# Online-Learning for min-max discrete problems

Beyond COmpetitive Analysis and On-line Learning (BeCOOL)

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# Context

For $t = 1, 2, \ldots, T$:

1. The player picks any feasible action $x^t \in \mathcal{X}$.
2. The adversary reveals any objective function $f^{\,t} \in \mathcal{F}$.
3. The player suffers loss $f^{\,t}(x^t)$.

Cost: $loss(\mathcal{A}) = \sum_{t=1}^{T} f^{\,t}(x^t)$

Benchmark: Minimize the *regret*, defined as:

$$R_T(\mathcal{A}) = |loss(\mathcal{A}) - \sum_{t=1}^{T} f^{\,t}(x^*)|$$

where $x^* = \arg\min_{x \in \mathcal{X}} \sum_{t=1}^{T} f^{\,t}(x)$.

# Motivation

1. For linear objectives, any algorithm for the underlying problem translates to a vanishing-regret algorithm for its On-line Learning variant.

2. **Question:** Can similar results be achieved for non-linear objectives?

   e.g. On-line Learning MinMax Vertex cover.
   Action $\Longrightarrow$ Decide a vertex cover on a static graph.
   Adversary $\Longrightarrow$ Reveals the weights of the vertices.
   Cost $\Longrightarrow$ The weight of the "heaviest" vertex on the cover.

# What we have done:

1. Hardness results on the regret of any algorithm for the family of non-linear On-line Learning problems, called *MinMax problems*.
2. Separate the power of $FTL$-based methods and $GD$-based methods for On-line Learning in the case of non-linear objectives.
3. Apply the FTL method for an On-line Learning version of the *Knapsack Problem*.

Thank you!