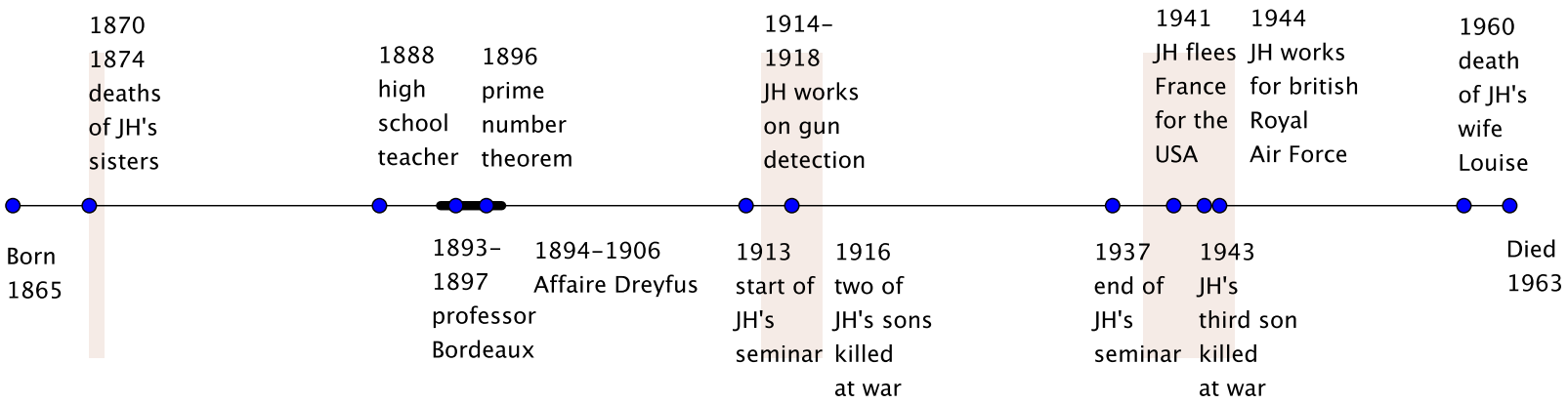
Jacques Hadamard

Pierre Pansu, Université Paris-Saclay

October 18th, 2024

Jacques Hadamard (1865-1963) has lived almost a century. He was born in a jewish family originating from the east of France (Lorraine), but always lived in Paris (except for 4 years as a professor in Bordeaux).

During his lifetime, he experienced three wars, each of which brought its own share of misfortune.



Hadamard's PhD

In 1888, Hadamard became a part-time high-school teacher. Simultaneously, he worked on his PhD under Émile Picard's remote supervision. The defense took place in 1892.

Hadamard gives a general sufficient condition for a power series not to extend beyond its disk of convergence. Then he proves an expression conjectured by Riemann for ζ in terms of its zeroes.

For this work, he won a prize from the Académie des Sciences, and a lecturer position at the Université de Bordeaux.

Essai sur l'étude des fonctions données par leur développement de Taylor;

PAR M. J. HADAMARD,

Ancien élève de l'École Normale supérieure.

INTRODUCTION.

Le développement de Taylor rend d'importants services aux mathématiciens, en raison de sa grande généralité. Lui seul, en effet, permet de représenter une fonction analytique quelconque, à certains cas singuliers près.

Depuis les travaux d'Abel et de Cauchy, on sait qu'à toute fonction régulière dans un certain cercle correspond un développement de Taylor, et réciproquement. C'est même ce développement que M. Weierstrass, et, en France, M. Méray emploient pour définir la fonction.

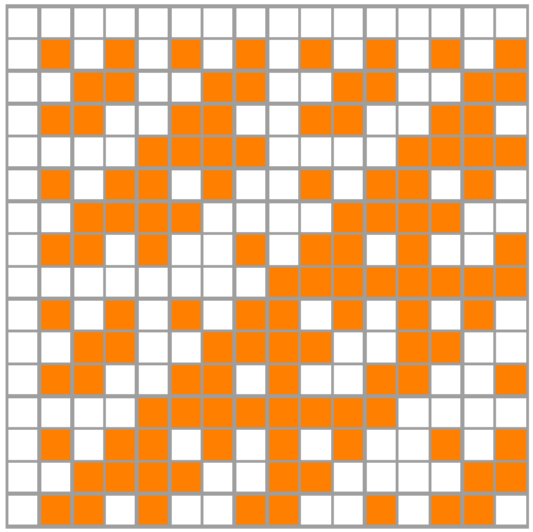
Un point a étant donné au hasard, on pourra, en général, former une série ordonnée suivant les puissances entières et positives de $x-a$ et qui représentera notre fonction dans le voisinage du point a. Il pourra y avoir exception pour certaines positions particulières du point a. C'est à ces points particuliers que l'on donne le nom de *points singuliers*.

