Report on Sorin Popa's scientific activites during his Chaire d'Excellence FSMP-FMJH 2016-2017

Sorin Popa spent the academic year 2016-2017 as a guest of the Institute de Mathematique de Jussieu (IMJ) and the Math Department of University of Paris Sud (Orsay), under a joint Chaire d'Excellence of the Fondation des Sciences Mathmatiques de Paris (FSMP) and of the Fondation Mathmatique Jacques Hadamard.

During this period Popa finished writing up six research articles ([AP], [PSV], [APV], [P5], [HP], [P6]) and a book ([AnP]).

In the first part of his visit, he finalized two long articles [AP], [PSV]. Both projects were mentioned in his research proposal, with a first draft of the papers being posted on arXiv in early 2016, just before his stay in Paris effectivley started. However, he continued to work on these papers during his stay in Paris, until end of 2016, augmenting and improving them for submission. Both papers have meanwhile been published.

The first of these papers, [AP], is a joint work with his PhD student Andreas Aaserud. It introduces several weaker versions of the notion of conjugacy and orbit equivalence (OE) of measure preserving actions of countable groups on probability spaces, involving equivalence of the ultrapower actions and asymptotic intertwining conditions: *weak conjugacy*, already considered in an equivalent form in [Ke], and *weak-OE*; *app-conjugacy* and *app-OE*; *ω-conjugacy* and and *ω-OE*. These notions are compared one with another and the usual type of rigidity questions are addressed, with many such results being obtained. This 45 pages long paper has meanwhile appeared in Ergodic Theory and Dynamical Systems.

The paper [PSV], written jointly with Dima Shlyakhtenko and Stefaan Vaes, is a substantial 70 pages long article developing notions of L^2-Betti numbers, homology and cohomology theories for subfactors and their standard invariants, through the corresponding quasi-regular symmetric enveloping inclusion of II_1 factors (which Popa has introduced 30 years ago, in [P3]). They actually develop a (co)homology theory for arbitrary quasi-regular inclusions of finite von Neumann algebras $Q \subset P$ satisfying the condition $Q' \cap P \subset Q$, and relate it to the Hochschild (co)homology for the tube $*$-algebra that we define for every such inclusion. For crossed products by countable groups Γ, they recover the ordinary (co)homology of Γ. For Cartan subalgebras, they recover Gaboriau's well known L^2-Betti numbers for the associated equivalence relation in [G]. In this common framework, which covers groups, equivalence relations and subfactors, it is shown that the L^2-Betti numbers vanish for amenable inclusions and one gives cohomological characterizations of property (T), the Haagerup property and amenability. The L^2-Betti numbers for the standard invariants of the Temperley-Lieb-Jones subfactors and of the Fuss-Catalan subfactors are computed, as well as for free products and tensor products. This work opened up a whole new direction of research in subfactor theory, which will certainly lead to many interesting results. The paper appeared in the International Mathematical Research Notices.

In a follow up joint work [APV], Andreas Aaserud, Sorin Popa and Stefaan Vaes, are studying inclusions of the form $Q \subset P$ with P a II_1 factor and Q a von Neumann subalgebra with $Q' \cap P = \mathcal{Z}(Q)$ and whose normalizer generates P (i.e., Q is a *regular* subalgebra). We obtain a complete classification of such inclusions in the case P is amenable, thus generalizing famous results of Connes, Ornstein-Weiss, Connes-Feldman-Weiss, Jones, Ocneanu.

Another direction of research that Popa worked on during the year 2016-2017 has been the construction of maximal abelian subalgebras (MASAs) and hyperfinite subfactors with prescribed properties. This resulted in two papers: [P5], written by himself, and [HP] in collaboration

with Cyril Houdayer. Both papers have already been accepted for publication: the paper [P5] will appear in Kyoto Journal of Mathematics and [HP] in the Proceedings of the 9th MSJ-SI "Operator Algebras and Mathematical Physics" held in Sendai, Japan (2016).

Thus, in [P5], Popa succeeded to give an intrinsic, local characterization of *s-MASA*, i.e., maximal abelian subalgebra A of a II_1 factor M with the property that L^2M is single generated as an A-bimodule. Using this, he is able to prove that if a II_1 factor M has an s-MASA (for instance, if M has a Cartan subalgebra), then it has uncountably many non-intertwinable singular (respectively semi-regular) s-MASAs. Thus, while such M may have a unique Cartan subalgebra up to conjugacy (e.g., as in [OP], [PV], etc), it always has many semi-regular s-MASAs.

This paper had an important follow up in work by Anna Krogager and Stefaan Vaes [KV], where they show that there exist II_1 factors with s-MASAs but without Cartan subalgebras, thus solving a problem raised in [P5].

The paper [HP], written jointly by Cyril Houdayer and Popa, was a direct consequence of Popa's stay in Paris and his weekly visits to Orsay Math Dept. Thus, while Popa has already shown in the early 1980s (cf. [P2]) that any separable diffuse semifinite von Neumann algebra and any separable type III_λ factor with $0 \leq \lambda < 1$ has a singular MASA, the problem of whether any type III_1 factor has a singular MASA remained open. In the paper [HP], Houdayer and Popa prove that if a separable type III_1 factor M satisfies *Connes bicentralizer property* (CBP) then indeed M contains singular MASAs with normal conditional expectation. The paper also provides several results about the stability of CBP for inclusions of type III_1 factors with normal conditional expectation, under the assumption that the inclusion has finite index or more generally is co-amenable.

