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« On the number of real zeros of structured random polynomials »

We plan to report on two recent results stating that structured (systems of) random polynomials typically only have few real zeros.

The first result is on random fewnomials: it says that a system of polynomials in n variables with a prescribed set of t terms and independent centered Gaussian coefficients has an expected number of positive real zeros bounded by $2 \binom{n}{t}$.

The second result is on Koiran's Real Tau Conjecture, which claims that the number of real zeros of a sum of m products of k real sparse univariate polynomials, each with a fixed set of at most t terms, is bounded by a polynomial in m, k, t . The Real Tau Conjecture implies Valiant's Conjecture $VP \neq VNP$. We have confirmed the conjecture on average: if the coefficients in these structured polynomials are independent standard Gaussians, then the expected number of real zeros is bounded by $O(mkt)$.

The proofs are based on the Rice formula and methods from integral geometry.

Joint work with Alperen Erguer, Josue Tonelli-Cueto and Irene Briquel