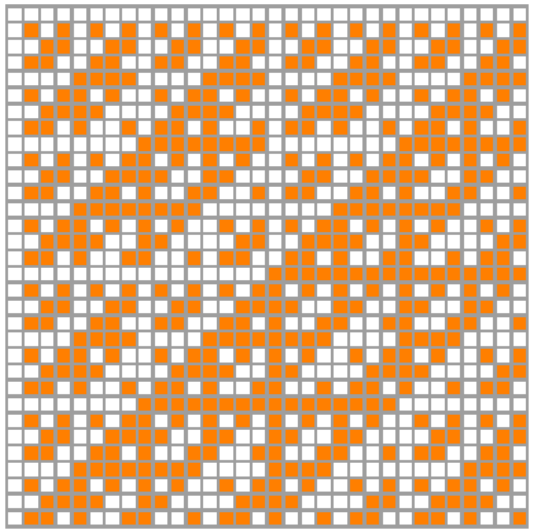
On peut donc dire que se donner une fonction analytique non singulière au point $x=0$, c'est se donner une suite de coefficients a_n,

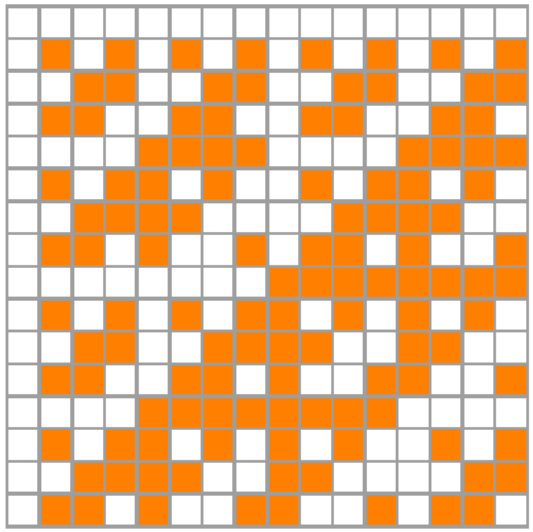
Journ. de Math. (4e série), tome VIII. — Fasc. II, 1892. 14

