provision of ancillary services. Moreover, they do not naturally contribute to the system inertia and they produce a variable power output that has low controllability. Even more, these sources are only partially predictable, increasing the level of uncertainty in power system operation. Therefore, classic optimization models that are used to establish production portfolios, from planning to real-time operation, are being reviewed to conciliate economics with security in power supply. This work is focused on the co-optimization of primary reserve and energy to ensure power system security in spite of the increasing share of VG.

Security will be understood here as the ability of the power system to maintain power balance avoiding load shedding. Traditionally, this has been achieved by the specification of a certain amount of reserve. Spinning Reserve (SR) is the difference between the power ratings of all the operating units and the actual load. It will ensure power balance following VG and load forecasting error, and unit failures. Depending on response time and its main function, SR can be classified into primary, secondary and tertiary reserve. In general, Primary Reserve (PR) is meant to avoid load shedding after sudden disturbances and stabilize power system frequency in seconds (approx. 30 s) [5]. It is also called frequency containment reserve by the ENTSO-E [4]. Then, lower reserves are used to bring the frequency back to its nominal value in some minutes and to compensate for very short term demand variability. Regarding the system inertia, it will determine the gradient of frequency following power unbalances.

If primary reserve fails to stabilize the system frequency following a large disturbance, Under Frequency Load Curtailment (UFLC) might be automatically activated in some seconds to prevent system collapse. On the other hand, slow frequency instability processes triggered by unexpected net demand ramp for example, can be handled in some minutes by non-spinning reserves or even selective curtailment of responsive demand, with a more controlled impact [3]. Therefore, to ensure power system security, first enough primary reserve must be committed, and its allocation should be optimized respecting an expected performance of system dynamics. This work compares different UC models to include these considerations in order to examine the cost and the risk that each one entails for a specific system.

PR has been dimensioned in a deterministic way, to cover for a reference incident. In large-scale power systems, enough kinetic energy is stored in synchronous machines to prevent UFLC if a sufficient volume of PR has been specified to establish power balance in the post-contingency steady-state condition. However, in small power systems with an important share of VG connected through power electronics, this criterion may be insufficient since UFLC might occur in the frequency transient. Thus, additional constraints must be considered in optimization models to guarantee an economical and secure schedule.

2 Consideration of primary frequency regulation in Unit Commitment (UC)

PR consideration was first proposed in literature for the Economic Dispatch problem (ED). The ED is defined here as the solution of the single-period optimization, where the active power levels of on-line units are allocated to supply the load respecting generation limits and satisfying certain security criterion at minimal cost [6] [7] [1]. Afterwards, it was proposed in [8] the inclusion of a set of additional constraints in the UC MILP model to account for primary frequency regulation as an explicit function of steady-state frequency deviation. More recent work includes linear representation of several dynamic aspects in a market design for the primary frequency response ancillary service [2]. Main modelling details of published constraints accounting for frequency regulation will be included in the full paper.

3 The proposed approach - A theoretical 10 unit test system

For illustrative purposes, lets' consider a theoretical 10 unit test system. Figure 1 and 2 present the schedules according to two different UC models. In Figure 1 the total amount of primary reserve is defined by fixed requirement. In Figure 2, the primary reserve is co-optimized with the energy dispatch and an inertia constraint has been included as a function of the maximal rate of change of frequency [1].

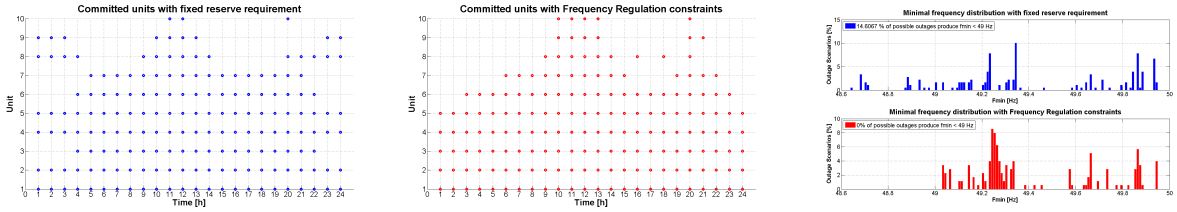

Figure 3 shows the histograms of minimal frequency values over all simulated scenarios to characterize the dynamic performance of each proposed schedule. For the test system, both UC schedules have equivalent production costs, but UC model 2 ensures that no UFLC will occur (fmin >49 Hz for all scenarios).

4 Conclusions

In this work, different security criteria to allocate primary reserve in UC models have been compared. The security criteria are defined by adding a specific set of constraints in a classic UC model. Then, dynamic simulations allow to evaluate the response of the system after unit outages, considering different schedules for the same forecasted load. Finally, the overcost incurred to improve dynamic security and the risk level associated for each optimization model have been quantified for a theoretical power system. The preliminary results presented are strongly dependent on the size and energy mix of the proposed test system. The full paper will focus on the advantages and limitations of linear constraints to tackle the deterioration of the power system dynamic response due to VG integration.

