Topologie et géométrie hyperbolique classique et quantique
[Classical and quantum hyperbolic geometry and topology]

en l’honneur de

Francis Bonahon (University of Southern California)

organisée par le GDR Platon 3341 CNRS et le Laboratoire de mathématique d’Orsay (UMR 2886 CNRS) à

Université Paris-Sud

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Comité scientifique : Stéphane Baseilhac (Université de Montpellier), Richard Canary (University of Michigan at Ann Arbor), David Gabai (Princeton University), François Labourie (Université Nice Sophia-Antipolis), Feng Luo (Rutgers University), Hyam Rubinstein (University of Melbourne).

Comité d’organisation : Michel Boileau (Aix-Marseille Université), Louis Funar (CNRS, Institut Fourier), David Futer (Temple University), François Guéritaud (CNRS, Université de Lille I), Ko Honda (UCLA), Frédéric Paulin (Université Paris-Sud, Orsay), Helen M. Wong (Carleton College, Northfield)

Cette conférence internationale de très haut niveau, dont le site internet est

http://www.math.u-psud.fr/~paulin/Bonahon2015.html,
a attiré un public important des deux côtés de l’océan atlantique, reflétant l’importance scientifique et les qualités relationnelles de Francis Bonahon, Professeur à l’University of Southern California après avoir été Chargé de recherches CNRS à Orsay. Elle s’est placée à l’interface entre :
— la topologie de basse dimension,
— l’étude des structures géométriques sur les variétés (géométrie hyperbolique notamment),
— la théorie des représentations et espaces de modules,
— les groupes quantiques et les TQFT.
Au cours des 35 dernières années un mouvement de fond, ponctué aussi de succès spectaculaires, a tendu à rapprocher ces différents domaines. La conférence a été l’occasion de faire le point, tout en célébrant l’anniversaire de Francis Bonahon (University of Southern California), qui depuis sa formation à Orsay a apporté de nombreuses contributions à chacun de ces sujets. La photo ci-dessous le représente, avec ses célèbres bretelles, entouré de ses anciens doctorants et post-doctorants qui ont pu faire le voyage.

Le secrétariat de la conférence a été assuré avec brio, à la satisfaction de tous les participants, par madame Fabienne Jacquemin, assistée de madame Isabelle Souriou, gestionnaires CNRS au Laboratoire de mathématique d’Orsay.
Grâce au travail de Helen Wong et de Giuseppe Martone, qui ont filmé les exposés et ont assuré leur traitement informatique, les exposés sont maintenant disponibles sur Youtube

https://www.youtube.com/channel/UCm9O4cWAkvghXt1uSncAxew

La liste des résumés des exposés est la suivante (voir l’annexe pour le programme de la semaine)
Ian Agol (University of California, Berkeley)  
*Twisted Alexander polynomials and Thurston norm*

We'll discuss a conjecture of Dunfield-Friedl-Jackson that implies that the twisted Alexander polynomial of a hyperbolic knot detects the genus, twisted by the discrete faithful representation into $SL_2(\mathbb{C})$. We explain a reduction to sutured manifolds which are twisted homology products, and prove this for a class of knots which include knots with trivial Alexander polynomial. This is joint work with Nathan Dunfield.

Martin Bridgeman (Boston College)  
*The Isometry group of the Quasifuchsian Intersection Pairing*

Introducing geodesic currents, Bonahon described a pairing $I : T(S) \times T(S) \to \mathbb{R}$ on the Teichmüller space $T(S)$ of a closed surface $S$. We consider a natural generalization to a pairing $J : QF(S) \times QF(S) \to \mathbb{R}$ on quasifuchsian space $QF(S)$. By naturally, it follows that the extended mapping class group of the surface $S$ preserves $J$. We prove that the group of maps that preserve $J$ not only contains, but is equal to the extended mapping class group of the surface. This is joint work with R. Canary.

Jeffrey Brock (Brown University)  
*Volumes of fibered 3-manifolds, systoles of moduli space, and the Weil-Petersson distance on the rational numbers*

Following work of Schlenker, and inspired by Kojima and Macshane, we recount new developments illustrating new connections between Weil-Petersson geometry and 3-dimensional hyperbolic volume. These connections give the first explicit lower bounds on the length of the systole of moduli spaces of Riemann surfaces with the Weil-Petersson metric in terms of volumes of hyperbolic mapping tori, and on the diameters of moduli spaces of Riemann surfaces with the Weil-Petersson metric. The results also give explicit upper and lower bounds on Weil-Petersson distances between rational points in the boundary of moduli space in terms of hyperbolic volume and the length of their continued fraction expansions. This is joint work with Ken Bromberg.

François Costantino (Université Paul Sabatier, Toulouse)  
*Non semi-simple invariants and TQFTs*

We will start by reviewing the notion of Topological Quantum Field Theory (TQFT) and its application to the study of three-manifolds and of mapping class groups. Then we will outline the properties of the "Non semi-simple TQFTs" we constructed jointly with Christian Blanchet, Nathan Geer and Bertrand Patureau. We will also discuss some of the analogies between our constructions and the recent work by Bonahon and Wong on the representations of the skein algebra of a surface.

Tudor Dan Dimofte (Institute for Advanced study, Princeton)
Categorified hyperbolic geometry

L’orateur n’a pas donné de résumé / The speaker has not given an abstract.

Vladimir Fock (Université de Strasbourg)
Towards higher Teichmüller spaces for affine groups

L’orateur n’a pas donné de résumé / The speaker has not given an abstract.

François Guéritaud (Université de Lille 1)
Geometry and combinatorics of veering triangulations

Veering triangulations of 3-manifolds are a class of ideal triangulations introduced by Agol in 2010 which satisfy a local combinatorial property linked, in some sense, to hyperbolic geometry. I will present recent results relating, in particular, veering triangulations to the combinatorics of the Cannon-Thurston sphere-filling curves.