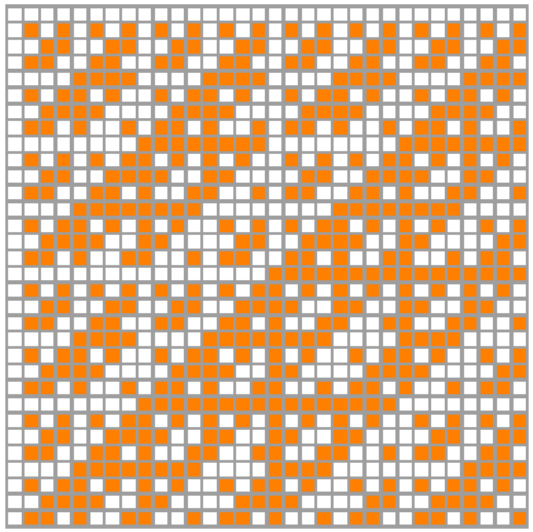
Mathematical landmarks

- 1893
- 1896
- 1898
- 1905
- Papers





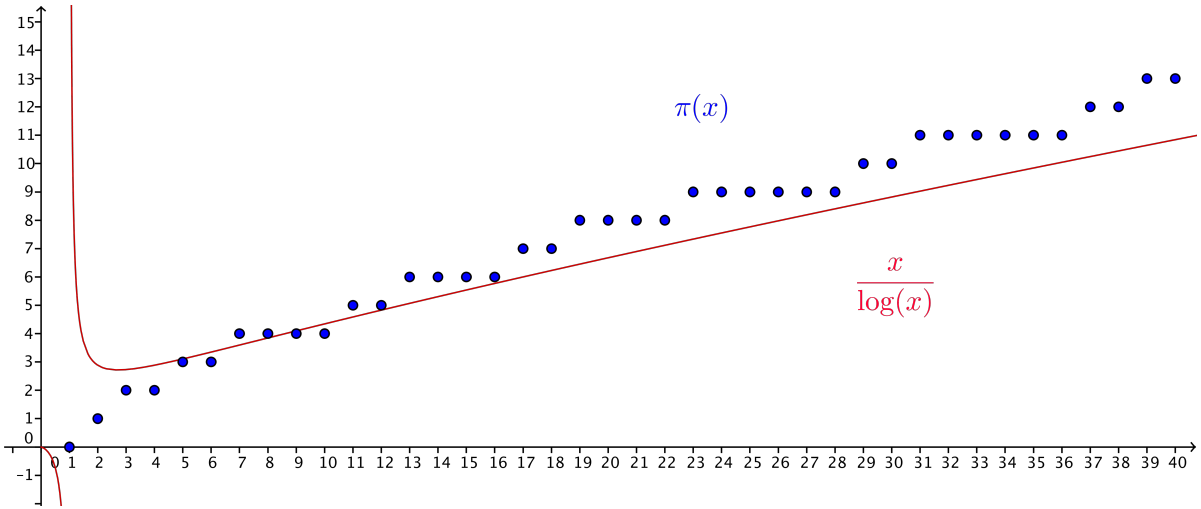


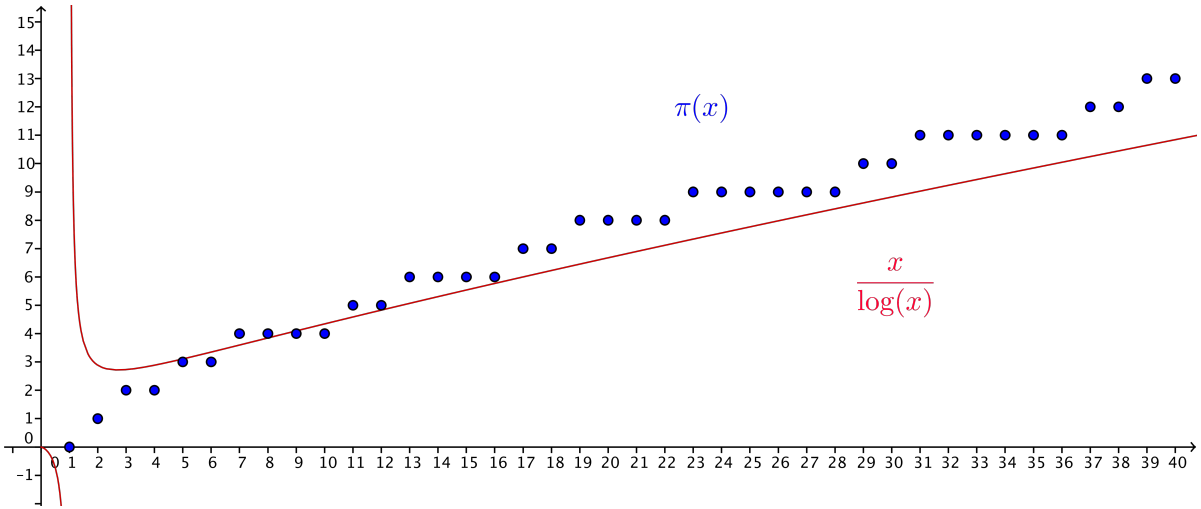


In 1893, Hadamard proved an inequality on determinants. It is sharp for a class of square ± 1 matrices now known as Hadamard matrices.

His conjecture, that there exist Hadamard matrices of any size divisible by 4 is still open. Hadamard matrices yield efficient error-correcting codes, like those used by the Mariner and Voyager NASA missions in the 1970s.