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References

1. R. Doherty, G. Lalor, and M. O'Malley. Frequency Control in Competitive Electricity Market Dispatch. *IEEE Transactions on Power Systems*, 20(3):1588–1596, 2005.
2. E. Ela, V. Gevorgian, A. Tuohy, B. Kirby, M. Milligan, and M. O'Malley. Market Designs for the Primary Frequency Response Ancillary Service Part I : Motivation and Design. *Power Systems, IEEE Transactions on*, 29(1):421–431, 2014.
3. E. Ela and B. Kirby. ERCOT Event on February 26 , 2008 : Lessons Learned. Technical report, National Renewable Energy Laboratory, Golden, Colorado, 2008.
4. ENTSO-E. Operational Reserve Ad Hoc Team Report. Technical report, ENTSO-E, Brussels, Belgium, 2012.
5. J. Machowski, J. W. Bialek, and J. R. Bumby. *Power System Dynamics: Stability and Control*. Wiley, Chichester, U.K., 2008.
6. J. W. O'Sullivan and M. J. O'Malley. A new methodology for the provision of reserve in an isolated power system. *IEEE Transactions on Power Systems*, 14(2):519–524, 1999.
7. K. A. Papadogiannis and N. D. Hatziargyriou. Optimal allocation of primary reserve services in energy markets. *IEEE Transactions on Power Systems*, 19(1):652–659, 2004.
8. J. F. Restrepo and F. D. Galiana. Unit commitment with primary frequency regulation constraints. *IEEE Transactions on Power Systems*, 20(4):1836–1842, 2005.

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Chance-constrained optimal management of a hydropower dam

P. Carpentier · M. De Lara

Abstract We consider the management of an hydroelectric dam subject to uncertain inflows and electricity prices and to a so-called "tourism constraint": the water storage level must be high enough during the tourist season with high enough probability. We formulate a chance-constrained stochastic optimal control problem, that is, we maximize the expected gain while guaranteeing a minimum storage level with a minimal prescribed probability level. Dualizing the chance constraint by a multiplier, we propose an iterative algorithm "à la Uzawa", alternating dynamic programming resolutions and updates of the multiplier value. Our numerical results reveal that the random gain is very dispersed around its expected value: in particular, low gain values have a relatively high probability to materialize. To conclude, we discuss the extension of the proposed methods to multiple probability constraints, and the so-called stochastic viability approach that jointly ensures a minimum gain and a minimum storage level during the tourist season.

Keywords Chance Constraint · Dynamic Programming · Dam Management

Mathematics Subject Classification (2000) MSC 49N15 · MSC 90B05 · MSC 90C39

1 Introduction

Hydroelectricity is the main renewable energy in many countries. It emits no greenhouse gases and provides fast-usable energy, cheap and substitutable for the thermal one. On the other hand, dam management has to deal with uncertain water inflows and uncertain electricity prices, and multiple uses (agriculture, tourism, flood prevention). We consider the situation where the following tourist chance constraint is prescribed by the local authorities: *a given reference water storage level must be guaranteed during the tourist season with a minimum probability level.* Modeling the problem using chance-constrained programming is a way to mix the economic objective (maximizing a gain) and the tourist objective (ensuring the tourist satisfaction) under uncertainty.

In §2, we present the dam hydroelectric dynamics and the economic objective. We aim to maximize the expectation of the economic gain while satisfying the tourist constraint, and we model the problem as a so-

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called chance-constrained stochastic optimal control one (references for chance-constrained programming are [Prékopa(1995)], [Prékopa(2003)] and [Dentcheva(2009)]). In §3, we reformulate the problem by adding a binary random variable to the storage level of the dam to form an extended dynamic state. In this new formulation, the probability constraint becomes a constraint in expectation. After dualizing this expectation constraint, we can apply dynamic programming for every fixed value of the multiplier, and the multiplier is iteratively updated until convergence. We provide real case based numerical results, and observe that the random gain is noticeably dispersed around its expected value; in particular, low gain values have a relatively high probability to materialize.

2 Dam Modeling and Problem Formulation

Let time t be an integer in $\{0, \ldots, T\}$ and let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. We consider the following real valued random variables:

- $\mathbf{X}_t$, the water storage level in the dam reservoir at the beginning of period $[t, t+1[$,
- $\mathbf{U}_t$, the dam turbined outflow during $[t, t+1[$,
- $\mathbf{A}_t$ and $\mathbf{C}_t$, the dam inflow and the electricity price during $[t, t+1[$, and $\mathbf{W}_t = (\mathbf{A}_t, \mathbf{C}_t)$.

The dynamics of the state process $(\mathbf{X}_t)_{t\in\{0,\ldots,T\}}$ reads

$$\mathbf{X}_{t+1} = f_t^{\mathbf{X}}\left(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_t\right) . \tag{1}$$

The control process $(\mathbf{U}_t)_{t\in\{0,\ldots,T-1\}}$ is required to be non-anticipative:

$$\mathbf{U}_t \text{ is measurable w.r.t. } \left(\mathbf{W}_0, \ldots, \mathbf{W}_t\right) . \tag{2a}$$

Moreover, each control $\mathbf{U}_t$ is subject to bound constraints:

$$0 \leq \mathbf{U}_t \leq \min\{\mathbf{X}_t + \mathbf{A}_t\ ,\ \overline{u}\} . \tag{2b}$$

Let the subset $\mathcal{T} \subset \{1, \ldots, T-1\}$ denote the tourist season period. The tourist chance constraint consists in ensuring a minimal reference storage level x_{ref} during the tourist season $\mathcal{T}$ at a probability level p_{ref}:

$$\mathbb{P}\left(\mathbf{X}_\tau \geq x_{\text{ref}}\ \forall \tau \in \mathcal{T}\right) \geq p_{\text{ref}} . \tag{3}$$

Denoting by $L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_t)$ the hydroelectric production gain at time t and $v_f(\mathbf{X}_T)$ the valorization of the stock at the final time$T$, the production management problem is:

$$\max_{\mathbf{X},\mathbf{U}} \mathbb{E}\left(\sum_{t=0}^{T-1} L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_t) + v_f(\mathbf{X}_T)\right) \quad \text{subject to} \quad (1) - (2) - (3) . \tag{4}$$

Problem (4) is a so-called chance constrained stochastic optimal control problem. Such problems raise theoretical and numerical difficulties: indeed, it is mathematically difficult to guarantee the closedness and the convexity of the set induced by the chance constraint (3). Thus, besides being of a practical interest, the resolution of the optimization problem (4) also represents a theoretical challenge.