Rinat Kashaev (Université de Genève)
Dilogarithm, Pachner moves in 4D and self-dual LCA groups

The Rogers five term dilogarithm identity can be given a form of an equation involving a set-theoretical mapping which realizes the Pachner moves in four dimensions. By using hyperbolic geometry and principles of quantum theory, one can construct quantum realizations of that mapping and the dilogarithm and interpret them within a broader context of Pontryagin self-dual locally compact abelian (LCA) groups.

Fanny Kassel (Université de Lille 1)
Anosov representations and proper actions

Anosov representations of word hyperbolic groups into reductive Lie groups provide a generalization of convex cocompact representations to higher real rank. They include for instance Hitchin representations of surface groups into $SL_n(\mathbb{R})$. I will explain how these representations can be used to construct properly discontinuous actions on non-Riemannian homogeneous spaces. In certain cases, all proper actions of quasi-isometrically embedded groups come from this construction. This is joint work with F. Guéritaud, O. Guichard, and A. Wienhard.

Thang Le (Georgia Institute of Technology, Atlanta)
On the quantum trace map of Bonahon and Wong

We show that the quantum trace map of Bonahon and Wong can be defined in a natural way in the framework of skein module theory.

Julien Marché (Université Pierre et Marie Curie, Paris)
Dynamics of the mapping class group on characters varieties in $SL_2(\mathbb{R})$ in genus 2
One component in the character variety of a genus 2 surface is the Teichmüller space, with proper action of the mapping class group. We exhibit another one which may be viewed as a configuration space of six points in the Poincaré disc and show that the mapping class group acts ergodically on it (joint work with M. Wolff).

Vladimir Markovic (Caltech)

*Harmonic Maps Between Poincaré Planes and Spaces*

I will talk about problems arising in the study of harmonic mappings between hyperbolic manifolds, focusing on the Schoen Conjectures about the existence of harmonic maps between hyperbolic planes (spaces) with quasisymmetric (quasiconformal) boundary values.

Yair Minsky (Yale University)

*Windows, cores and skinning maps*

Thurston’s skinning map played an important role in his original hyperbolization theorem. His Bounded Image Theorem, in the acylindrical case, gave control of the fixed-point problem on which the proof depends. We generalize this theorem to a relative version which holds for manifolds with cylinders, and along the way refine our understanding of how one constructs compact cores with uniform topological and geometric control in degenerating sequences of hyperbolic structures. This is joint work with J. Brock, K. Bromberg and R. Canary.

Jessica Purcell (Brigham Young University, Provo)

*Cusp volumes of alternating knots*

We show that the cusp volume of a hyperbolic alternating knot can be bounded above and below in terms of the twist number of an alternating diagram of the knot. This answers a question asked by Thistlethwaite on the cusp geometry of these knots. In addition to giving diagrammatical estimates on cusp volume, this also leads to geometric estimates on lengths of slopes, in terms of a diagram of the knot. All these estimates are explicit. This is joint work with Marc Lackenby.

Dragomir Saric (Queens College, New-York)

*Limits of Teichmüller geodesics in the universal Teichmüller space*

The universal Teichmüller space consists of all quasisymmetric maps of the circle that fix three points. Following an idea of Bonahon, the universal Teichmüller space embeds into the space of geodesic currents of the hyperbolic plane. Thurston’s boundary to the universal Teichmüller space is identified with the space of projective bounded measured laminations of the hyperbolic plane. In a joint work with Hrant Hakobyan, we study the limit points of geodesic rays on Thurston’s boundary.

Saul Schleimer (Warwick University)

*End invariants of splitting sequences*

Thurston introduced train tracks and geodesic laminations as tools to study surface diffeomorphisms and Kleinian groups. We’ll start the talk with a relaxed
introduction to these. Then, in analogy with the end invariants of Kleinian groups and Teichmüller geodesics, we will define the end invariants of an infinite splitting sequence of train tracks. These end invariants determine the set of laminations that are carried by all tracks in the infinite splitting sequence. If there is time, we'll use these ideas to sketch a new proof of Klarreich’s theorem, determining the boundary of the curve complex.

Jean-Marc Schlenker (University of Luxembourg)

The renormalized volume of quasifuchsian manifolds

Quasifuchsian hyperbolic manifolds have infinite volume, but they have a well-defined "renormalized" volume, closely related to the Liouville functional. It has interesting "analytic" properties, in particular it provides a Kähler potential for the Weil-Petersson metric of the boundary at infinity, but also interesting "coarse" properties since it differs by at most additive constants from the volume of the convex core, and therefore from the Weil-Petersson distance between the conformal structures at infinity. We will survey the construction and main properties of this renormalized volume, as well as some recent applications.

Anna Wienhard (Ruprecht-Karls Universität, Heidelberg)

Projective structures and higher Teichmüller spaces

The Hitchin component for $SL(3,R)$ parametrizes convex real projective structures on a given topological surface. In this talk I will explain that many Hitchin components parametrize real projective structures on higher dimensional manifolds, and how they can be decomposed with respect to a pair of pant decomposition of the surface. This is joint work with O. Guichard.

Helen M. Wong (Carleton College, Northfield)

Skeins and Arcs

The Kaufman bracket skein algebra of a surface is one of few constructions for which the relationship between quantum topology and hyperbolic geometry is relatively concrete and well-understood. Broadly speaking, the Kaufman bracket skein algebra is the quantum analog of the space of hyperbolic metrics on a 3-dimensional thickening of the surface. In this talk we will survey two recent generalizations, defined by Roger and Yang and by Muller, which include both arcs and skeins on the surface and which can be interpreted in terms of the decorated Teichmüller space.