Back

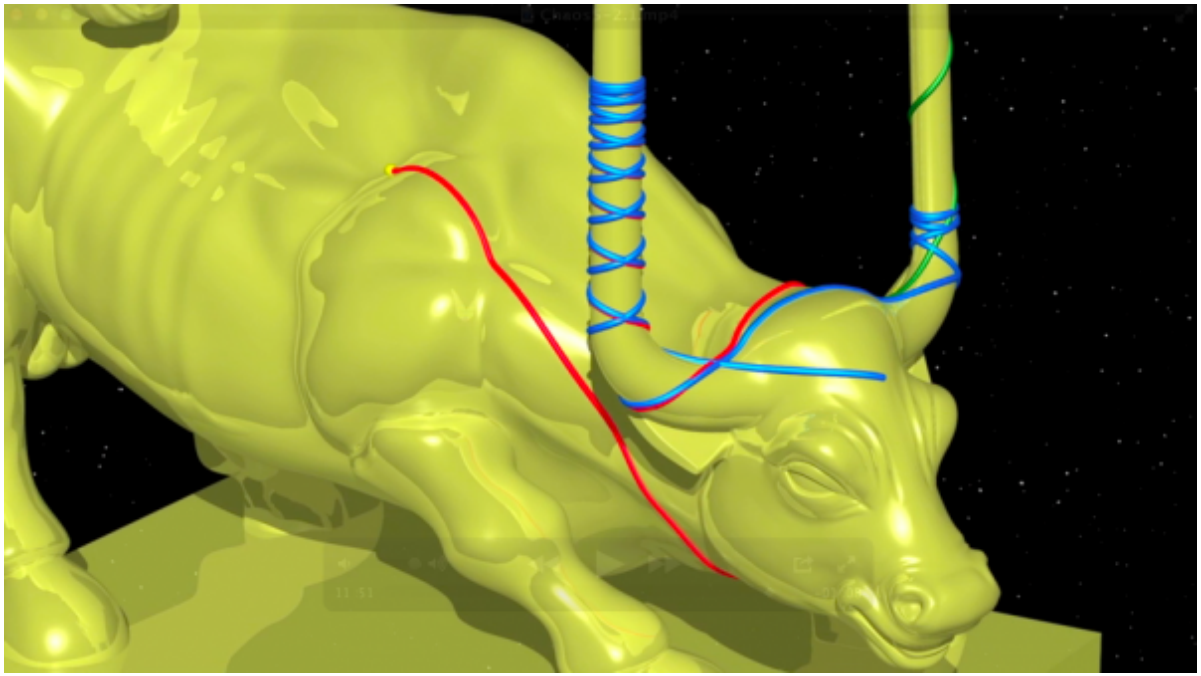

$\pi(x)$
$\frac{x}{\log(x)}$

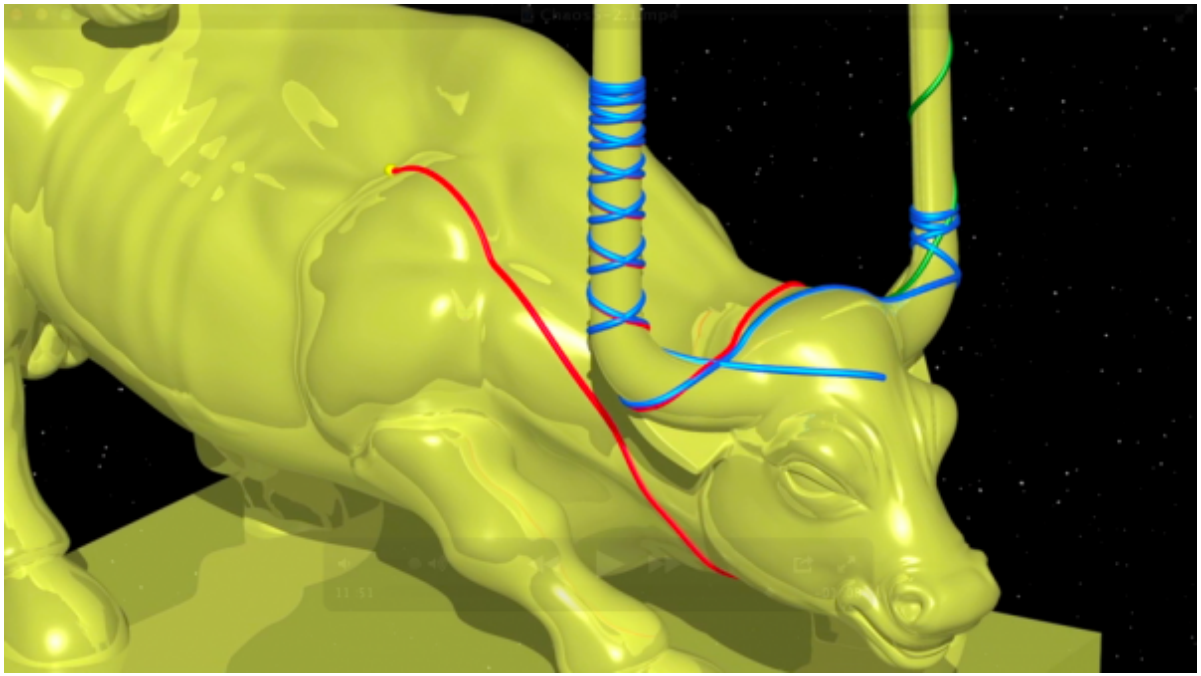


In 1896, Hadamard and de la Vallée-Poussin independently proved the prime number theorem $\pi(x) \sim \frac{x}{\ln x}$.

Back

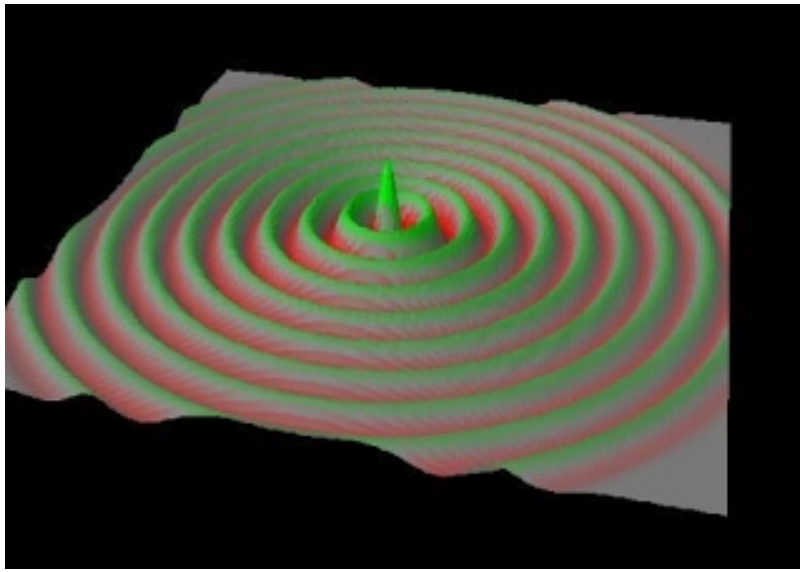
Mathematical landmarks

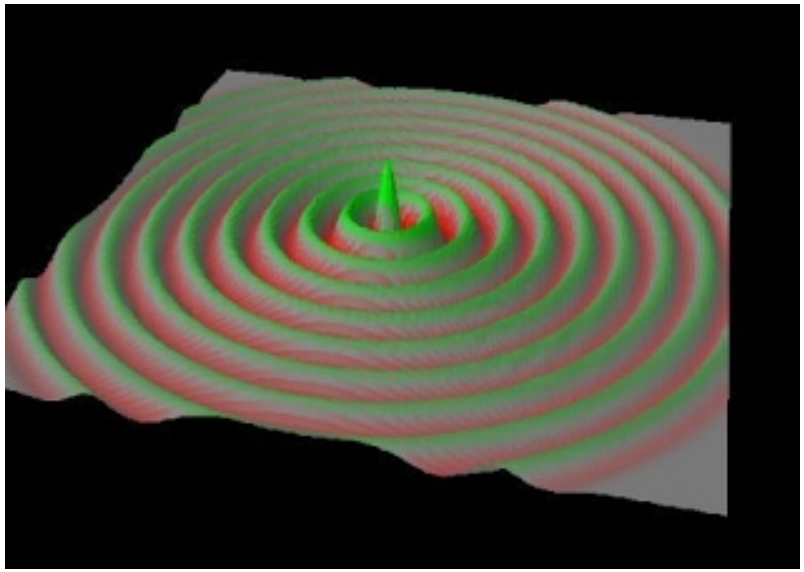




In 1898, Hadamard described the behaviour of geodesics on negatively curved surfaces: uniqueness in homotopy classes, sensitivity on initial position, structural stability. This is the starting point of hyperbolic dynamics, a cornerstone of the analysis of chaos.