3 Reformulation and Resolution

We introduce a new binary valued random process $(\boldsymbol{\pi}_t)_{t\in\{0,\dots,T\}}$ driven by the dynamics:

$$\boldsymbol{\pi}_{t+1} = f_t^{\boldsymbol{\pi}}\left(\mathbf{X}_t,\, \boldsymbol{\pi}_t,\, \mathbf{U}_t,\, \mathbf{W}_t\right) = \begin{cases} \boldsymbol{\pi}_t.\mathbf{1}_{\{\mathbf{X}_{t+1}\geq x_{\text{ref}}\}} & \text{if } t \in \mathcal{T}\ , \\ \boldsymbol{\pi}_t & \text{otherwise}\ . \end{cases} \tag{5}$$

Using this binary process, the chance constraint (3) can be written as an expectation constraint:

$$\mathbb{E}\left(\boldsymbol{\pi}_T\right) \geq p_{\text{ref}}\ . \tag{6}$$

The initial optimization problem (4) is thus reformulated as:

$$\max_{\mathbf{X},\boldsymbol{\pi},\mathbf{U}} \mathbb{E}\left(\sum_{t=0}^{T-1} L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_t) + v_f(\mathbf{X}_T)\right) \quad \text{subject to} \quad (1)-(5)-(2)-(6)\ . \tag{7}$$

Dualizing the (real-valued) expectation constraint by a multiplier λ, we are able to apply an algorithm "à la Uzawa" to obtain a solution of the problem. At an iteration of the algorithm,

- the primal maximisation is solved by Dynamic Programming with the "extended" state $(\mathbf{X}_t, \boldsymbol{\pi}_t)$,
- the multiplier update is done by computing the expected value $\mathbb{E}\left(\boldsymbol{\pi}_T\right)$.

Numerical experiments are conducted on a realistic dam model. Results show that the method converges toward a solution which satisfies the probability constraint (Figure 1, left). The theoretical justification of the experimentally observed convergence is difficult to obtain because the usual convexity assumptions are not fulfilled, and because the system encompasses a discrete dynamical process. However, the Everett's theorem [Everett(1963)] guarantees that the strategy obtained at convergence is a solution to the initial optimization problem.

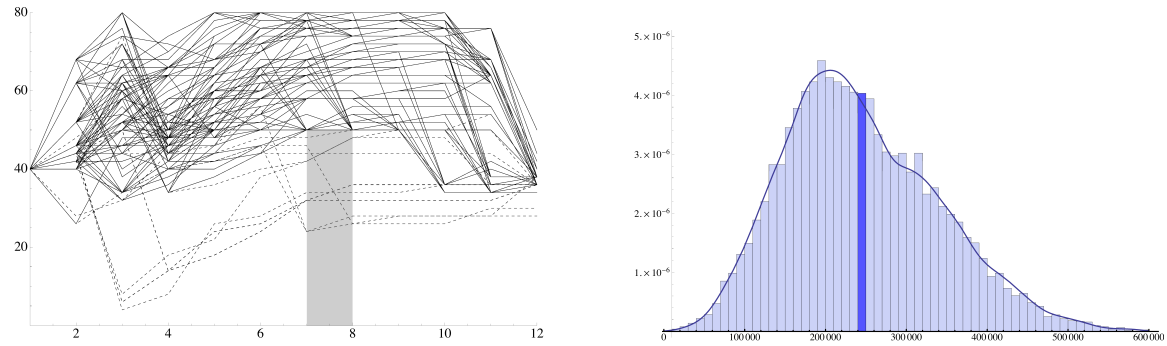

Fig. 1 Realizations of the storage level process $\mathbf{X}^\sharp$ (left) and empirical probability distribution of the gain (right)

A noticeable feature of the method is that the deviations of the realizations of the gain from its expected value are substantial (Figure 1, right). This might disappoint a dam manager who would expect a gain of the magnitude of its mean.

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References

[Alais (2013)] Alais, J.-C. 2013 *Risque et optimisation pour le management des énergies : application à lhydraulique*. Thèse de l'Université Paris-Est.

[Dentcheva(2009)] Dentcheva, D. 2009. Optimization models with probabilistic constraints. *In Lecture Notes on Stochastic Programming Modeling and Theory*, 87–153.

[Everett(1963)] Everett, H. 1963. Generalized Lagrange multiplier method for solving problems of optimum allocation of resources. *Operations Research*, vol. 11, 399-417.

[Prékopa(1995)] Prékopa, A. 1995. *Stochastic Programming*. Mathematics and Its Applications, Kluwer Academic Publisher.

[Prékopa(2003)] Prékopa, A. 2003. Probabilistic programming. A. Ruszczynski, A. Shapiro, eds., *Stochastic Programming. Handbooks in Operations Research and Management Science*, vol. 10. Elsevier, 267–351.

