

Interactive Methods and Preference Elicitation for Solving Hard Multi-Objective Combinatorial Optimization Problems

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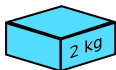
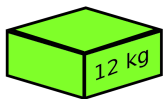
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Multi-Objective Knapsack Problem

- ▶ one knapsack:



- ▶ set of p items $\{1, 2, \dots, p\}$:



- ▶ one knapsack constraint:
$$\sum_{i=1}^p w_i x_i \leq W$$

- ▶ n objectives to maximize:
$$y_j(x) = \sum_{i=1}^p x_i y_j(i) \quad j = 1, \dots, n$$

- ▶ performance vector: $(y_1(x), y_2(x), \dots, y_n(x))$

Aggregation Functions

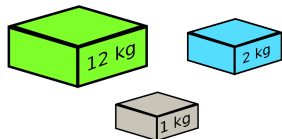
▶ Weighted Sums (WS): $WS(y(x), \omega) = \sum_{j=1}^n \omega_j y_j(x)$

▶ Ordered Weighted Averages (OWA):

$$OWA(y(x), \omega) = \sum_{j=1}^n \omega_j y_{(j)}(x)$$

$$\omega = (0.7, 0.2, 0.1)$$

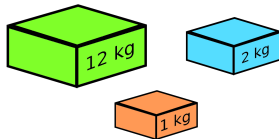
$$x^1 = (1, 1, 1, 0, 0) :$$



$$y(x^1) = (12, 3, 9)$$

$$OWA(y(x^1)) = 0.7 * 3 + 0.2 * 9 + 0.1 * 12 = 5.1$$

$$x^2 = (1, 1, 0, 0, 1) :$$



$$y(x^2) = (8, 7, 8)$$

$$OWA(y(x^2)) = 0.7 * 7 + 0.2 * 8 + 0.1 * 8 = 7.3$$

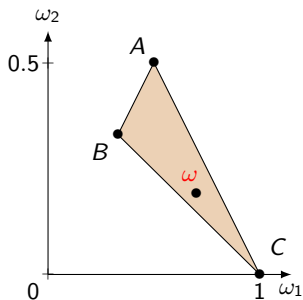
▶ Choquet Integrals

Preference imprecision

- ▶ The weight ω is unknown
- ▶ Θ : set of preference statements learned from the DM
→ a solution x is preferred to x' iff $f_\omega(y(x)) \geq f_\omega(y(x'))$.
- ▶ $\Omega_\Theta: \{\omega: \forall (a, b) \in \Theta, f_\omega(a) \geq f_\omega(b)\}$

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Elicitation

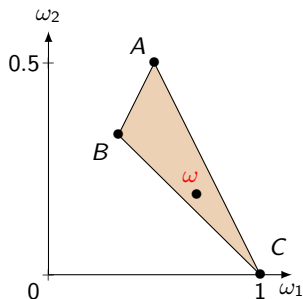
Incremental Elicitation e.g., [Boutilier et al., 2006]

Asking preference queries iteratively to reduce the set Ω_{Θ} until we can determine the best solution for the DM.

Elicitation

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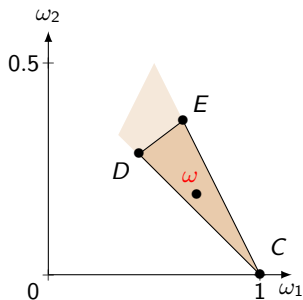
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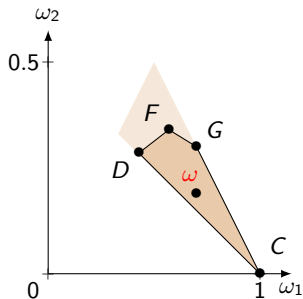
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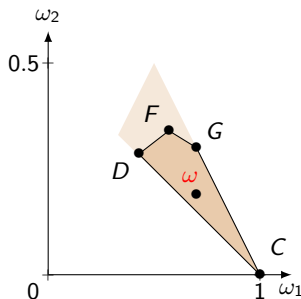
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Elicitation

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Asking preference queries iteratively to reduce the set Ω_Θ until we can determine the best solution for the DM.



Main novelty

To combine search and regret-based incremental elicitation for rank-dependent aggregators in combinatorial domains

Related papers

-  Nawal Benabbou, Cassandre Leroy, Thibaut Lust: An Interactive Regret-Based Genetic Algorithm for Solving Multi-Objective Combinatorial Optimization Problems. AAI 2020: 2335-2342
-  Nawal Benabbou, Cassandre Leroy, Thibaut Lust: Regret-Based Elicitation for Solving Multi-Objective Knapsack Problems with Rank-Dependent Aggregators. ECAI 2020: 419-426
-  Nadjat Bourdache, Patrice Perny, Oliveir Spanjaard : “Élicitation incrémentale de préférences par mise à jour Bayésienne sur des zones d’optimalité”, ROADEF 2020 - 21e congrès annuel de la société Française de Recherche Opérationnelle et d’Aide à la Décision, Montpellier, France (2020)