Back

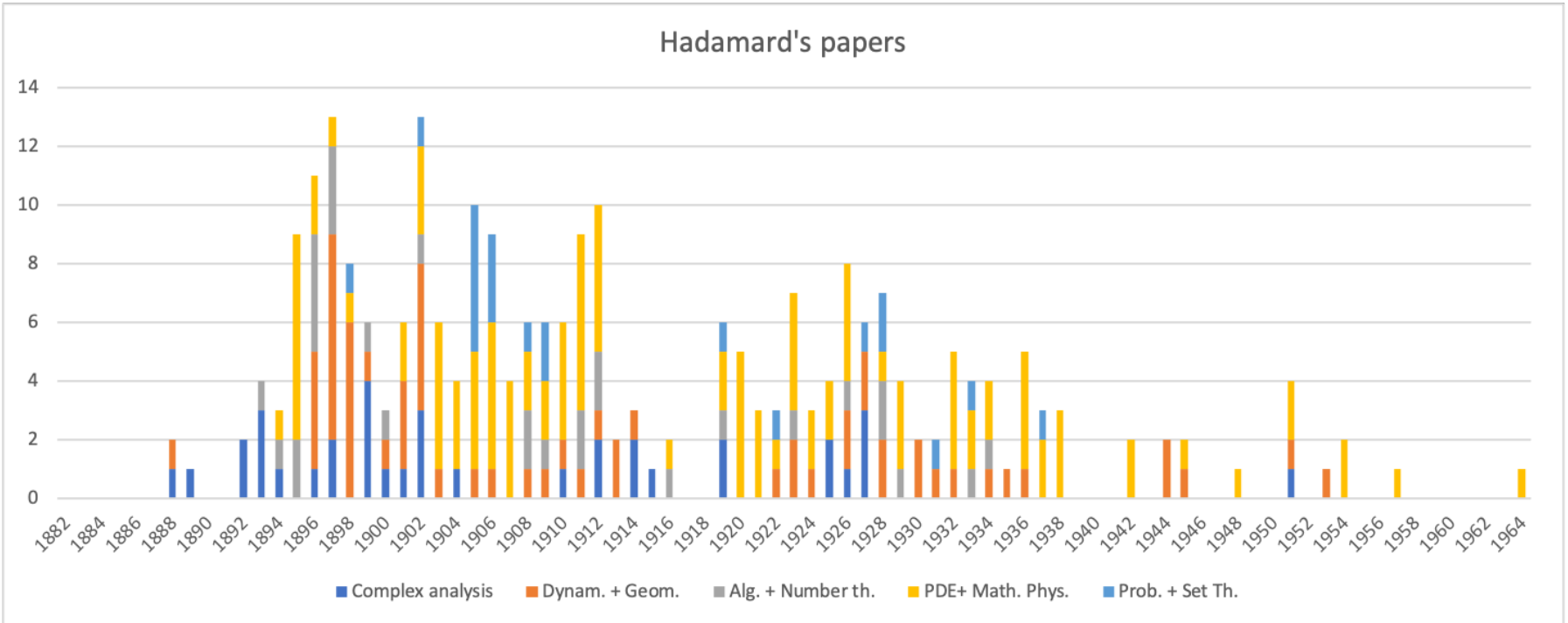




In 1905, Hadamard solved the Cauchy problem for general hyperbolic linear PDEs with variable coefficients. Along the way, he introduced the notion of "partie finie" for certain diverging integrals.

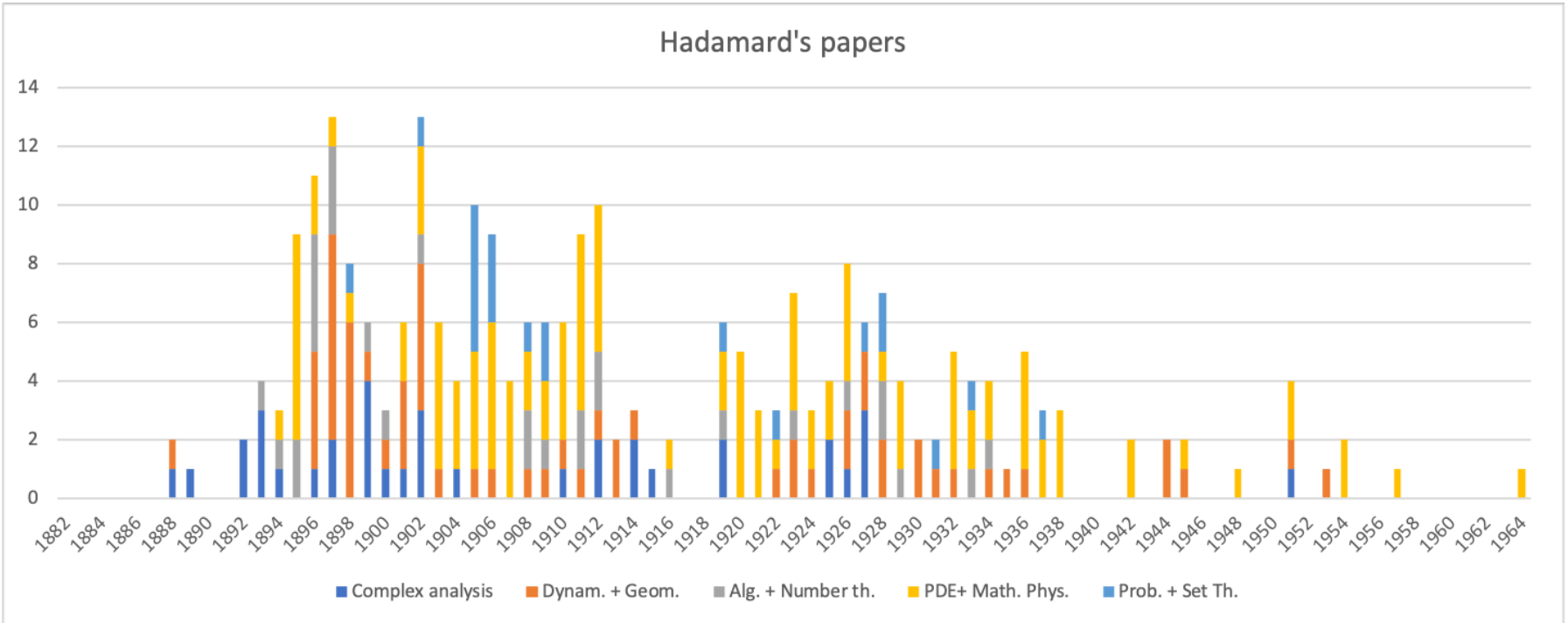
According to Hadamard, an initial value problem is well-posed if the (existing, unique) solution depends continuously on the initial data. This raises the issue of choosing a topology on the set of initial data. This challenge triggered the development of function spaces.