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A Modified Sample Approximation Method for Chance Constrained Problems

Jianqiang Cheng · Abdel Lisser · Céline Gicquel

Abstract In this talk, we present a new scheme of a sampling method to solve chance constrained programs. First of all, a modified sample average approximation is presented. With the modified sample method, despite that some other chance constraints arise, the corresponding problem has no binary variable whereas there are binary variables based on the traditional sample average approximation (SAA). Second, we show that, for the new chance constraints, it is easy to handle these chance constraints in some cases. Finally, numerical experiments are conducted to compare the proposed approximation to SAA in order to show the strength of the new sample method.

Keywords Stochastic programming · Chance constraints · Sampling approximation

Mathematics Subject Classification (2000) 90C15 · 90C59

1 Introductions

In this paper, we focus on the following chance constrained problems:

$$(CCP) \quad \begin{aligned} \min \quad & f(x) && \text{(1a)}\\ s.t. \quad & \mathbb{P}\{G(x,\xi) \geq 0\} \geq 1-\alpha && \text{(1b)}\\ & x \in X, && \text{(1c)} \end{aligned}$$

where $f : R^n \to R$ is the objective function, $X \subset R^n$ represents a set of additional deterministic constraints, $\xi \in R^d$ is random vector with a distribution F, $G : R^n \times R^d \to R^m$ are given constraint mapping, and $\alpha \in (0, 1]$ is a confidence parameter. Constraint (1b) is called chance (or probabilistic) constraint. Furthermore, in (1b), we only use a single probability constraint on all the rows in the

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constraints $G(x,\xi) \geq 0$ rather than requiring each row to be satisfied with high probability individually. Such a constraint is known as a *joint chance constraint*.

Since chance constrained programming was firstly introduced by Charnes, Cooper and Symonds in [1], it has attracted significant attention of many researchers and practitioners as it plays an important role in engineering, telecommunication, finance, etc. However, little progress was made until recently because of two reasons. One main reason is that the feasible set of CPP is generally nonconvex even if the set X is convex and the function $G(x,\xi)$ is concave in x. The other reason is that $\mathbb{P}\{G(x,\xi) \geq 0\}$ with a fixed $x \in X$ is generally hard to compute, as it requires multi-dimensional integrations. For a comprehensive overview on theory and applications of chance constrained problems, we refer the reader to the books of Prékopa [4]and Shapiro et al. [5].

As chance constrained problems are generally intractable, it leads to the development of solving methods in two directions. One is to apply convex (or tractable) approximations (see, e.g., Nemirovski and Shapiro [3]). The other approach is to use sampling methods to approximate original problems (see, e.g., sample average approximation (SAA) of Luedtke and Ahmed [2]).

In this paper, we put forward a modified sample average approximation to solve the chance constrained problems. With the modified sample method, the corresponding problem has no binary variable whereas there are binary variables based on the SAA of Luedtke and Ahmed [2]. Despite that some other chance constraints arise, we will show that it is easy to handle these chance constraints in many cases in the full paper. Meanwhile, the numerical tests to compare the proposed approximation to SAA are in preparation.

2 Partial SAA

Before we give the scheme of our approach, the key assumption on the modified sample method is presented firstly.

Assumption 1 *We assume that $\xi = (\xi_1, \xi_2)$, further ξ_1 and ξ_2 are independently distributed.*

We let $\xi_1^1, ..., \xi_1^N$ be an independent Monte Carlo sample of the random variable ξ_1. Then we have the SAA of CPP as follows:

$$\min \quad f(x) \tag{2a}$$

$$s.t. \quad \frac{\sum_{t=1}^{N} \mathbb{P}\{G(x, \xi_1^t, \xi_2) \geq 0\}}{N} \geq 1 - \eta \tag{2b}$$

$$x \in X, \tag{2c}$$

which is equivalent to

$$\min \quad f(x) \tag{3a}$$

$$s.t. \quad \mathbb{P}\{G(x, \xi_1^t, \xi_2) \geq 0\} \geq y_t, \ t = 1, ..., N \tag{3b}$$

$$\frac{\sum_{t=1}^{N} y_t}{N} \geq 1 - \eta \tag{3c}$$

$$y_t \geq 0; t = 1, ..., N, x \in X, \tag{3d}$$

The sampling method is called "Partial SAA" (PSAA). Compared to the SAA of Luedtke and Ahmed [2], there is no binary variable in Problem (3) despite that there still exists joint chance constraints. However, we will show that the new chance constraints can be well approximated in some cases through numerical experiments. Further, the numerical results will be presented in the talk to show the strength of the proposed method.

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References

1. A. Charnes, W.W. Cooper, G.H. Symonds, Cost horizons and certainty equivalents: an approach to stochastic programming of heating oil, Manag. Sci. 4 (1958) 235–263.
2. J. Luedtke and S. Ahmed, *A sample approximation approach for optimization with probabilistic constraints*, SIAM Journal of Optimization 19(2) (2008), pp. 674–699.
3. A. Nemirovski and A. Shapiro, *A. Convex Approximations of Chance Constrained Programs*, SIAM Journal on Optimization 17(4) (2006), pp. 969–996.
4. A. Prékopa, *Stochastic programming*, Kluwer Academic Publishers, 1995.
5. A. Shapiro, D. Dentcheva and A. Ruszczyński, *Lectures on Stochastic Programming: Modeling and Theory*, Cambridge University Press, Cambridge, UK, 2009.

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Scheduling independent parallel machines with convex programming

Stéphane Chrétien · Nathalie Herr · Jean-Marc Nicod · Christophe Varnier†

Abstract In the field of production scheduling, this paper adresses the problem of maximizing the production horizon of a set of independant parallel machines. Convex optimization is used to define the contribution of each machine to a global needed throughput. A coupled Alternative Direction Method of Multipliers and Bregman-Proximal scheme is proposed to cope with the assignment problem.

