Localization phenomena in scattering and inverse problems for wave absorption by rough boundaries

It is well-known that rough (fractal) boundaries have improved noise absorption properties, and it is conjectured that for certain problems such boundaries may even be optimal absorbers. The main principle has many applications in acoustics (absorbing walls, anechoic chambers) and has already led to patented innovations\(^1\). While this ‘global’ effect of increased absorption by rough boundaries is understood to some extent, a detailed understanding of ‘local’ effects is still missing. On the other hand, the understanding of local effects would be extremely helpful in engineering applications, because it would allow to decompose the noise into parts and to predict which of these parts get absorbed with particularly high efficiency at which parts of the boundary. In concrete applications and building one could then focus on the relevant parts and reduce computation, production or building costs.

We wish to achieve three goals: The first goal is to provide a suitable decomposition of the noise signal. The second goal is to express the temporal decay of acoustical energy in terms of this decomposition and to see which configurations might be particularly efficient. The third goal is to analyze where the bulks of the separate parts are located; this is referred to as energy localization.

For self-adjoint operators, such as Laplacian with homogeneous Dirichlet boundary conditions\(^2\), comparable localization phenomena are well-known. However, for acoustical damping one needs to study Robin and mixed boundary value problems involving complex dissipation coefficients, and this requires the study of non-self-adjoint operators. In addition, the study of such boundary value problems on rough domains is quite nontrivial, because they involve integrals and operators on fractal boundaries. However, the tools we developed recently\(^3\) permit a suitable analysis.

We will develop a spectral theory for the non-self-adjoint dissipative operators corresponding to wave absorption (involving complex-valued coefficients) by a rough boundary. This involves the study of operators and their adjoints, and related modes, which can then be combined to obtain a Riesz basis. The temporal energy decay can then be expressed in terms of related mode expansions. We will investigate the mechanism of the astride localization\(^4\), i.e., the existence of modes both in lossy and lossless regions, an effect that makes the absorption particularly efficient. The study will be complemented by numerical linear algebra and numerical investigations on various geometry configurations including several material properties. We plan to extend Poincaré-Steklov operator on d-sets\(^5\) to larger classes of domains with rough boundaries both in the continuous space and in the discrete space. The understanding of the properties of these operators is expected to lead to new tools in scattering theory and imaging methods for irregular objects or inclusions.

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1 Fractal wall (TM), Brevet Colas - École Polytechnique 03/00881, noise abatement wall, US patent 7308965b2.