Back



Hadamard did a lot of work in mechanics and in optics, following discussions with physicists like Duhem. This triggered his discoveries in dynamical systems and PDEs.

Although not a very successful teacher, Hadamard wrote > 40 papers and books on education, and > 60 philosophical, historical or political papers.



Hadamard did a lot of work in mechanics and in optics, following discussions with physicists like Duhem. This triggered his discoveries in dynamical systems and PDEs.

Although not a very successful teacher, Hadamard wrote > 40 papers and books on education, and > 60 philosophical, historical or political papers.

Hadamard's first paper deals with the radius of convergence of power series. Can you state his result?

Hadamard's seminar

In 1913, Hadamard started the first mathematics seminar ever organized in France (and the only one for decades), in Paris (Collège de France). It met twice a week. It was a working seminar: Hadamard used to distribute recent articles for participants to read and read out. Occasionally, there were talks by visitors on their own work too.

The seminar ended in 1937, when Hadamard, aged 72, had to retire.

Broad, but not sufficiently universal, the Hadamard Seminar encouraged young listeners (Cartan, Dieudonné, Weil...) to embark on an ambitious publishing project, Nicolas Bourbaki's *Éléments de Mathématiques*.

The Bourbaki Seminar (next session November 23rd, 2024) is the heir to the Hadamard Seminar.

L'Affaire Dreyfus

... Surtout ! ne parlons pas de l'affaire Dreyfus !

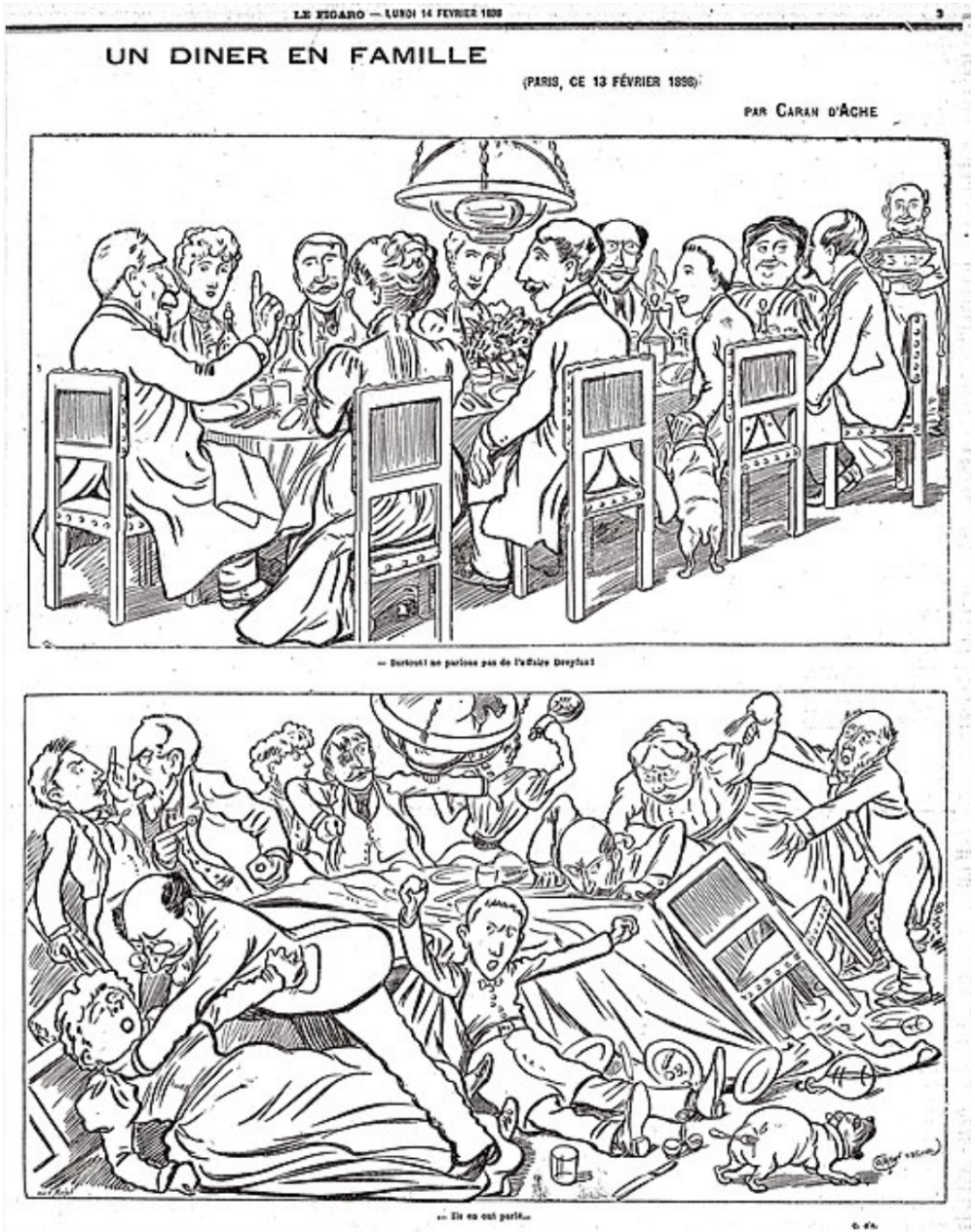
...Ils en ont parlé...