Keywords Convex optimization · l_1 trend filtering · Production scheduling · Parallel machines

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

The problem tackled here concerns the scheduling of m heterogeneous parallel machines M_j $(1 \leq j \leq m)$, performing independent and identical tasks. All the machines are supposed to be of similar type and independent. A subset of machines has to be used in parallel to reach a target throughput $\sigma(t)$. The total provided throughput corresponds to the sum of the contributions of machines that are currently running. All the machines are not supposed to be in use at any time because the target throughput can be reached by using only a subset of the machines within the platform or because some machines are not available. Machines are indeed assumed to suffer from wear and tear. Their lifetime is then limited and maintenance is required. Many reasons justify to postpone maintenance operations as late as possible and to maintain all the machines at the same time. Maintenance can for instance be challenging and costly [3]. Isolated or embedded equipment can also require to wait for the end of a global task before performing maintenance [1], for example in the aerospace, the railway or the automobile domain. One challenging objective is then to maximize the production horizon of the set of machines between two maintenance periods. This production horizon corresponds to the lifetime of the whole set of machines. This global lifetime depends on each machine lifetime, but also on the schedule of the machines. A machine lifetime is indeed assumed to be variable and dependent on its use. For each machine M_j, the provided throughput ρ_j can vary continuously and take any value between a minimal and a maximal

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one ($\rho_{min,j} \leq \rho_j \leq \rho_{max,j}$). The maximal throughput is more efficient in terms of output, but is associated to a minimal lifetime. A lower throughput is less efficient, but allows to reach a longer operational time. Considering discretized time, the problem consists then in selecting, for each period of time t, a subset of machines to be used and an associated throughput ρ_j for each of them, with the maximization of the production horizon as objective.

2 Model

For each machine M_j ($1 \leq j \leq m$), we define a function $f_j(t)$, $j = 1, \ldots, m$, which is the throughput that the j^{th} machine contributes. Our main constraint is that

$$\sum_{j=1}^{m} f_j(t) \geq \sigma(t) \quad \text{for all } t \text{ over the time span } \{0, \ldots, T\} \tag{1}$$

Let us assume that each $f_j(t)$, $j = 1, \ldots, m$, can be decomposed as

$$f_j(t) = f_{1,j}(t) + f_{2,j}(t) \tag{2}$$

where $f_{1,j}(t)$ is piecewise constant and $f_{2,j}(t)$ is piecewise linear Each time where $f_{2,j}(t)$ changes its slope will be called a breackpoint. Each function $f_{c,j}$ satisfies

$$f_{c,j}(t) \geq 0 \quad \text{for } c \in \{1, 2\} \text{ and for all } t \in \{0, \ldots, T\} \tag{3}$$

We will also impose the upper bound

$$f_j(t) \leq f_{\max,j}(t) \quad \text{for all } j = 1, \ldots, m \text{ and for all } t = 0, \ldots, T. \tag{4}$$

This upper bound corresponds to a maximal throughput, which typically declines gradually during the use of a machine M_j.

A certain consumption rate constraint is set for each machine $j = 1, \ldots, m$ and may be written as

$$\sum_{t=0}^{T} \Phi(f_j(t)) \leq 1 \tag{5}$$

with Φ a given convex function. Theses consumption constraints express the limited lifetime of each machine.

3 Finding well-structured solutions via convex optimization

3.1 Main idea

Our goal is to find the functions $f_{c,j}$, $c = 1, 2$ and $j = 1, \ldots, m$ using convex optimization, so that the solution can be found in polynomial time. The main idea is to use an approach which was recently promoted in signal processing and computational statistics. In [2], Kim, Koh, Boyd, and Gorinevsky showed the practical interest of minimizing the ℓ_1-norm for obtaining sparsity in the context of function modelling over time. More precisely, they showed through multiple experiments that minimizing the ℓ_1-norm of the finite differences of a vector leads, under very mild conditions, to a vector which is piecewise constant. The same idea can be used to obtain polynomially shaped (of any order) vectors which can be interpreted as the discretized version of a polynomial function of time.

The main ingredient in our proposal is to model the functions f_j by a sum $f_{1,j} + f_{2,j}$ of a piecewise constant function and a function which has uniformly controlled slopes. Using the ℓ_1 penalization approach, one obtains that our problem can be addressed via optimizing the composite function

$$\phi(f) = \sum_{j=1}^{m} \lambda_{1,j} \|\Delta f_{1,j}\|_1 + \lambda_{2,j} \|\Delta f_{2,j}\|_\infty + \lambda_{2',j} \|\Delta^2 f_{2,j}\|_1, \tag{6}$$

subject to the constraints (1), (2), (3), (4) and (5). Each term in this objective function is a penalty for imposing a certain sparsity on $\Delta f_{c,j}$ or $\Delta^2 f_{c,j}$, $c \in \{1,2\}$, $j = 1,\dots,m$. $\|\Delta f_{1,j}\|_1$ is used to minimize the discontinuity of the final solution, $\|\Delta f_{2,j}\|_\infty$ minimizes the slopes of the linear parts of f_j and $\|\Delta^2 f_{2,j}\|_1$ minimizes the number of slope changes. The use of ℓ_1-type penalties as surrogates for non-convex penalties like the ℓ_0-"norm", i.e. the number nonzero components, has been much investigated in the recent years due to the success of such approaches for Compressed Sensing and related fields in Inverse Problems Theory and Signal Processing. The main interest in using such penalties is that they are convex and are thus amenable to efficient methods of convex optimization.

