Dynamical Geometry and Analysis in Orsay

One of the main research areas of contemporary geometric analysis concern the study of the dynamics of geometric flows. Their use lead to breakthrough results such as the solution of the Poincaré conjecture by Grigory Perelman. Geometric flows such as the Ricci flow, the Kähler-Ricci flow, its modifications and generalisations such as the Chern-Ricci and the pluriclosed flows play a key role in the field. Recent progress have been made by Bamler and Tian-Zhang on the convergence of the Kahler-Ricci flow over complex manifolds with positive first Chern class. These advances concern the celebrated Hamilton-Tian conjecture. A key technique needed for the solution of this conjecture is the Cheeger-Colding-Tian and Cheeger-Naber theory on the structure of the limit spaces and the uniqueness of their tangent cones. A major advance in this theory has been made by Cheeger-Naber with their solution of the codimension four conjecture on the Gromov-Hausdorf limit of noncollapsed manifolds with bounded Ricci curvature.

The theory initiated by Cheeger-Colding-Tian plays also a fundamantal role on the existence of Kähler-Einstein metrics over complex manifolds with positive first Chern class. It is expected that the Hamilton-Tian conjecture will lead to a different proof of their existence. For this purpose an important notion of modified algebraic stability have been introduced by B.Zhou-X.Zhu.

On a different direction Song-Tian and La Nave-Tian initiated a promising Analytical Minimal Model Program for the solution of the abaundance conjecture on algebraic varieties with intermediate Kodaira dimension. Their analytic program has attracted many researchers working in the field of complex analysis.

One objective of the workshop were to bring together leading experts from France, China as well as from International institutions. The other main objective was to bring together PhD students, post-docs and other young researchers from France and China. Some 60 people, from a great varieties of countries (U.S., Germany, Italy, Japan, Iran, Algeria... besides France and China), attended the workshop; more than a half of them were PhD students or post-docs. Fifteen high-quality talks were given, both in the Mathematics Department of Paris-Sud University in Orsay, and in IHES.

Dynamical Geometric Analysis in Orsay

(June 27th to June 30th, 2017)

Titles and Abstracts

- ***Steady Kaehler-Ricci solitons with nonnegatively holomorphic bisectional curvature*, by Xiaohua Zhu**, SMS and BICMR-Peking University.

Abstract: In this talk, we discuss a rigidity problem of steady Kaehler-Ricci solitons with nonnegatively bisectional curvature. By using the Ricci flow method, we show that any n-dimensional κ-noncollapsed steady Kaehler-Ricci soliton with nonnegatively holomorphic bisectional curvature must be flat.

- ***Generalized Kahler Ricci flow and a generalized Calabi conjecture*, by Jeffrey D. Streets**, University of California.

Abstract: Generalized Kahler geometry is a natural extension of Kahler geometry with roots in mathematical physics, and is a particularly rich instance of Hitchin's program of 'generalized geometries.' In this talk I will discuss an extension of Kahler-Ricci flow to this setting. After introducing the equation, I will formulate a natural Calabi-Yau type conjecture based on Hitchin/Gualtieri's definition of generalized Calabi-Yau equations. The main result is a global existence and convergence result for the flow which yields a partial resolution of this conjecture, and which classifies generalized Kahler structures on hyperKahler backgrounds.

- ***Quasi-local mass and isoperimetric inequality in General relativity*, by Yuguang Shi**, BICMR-Pekin University.

Abstract: Quasi-local mass is very important notion in General Relativity. Geometrically, it can be regarded as a geometric quantity of a boundary of a 3-dimensional compact Riemannian manifold. Usually, it is in terms of area and mean curvature of the boundary. It is interesting to see that some of quasi-local masses, like Brown-York mass, Hawking mass and isoperimetric mass have deep relation with classical isoperimetric inequality in Riemannian manifolds. I will discuss these problems in this talk which is based on some of my recent joint works in this direction.

- ***Sasaki-Einstein metrics and normalized volumes*, by Chi Li**, Purdue University.