In 1897, Hadamard supported the defense in this case, which divided France and unleashed an unprecedented wave of anti-Semitism*.

J.H. was one of the founders of the *Ligue des Droits de l'Homme*, created in 1898 to defend the individual against the abuses of the state.

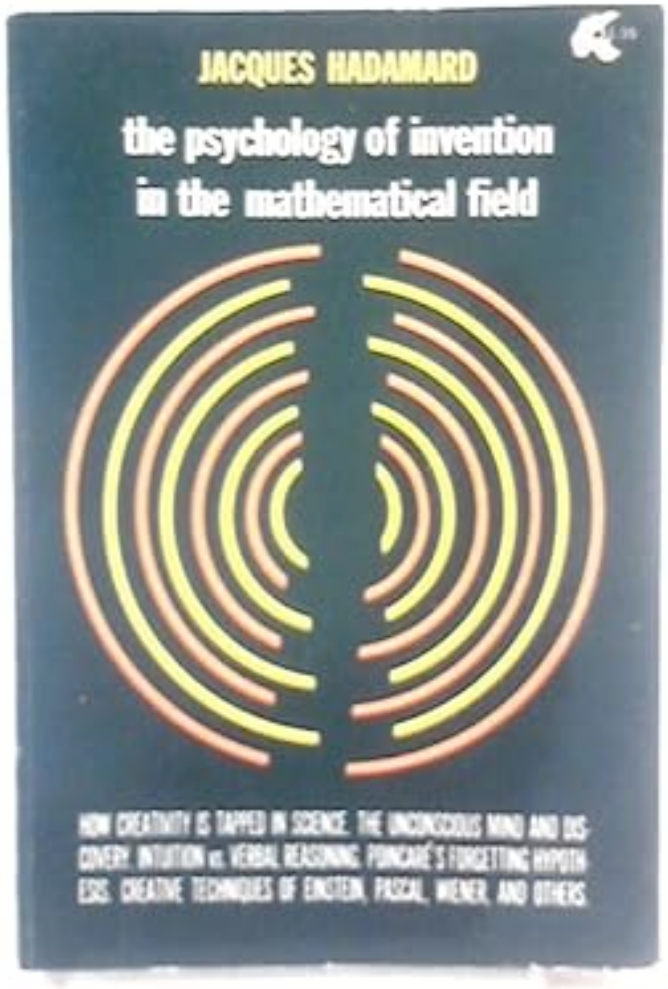
Throughout his life, Hadamard took part in actions in defense of human rights and for peace between peoples, seeking the support of scientists like Einstein.

(*) See R. Polanski's movie *An Officer and a Spy*.