In order to enforce that the $f_{c,j}$ equal zero more often than would lead the previous objective, one can propose the following objective function

$$\mathcal{F}(F) = \|F\|_1 + \sum\nolimits_{j=1}^{m} \lambda_{1,j}\|\Delta f_{1,j}\|_1 + \lambda_{2,j}\|\Delta f_{2,j}\|_\infty + \lambda_{2',j}\|\Delta^2 f_{2,j}\|_1 + \lambda\, D_h^{\psi}(F^{(l)}, F) \tag{7}$$

3.2 Algorithm

A Bregman-Proximal method is used to minimize ϕ defined in Equation (6). Let us now introduce the following functions which adequately describe our constraints:

$$\psi_0 : \mathbb{R}^{2m(T+1)} \mapsto \mathbb{R}^{T+1} \qquad \text{as} \quad \psi_0(F) = \sum\nolimits_{j=1}^{m} f_j - \sigma \tag{8}$$

$$\psi_{c,j} : \mathbb{R}^{2m(T+1)} \mapsto \mathbb{R}^{T+1} \qquad \text{as} \quad \psi_{c,j}(F) = f_{c,j}, \quad c \in \{1,2\},\ j = 1,\dots,m \tag{9}$$

$$\psi_{3,j} : \mathbb{R}^{2m(T+1)} \mapsto \mathbb{R}^{T+1} \qquad \text{as} \quad \psi_{3,j}(F) = f_{\max,j} - f_j \tag{10}$$

$$\psi_{4,j} : \mathbb{R}^{2m(T+1)} \mapsto \mathbb{R}^{m} \qquad \text{as} \quad \psi_{4,j}(F) = 1 - \sum\nolimits_{t=0}^{T} \Phi(f_j(t)) \quad j = 1,\dots,m \tag{11}$$

Then, a simple Bregman-Proximal scheme would consider iterations of the form

$$\begin{aligned} F^{(l+1)} = \operatorname{argmin}_{F\in\mathbb{R}^{2\times m\times(T+1)} \text{ and } \psi_4(F)\geq 0} \ & \mathcal{F}(F) + \lambda\Big(D_h(\psi_0(F^{(l)}),\psi_0(F)) \\ & + D_h(\psi_1(F^{(l)}),\psi_1(F)) + D_h(\psi_2(F^{(l)}),\psi_2(F)) + D_h(\psi_3(F^{(l)}),\psi_3(F))\Big). \end{aligned}$$

In order to take the constraint $\psi_4(F) \geq 0$ into account, we introduce a Lagrange function

$$L(F,U) = \mathcal{F}(F) + \langle U, \psi_4(F)\rangle. \tag{12}$$

One can then perform a primal-dual scheme based on Uzawa's iterations:

$$\begin{aligned} F^{(l+1)} = \operatorname{argmin}_{F\in\mathbb{R}^{2\times m\times(T+1)}} \ & \nabla L(F,U) + \lambda\Big(D_h(\psi_0(F^{(l)}),\psi_0(F)) \\ & + D_h(\psi_1(F^{(l)}),\psi_1(F)) + D_h(\psi_2(F^{(l)}),\psi_2(F)) + D_h(\psi_3(F^{(l)}),\psi_3(F))\Big) \\ U^{(l+1)} = \ & \max\{0, U^{(l)} + \psi_4(F^{(l+1)})\}. \end{aligned}$$

The proposed algorithm allows to search for a solution F for a fixed horizon. Since the main objective is to maximize the production horizon, a dichotomic search approach is used to determine the maximal horizon for which a solution exists. Convergence results of the method and extensive computational experiments will be provided.

References

1. Edward Balaban, Sriram Narasimhan, Matthew Daigle, José Celaya, Indranil Roychoudhury, Bhaskar Saha, Sankalita Saha, and Kai Goebel. A mobile robot testbed for prognostic-enabled autonomous decision making. In *Annual Conference of the Prognostics and Health Management Society*, 2011.
2. S.-J. Kim, K. Koh, S. Boyd, and D. Gorinevsky. ℓ_1-trend filtering. *SIAM Review*, 51(2):339–360, 2009.
3. Andras Kovacs, Gabor Erdos, Laszlo Monostori, and Zsolt Janos Viharos. Scheduling the maintenance of wind farms for minimizing production loss. In *Proceedings of the 18th IFAC World Congress, Milano (Italy)*, 2011.

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Cabling optimization of a windfarm and capacitated K-Steiner tree

Cédric Bentz · Marie-Christine Costa · Alain Hertz

Abstract The original goal is to optimize the cabling of a wind energy farm. We show the relationship between this network design problem and the capacitated K-Steiner tree problem in graphs. We give some complexity results and we propose an approach based on integer linear programming to solve the problem.

Keywords Cabling optimization · Steiner tree · Integer linear programming

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

1 Introduction

A wind farm is composed of wind turbines producing energy and cables used to collect this energy and send it to a specific sub-station which then distributes power to customers. The cables laid between given connecting nodes. Engineering constraints impose that the energy flowing through a cable is unsplittable, i.e. the energy routed from turbines to a connecting node through different cables is then routed through a unique cable path from this node to the root. Knowing the location and production of turbines and the location, capacity and cost of all possible cables with their connecting nodes, the wind farm network design problem is to optimize the total length of required cables to install. A version of the problem including the possibility of parallel cables between connecting nodes and other constraints is studied in [4] where an approach based on integer linear programming is proposed to solve real-world instances.