One of the main projects in Popa's FSMP Research Proposal had to do with the problem of *simulating* algebras inside II_1 factors (or *approximate embeddings*), in particular in relation to Connes Approximate Embedding conjecture. In the paper [P6], Popa obtained new results in this direction, by showing that if a II_1 factor M is separable and $Q \subset M$ is a von Neumann algebra that has infinite Pimsner-Popa index under any projection $q \in Q' \cap M$, then there exists a sequence of unitary elements $u = (u_n)_n \subset M$ such that in any ultrapower M^ω we have $uMu^* \perp Q^\omega$. Moreover, one can take u_n to lie in any given irreducible subfactor $P \subset M$. This implies that if a II_1 factor N can be embedded into R^ω, then it can be embedded in a way that makes it orthogonal to Q^ω, for any given subalgebra with infinite index $Q \subset R$. The results in this paper were motivated in part by Popa's discussions with Francois Le Maitre and Pierre Fima (Paris 7), who in fact rely on [P6] to carry on a research project of their own.

During his year in Paris, Popa in joint work with Claire Anantharaman have finalized writing a text book entitled "An introduction to II_1 factors" (see [AnP]), a project on which they have been working for several years. There has recently been much demand for an introductory book on this subject, coming from both the operator algebra community, but also from the related areas of group theory, ergodic theory, logic, etc. The book will appear in the Cambridge University Press.

During the 2016-2017, Popa disseminated his *deformation rigidity* and *approximate embedding* ("simulation") techniques in a series of lectures and courses at College de France (November 2016), IMJ (October 2016) and University of Paris-Sud (March-April 2017).

Entitled "Approximating freeness under constraints, his graduate course at Orsay in Sring 2017 has been followed by 8-10 students and young mathematicians. lt presented a method for constructing Haar unitaries u in a subalgebra B of a von Neumann II_1 factor M that are "as independent as possible with respect to a given finite set of elements in M. This technique,

which is called *incremental patching*, had most surprising applications over the years, e.g., to Kadison-Singer type problems, to proving vanishing cohomology results for II_1 factors, as well as to subfactor theory.

Also, Popa was involved in organizing several activities during his 2016-2017 stay, notably the Operator Algebra Seminar at Paris 7 (jointly with A. Zuk and M. Hilsum) and the Conference "von Neumann Algebras and Measured Group Theory", July 3-7, 2017, at IHP in Paris (with Damien Gaboriau, Cyril Houdayer and Stefaan Vaes).

Thus, using the funds from his FSMP Chair, Popa was able to invite to talk in the Operator Algebra Seminar at Paris 7 several prestigious specialists in the areas he Operator Algebra, Group Theory and Ergodic Theory: Stewart White (December 2016), Wilhelm Winter and Joachim Cuntz (January 2017), Alain Valette and Wolfgang Luck (March 2017), Bachir Bekka and Andreas Aaserud (April 2017), Miguel Walsh (May 2017).

The Conference co-organized by Popa in July 2017 at IHP largely benefitted from funding from his FSMP Chair. The conference has been an immense success, with almost a hundred participants and lectures of exceptional quality (including four ICM 2018 speakers!).

References

[APV] A. Aaserud, S. Popa, S. Vaes: *On the classification of regular subalgebras in* II_1 *factors*, in preparation.

[AP] A. Aaserud, S. Popa: *Approximate equivalence of groups actions*, arXiv:1511.00307, to appear in Ergodic Theory and Dynamical Systems.

[AnP] C. Anantharaman, S. Popa: "An introduction to II_1 factors", http://www.math.ucla.edu/ popa/books.html

[G] D. Gaboriau: *Invariants* ℓ^2 *de rélations d'équivalence et de groupes*, Publ. Math. I.H.É.S. **95** (2002), 93-150.

[HI] C. Houdayer, Y. Isono: *Bi-exact groups, strongly ergodic actions and group measure space type III factors with no central sequences*, arXiv:1510.07987.

[HP] C. Houdayer, S. Popa: *Singular MASAs in type* III *factors and Connes' bicentralizer problem*, math.OA/1704.07255, to appear in Proceedings of the 9th MSJ-SI Operator Algebras and Mathematical Physics.

[KV] A-S. Krogager, S. Vaes: *Thin* II_1 *factors with no Cartan subalgebras*, arXiv:1611.02138

[OP] N. Ozawa, S. Popa: *On a class of* II_1 *factors with at most one Cartan subalgebra*, Annals of Mathematics **172** (2010), 101-137 (math.OA/0706.3623)

[P1] S. Popa: *On a problem of R.V. Kadison on maximal abelian *-subalgebras in factors*, Invent. Math., **65** (1981), 269-281.

[P2] S. Popa: *Singular maximal abelian *-subalgebras in continuous von Neumann algebras*, J. Funct. Analysis, **50** (1983), 151-166.

[P3] S. Popa: *Some properties of the symmetric enveloping algebras with applications to amenability and property* (T), Documenta Mathematica, **4** (1999), 665-744.

[P4] S. Popa: *Strong Rigidity of* II_1 *Factors Arising from Malleable Actions of w-Rigid Groups* I, Invent. Math., **165** (2006), 369-408.

[P5] S. Popa: *Constructing MASAs with prescribed properties*, math.OA/1610.08945, to appear in Kyoto Journal of Mathematics.

[P6] S. Popa: *Asymptotic orthogonalization of subalgebras in type* II_1 *factors*, math.OA/1707.07317

[PSV] S. Popa, D. Shlyakhtenko, S. Vaes: *Cohomology and* L^2*-Betti numbers for subfactors and quasi-regular inclusions*, math.OA/1511.07329 , to appear in the International Mathematical Research Notices.

[PV] S. Popa, S. Vaes: *Unique Cartan decomposition for* II_1 *factors arising from arbitrary actions of free groups*, Acta Mathematica, **194** (2014), 237-284