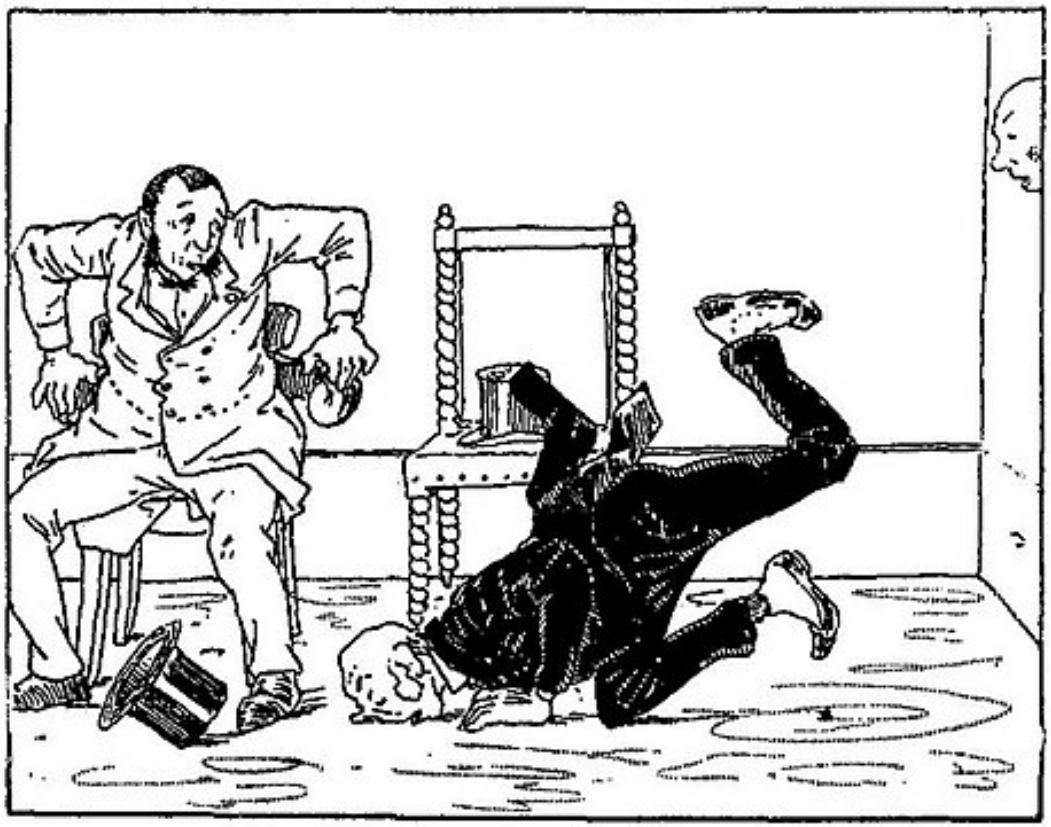


The Psychology of Invention in the Mathematical Field

In 1943, in New York, Hadamard lectured on a topic triggered by a famous lecture by H. Poincaré: discovery can be the result of unconscious work by the mind. Hadamard confronted his personal experience with that of other creators (scientists, artists) and various theories.



A popular figure?



Il ne faudrait pas croire, d'après la figure ci-dessus, que Cosinus, subitement atteint de folie, s'est cru transformé en chercheur de truffes et prend le cabinet du ministre pour une forêt périgourdine... Non ! il a simplement, dans son émotion, fait un faux pas.

Savant Cosinus, a character in Christophe's 1893 comic strip, might have been inspired by Jacques Hadamard. This used to be common belief in Hadamard's family. A more likely model was Henri Poincaré.

Hadamard at Saclay?

Jacques Hadamard was a professor at Collège de France (1897), École Polytechnique (1912) and École Centrale (1920), but he has visited neither Orsay, Palaiseau or Saclay. Rumor has it that his daughter, Jacqueline Hadamard, was once a member of the staff of Orsay's mathematical library, now named after Jacques Hadamard.

Go to Orsay's math building. Visit the math library and its small permanent exhibition on Jacques Hadamard!

To find out more:

https://images-archive.math.cnrs.fr/+-Jacques-Hadamard-+.html

V. G. Maz'ya, T. Shaposhnikova, *Jacques Hadamard, a universal mathematician*, History of Mathematics Vol. **14**, Amer. Math. Soc. (1998).
French translation by G. Tronel, EDP Sciences (2005).

Two thoughts of Jacques Hadamard

Pour inventer, il faut penser à côté, c'est-à-dire qu'il est important pour celui qui veut découvrir de ne pas se confiner à un seul chapitre de la science, mais de se tenir au courant de divers autres.

[To invent, you have to think outside the box, i.e. it's important for those who want to discover not to confine themselves to a single chapter of science, but to keep abreast of various others.]

Après avoir commencé un travail sur un certain ensemble de questions et voyant que plusieurs auteurs s'étaient mis à suivre cette direction, je l'ai souvent abandonnée et j'ai cherché quelque chose d'autre.

[After starting work on a certain set of questions, and seeing that several authors had started to follow this direction, I often abandoned it and looked for something else.]

Hadamard's seminar

In 1913, Hadamard started the first mathematics seminar ever organized in France (and the only one for decades), in Paris (Collège de France). It met twice a week. It was a working seminar: Hadamard used to distribute recent articles for participants to read and read out. Occasionally, there were talks by visitors on their own work too.

The seminar ended in 1937, when Hadamard, aged 72, had to retire.

In 1970, André Weil wrote:
Les jeunes gens d'aujourd'hui, saturés qu'ils sont de séminaires de toute sorte, imagineront difficilement combien, en ces temps reculés, leurs aînés furent livrés à eux-mêmes. De "séminaire", il n'y en avait qu'un ; il est vrai que c'était celui d'Hadamard, consacré à des analyses de travaux récents en tout domaine ; y prenaient part, dans une atmosphère de parfaite égalité exceptionnelle à cette époque, normaliens et mathématiciens "arrivés", rivalisant de zèle pour ces exposés où Hadamard ne manquait jamais d'intervenir avec sa vivacité incomparable.

[Today's young people, saturated as they are with seminars of all kinds, can scarcely imagine how much their elders were left to their own devices in those early days. There was only one "seminar", and it was Hadamard's, devoted to analyses of recent work in all fields. In an atmosphere of perfect equality that was exceptional at the time, normaliens and "senior" mathematicians took part, competing in zeal for these presentations in which Hadamard never failed to intervene with his incomparable vivacity.]

L'Affaire Dreyfus

In 1894, Alfred Dreyfus, a French officer, was convicted of espionage on flimsy evidence. His family was convinced that there had been a miscarriage of justice, but the Army refused to review the trial. Hadamard was able to convince Paul Painlevé, mathematician and politician, to join the action. France was inflamed by the affair, which unleashed a wave of anti-Semitism.

J.H. was one of the founders of the *Ligue des Droits de l'Homme*, created in 1898 to defend the individual against the abuses of the state.

Throughout his life, Hadamard took part in actions in defense of human rights and for peace between peoples, seeking the support of scientists like Einstein.