The problem is in closed relation with the well-know Steiner tree problem: given a weighted graph G, the K-Steiner tree problem is to find in G a minimum length tree S spanning a specified set T of K vertices called terminals. More precisely, the basic wind-farm network design problem corresponds to the capacitated rooted K-Steiner tree problem described in the next section. In our problem, the sub-station is the root of the graph and the wind turbines are the terminals. The possible cables are the edges of the graph and the connecting nodes are the Steiner nodes. Any solution of the network design problem corresponds to an anti-rooted Steiner tree.

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2 The capacitated K-Steiner tree problem

We are given a graph $G = (V, E)$, directed or undirected, with a set V of n vertices, a set $T \subseteq V$ of $K \leq n$ terminals, a set E of m edges and a length (or cost) function $l : E \to \mathbb{R}^+$. Let $r \in V - T$ be a special vertex considered as a root and let $c : E \to \mathbb{N}^*$ be a capacity function such that for each $e \in E$, $c(e)$ denotes the maximum number of terminals in the subtree disconnected from r when removing e from S, or equivalently the maximal number of paths from r to a terminal through e. We study the following problem :

Capacitated K-Steiner Tree Problem `CKSTP`

Given a graph $G = (V, E)$, an integer $K < n$, a set $T \subset V$ of K terminals, a root $r \in V - T$, a non-negative length function l and a positive capacity function c on its edges, determine, if it exists, a minimum length rooted tree S^* spanning all the vertices of T and respecting the capacity constraints.

In any solution S there is an implicit orientation of the edges along the path from r to the terminals. In the following we call arcs the edges of S according to this orientation: let $e = [i, j] \in S$, then $(i, j) \in S$ if i belongs to the path from r to j, otherwise $(j, i) \in S$. We also consider the associated decision problem `CKSTD`, i.e. `CKSTP` with $l(e) = 0$ for all $e \in E$: is there a tree S rooted at r such that for each arc $e = (i, j) \in S$, the number of terminals in the subtree rooted at j is not greater than $c(e)$?

`CKSTP` is a restricted version of the capacitated Steiner tree problem where each terminal has a positive integer requirement and the capacity of an edge is the maximal total requirement which can be served from r through e. This in turn is a generalization of the capacitated spanning tree problem where $K = n - 1$. Both these last problems are known to be NP-hard and have been extensively studied [1,2,3,5].

3 Results

We first give some complexity results or polynomial time algorithms for the decision problem `CKSTD` in different classes of graphs. We prove that, for G undirected, directed or even directed without circuit, `CKSTD` is NP-Complete, even if the capacity function is such that $c(e) = 1$ or 2 $\forall e \in E$. Then we show how solving `CKSTD` in polynomial time when the capacity function is such that $c(e) = 1$ $\forall e \in E$. We also prove that when G is a directed graph, `CKSTD` is NP-complete if K is a fixed parameter ($K \geq 3$).

Then we study the problem for any length function. We show that `CKSTP` is NP-Complete even if $l(e) = 1$ $\forall e \in E$ but is solvable in polynomial time if $K = 2$.

Finally, we propose an integer linear programming approach to solve the general problem in an undirected graph G. For any $[i, j] \in E$, y_{ij} is a 0-1 variable such that $y_{ij} = 1$ if and only if the arc (i, j) belongs to S, and if $y_{ij} = 1$ then x_{ij} denotes the number of terminals in the subtree of S rooted at j, otherwise $x_{ij} = 0$; in other words, x_{ij} is the number of paths from r to a terminal through (i, j) in S. Let l_{ij} and c_{ij} denote respectively the length and capacity of $[i, j]$ and $\Gamma(i)$ be the set of neighbors of i. The problem can be formulated as the following integer linear program :

$$
(CKSP)\quad
\left|
\begin{array}{ll}
\min & \displaystyle\sum_{[i,j]\in E} l_{ij}\ (y_{ij}+y_{ji}) \\
& \displaystyle\sum_{j\in\Gamma(r)} x_{rj} = K \qquad (1)\\
& \displaystyle\sum_{j\in\Gamma(t)} x_{jt} = 1,\ \forall t\in T \qquad (2)\\
& \displaystyle\sum_{i\in\Gamma(j)} x_{ij} = \sum_{k\in\Gamma(j)} x_{jk},\ \forall j\notin T\cup\{r\} \qquad (3)\\
& \displaystyle\sum_{j\in\Gamma(i)} y_{ji} \le 1,\ \forall i\neq r \qquad (4)\\
& x_{ij} \le c_{ij}\ y_{ij},\quad \forall [i,j]\in E \qquad (5)\\
& x_{ij}\ \in \mathbb{N},\quad y_{ij}\ \in\{0,1\},\quad \forall (i,j)\in E \qquad (6).
\end{array}
\right.
$$

We solve the problem by adding some valid inequalities and using CPLEX. We present the results for tested instances which have been either randomly generated or obtained from real-world network wind-farm data.

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References

1. C. Bousba, L.A. Wolsey (1991), *Finding minimum cost directed trees with demands and capacities*, Annals of operations research 33, 285-303.
2. D. Du, X. Hu, Steiner Tree Problems In Computer Communication Networks, *World Scientific Publishing* (2008).
3. M.R. Garey and D.S. Johnson, Computers and intractability, a guide to the theory of NP-completeness, *ed. Freeman, New York* (1979).
4. A. Hertz, O. Marcotte, A. Mdimagh, M.Carreau, F. Welt (2012), *Optimizing the Design of a Wind Farm Collection Network*, INFOR, 95-104.
5. C. H. Papadimitriou (1978), *The complexity of the capacitated tree problem*, Networks 8-3, 217-230.