Abstract: In this talk, we discuss a rigidity problem of steady Kaehler-Ricci solitons with nonnegatively bisectional curvature. By using the Ricci flow method, we show that any n-dimensional κ-noncollapsed steady

Kaehler-Ricci soliton with nonnegatively holomorphic bisectional curvature must be flat.

- ***Fully Nonlinear flows on complete graphs*, by Panagiota Daskalopoulos**, Colombia University.

Abstract: We will discuss the evolution of complete non-compact graphs by full-nonlinear geometric flows. Special emphasis will be given to the Gauss curvature flow, an example of degenerate diffusion and the Inverse Mean curvature flow, an example of ultra-fast diffusion.

- ***L^2 curvature estimates on manifolds with bounded Ricci curvature*, by Wenshuai Jiang**, Warwick University.

Abstract: In this talk, we will discuss the L^2 curvature estimates on noncollapsing Einstein manifolds. We also discuss some applications of the L^2 estimates. In order to show the L^2 estimates, we first introduce δ-neck region on the manifold, which is roughly the regular region of codimension four cone. Basing on several estimates on the neck region, we can decompose our manifolds into neck regions and regular balls. Moreover, the number of the neck regions and regular balls are well controlled. Combining the estimates on decomposition numbers with the curvature estimates on neck regions and regular balls, we finally prove the L^2 curvature estimates on noncollapsing Einstein manifolds. In this talk, we will discuss some details of the proof. This is a joint work with Professor Aaron Naber of Northwestern University.

- ***Asymptotic structure of self-shrinkers of mean curvature flow*, by Lu Wang**, University of Wisconsin Madison.

Abstract: Self-shrinkers are a special class of solutions to mean curvature flow and they are singularity models of the flow. In this talk, I will show that each end of a noncompact self-shrinker in three-dimensional Euclidean space of finite topology must be smoothly asymptotic at infinity to a regular cone or a round cylinder.

- ***Expanders of the harmonic map flow*, by Alix Deruelle**, Université Pierre et Marie Curie.

Abstract: Joint work with Tobias Lamm. Expanding self-similarities of a given evolution equation create an ambiguity in the continuation of the flow after it reached a first singularity. In this talk, we investigate the possibility of smoothing out any map from the n-sphere, n¿1, to another sphere, that is homotopic to a constant by a self-similarity of the harmonic map flow. To do so, in the spirit of Chen-Struwe, we introduce a one-parameter family of Ginzburg-Landau equations that exhibit the same homogeneity and once the existence of expanders for this family is granted, we pass to the limit. We also study the singular set of such solutions as well as the uniqueness issue when the initial map is already harmonic.

- ***Asymptotic expansions of holonomy*, by Erlend Grong**, University of Luxembourg.

Abstract: Given a principal bundle with a connection over a base manifold, we want to give asymptotic expansions for the holonomy of a loop given in terms of its length. The length of the loop is here determined by either a Riemannian or a sub-Riemannian metric on the base manifold. We give an explicit formula for how curvature and its covariant derivatives can be used to approximate the holonomy up to a certain order. Surprisingly, this order is larger in some sub-Riemannian manifolds compared to the Riemannian case. These results have been obtained in collaboration with professor Pierre Pansu.

- ***Global Generation in K-Stability*, by Sean T. Paul**, University of Wisconsin Madison.

Abstract: It is well known that the CM-line bundle lacks good positivity properties. The speaker will discuss his work on overcoming this by the introduction of a pair of globally generated line bundles on the base of any flat (smooth) family of projective varieties which cut out the "K-(semi)stable" locus. As one application we give a new proof of the main result of Ding and Tian's 1992 paper.

- ***Analytic Minimal Model Program through continuity method*, by Zhenlei Zhang**, Capital Normal University.

Abstract: In the talk, I will present an introduction to the La Nave and Tian continuity method to the Analytic Minimal Model Program (AMMP). It is in many ways similar to the Kahler-Ricci flow approach AMMP by Song and Tian. Then I will review some known results. The talk is based on the joint work with La Nave, Tian and Y. Zhang.