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The Role of Hydrologic Information in Stochastic Dynamic Programming applied to hydropower reservoir management

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Abstract

This paper presents a study describing the effect of various hydrological variables in stochastic dynamic programming (SDP) for solving the optimization problem of managing a hydropower system. We will show that the choice of hydrological variables can strongly affect management policies. This is especially true for the system studied here, namely the Kemano hydroelectric system located in British Columbia, Canada, which is subject to large streamflow volumes due to significant snow cover during winter. Several hydrological variables were tested to calibrate the model on this system including auto-regressive variables on past inflows, real-time snow water equivalent (SWE) and forecasted runoff volume from Ensemble of Streamflow Prediction (ESP). Results indicate that for the system in this study, using ESP scenarios to estimate the runoff volume is the best among the methods investigated for effective, safe management, compared to other hydrological variables.

Keywords: Stochastic Dynamic Programming, Hydrological Information, Hydropower Reservoir, Stochastic Optimization.

Mathematics Subject Classification (2000) MSC 49M37 · MSC 65K05 · MSC 90C15

When a hydropower system is only made up of one or two reservoirs, the stochastic dynamic programming (SDP) algorithm can be used to calculate an operating policy [1]. In this context, the success of SDP essentially relies on the ability to properly model the uncertainty associated with inflows. In some cases, the system receives large amounts of floodwaters or is subjected to prolonged droughts, making it crucial to properly model this uncertainty for the effective and safe management of the hydroelectric system. The Kemano hydroelectric system, managed by Rio Tinto Alcan (RTA), a multinational aluminium producer, is subject to a very particular hydrological regime. It is located in a mountainous region of northern British Columbia (BC), Western Canada, where deep snowpacks can occur. The Kemano powerhouse, located south of Kitimat in the North Coast of BC, has an installed capacity of 1000 MW and is fed by the Nechako reservoir via a 16 km tunnel. The watershed where the reservoir is located covers 14,000 km^2 and its hydrological regime is mainly driven by snow melt, which implies a fairly high degree of interseasonal variability with maximum inflows during the snow-melting period, i.e., summer (May-July). The precipitation gradient between the eastern and western parts of the watershed is also significant, with an average yearly precipitation of 2,000 mm/year on the west side and of400 mm/year on the east side. The town of Vanderhoof, located west and downstream from the reservoir, is subject to a significant risk of flood partly due to the uncontrolled flow of the Nautley River, which crosses the Nechako River downstream from the spillway. Managing the hydroelectric system is therefore a very delicate operation given the various environmental constraints on the system and the substantial spring runoff volume as compared to the actual size and storage capacity of the Nechako reservoir.

In SDP, the hydrological state variables must be carefully chosen to produce an optimal operating policy [2]. In many studies, the stochastic process of natural inflows to reservoirs is represented by a Markov model with a hydrological variable derived from inflow measurements or reconstructions made during the preceding period. However, in some cases, this approach does not yield consistent results and choosing other hydrological variables is essential for successfully using SDP [3-5]. Even if the role of hydrological variables in stochastic optimization is important, the optimal choice of these variables remains problem dependent and relies on the hydrology of the system. For the Kemano system, inflow volumes between April and August comprise about 67% of annual streamflow. This is due to large snow accumulations in the Coast Mountains in the west end of the watershed. Monitoring the snow cover is therefore of primary importance to system management.

Numerical simulation results will be presented that demonstrate choosing the right hydrological variable is fundamental to a successful application of the method. Four hydrological variables were tested: an auto-regressive model of order 1 AR(1), an auto-regressive model of order 5 AR(5), Snow Water Equivalent (SWE) and a forecasted volume using ESP. In the case of the Kemano system, using the ESP to forecast runoff volume has proven to be, to date, the best choice for its management policy. With this hydrological variable, it was possible to provide the remaining runoff during the freshet. It established a minimum level in the Kemano reservoir prior to the freshet, leading to a policy that offered a good compromise between the efficiency of the powerhouse, supplying the minimum power generation for the aluminium smelter and flood control. Moreover, AR(p) variables did not include enough information for safe management, i.e. avoiding flooding at Vanderhoof while limiting the probability of power generation shortage. An operating policy derived from AR(p) variables generated significant losses of water through the spillway, which reduced the long-term hydroelectric production as compared with using the other hydrological variables.

[1] Labadie, J.W., 2004. Optimal operation of multireservoir systems: State-of-the-art review. Journal of water resources planning and management 130, 93–111.

[2] Tejada-Guibert, J.., Johnson, S.., Stedinger, J.R., 1995. The value of hydrologic information in stochastic dynamic programming models of multireservoir system. Water Resources Research 31, 2571–2579.

[3] Turgeon, A., 2005. Solving a stochastic reservoir management problem with multilag autocorrelated inflows: solving a stochastic reservoir management problem. Water Resources Research 41, n/a–n/a. doi:10.1029/2004WR003846.

[4] Karamouz, M. and Vasiliadis, H.V. (1992). Bayesian stochastic optimization of reservoir operation using uncertain forecasts. Water Resources Research 28: doi: 10.1029/92WR00103. issn: 0043-1397.

[5] Côté, P., Haguma, D., Leconte, R., Krau, S., 2011. Stochastic optimisation of Hydro-Quebec hydropower installations: a statistical comparison between SDP and SSDP methods. Canadian Journal of Civil Engineering 38, 1427–1434.

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