

# Planning energy investment under uncertainty

## Primal and dual views

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Two-stage stochastic convex programming problem:

$$\begin{aligned} \min_{x,y} \quad & \mathbb{E}_{\xi}[f_{\xi}(x, y)] \\ \text{s.t.} \quad & (x(\xi), y(\xi)) \in C(\xi), \text{ a.e. } \xi \\ & (x, y) \in \mathcal{N} \rightarrow \text{nonanticipative space} \end{aligned}$$

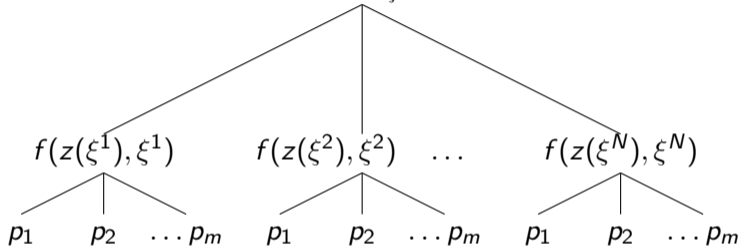
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**Objective:** decompose the problem using its structure.

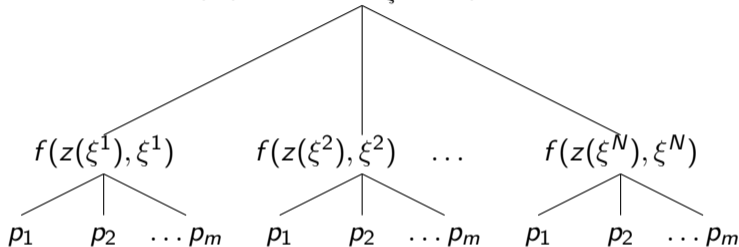
- Decomposition for different scenarios.
- Decomposition for different technologies.

$$\mathbb{E}_{\xi}[f_{\xi}(x, y)] = \sum_{\xi} \mathbb{P}(\xi) f_{\xi}(x, y)$$



- Progressive Hedging  $\rightarrow$  separate the problem for different scenarios. ([Roc18]).
  - Scenario subproblems with coupling constraints.

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- Solve the dual scenario subproblems (separable for different plants) with a proximal bundle method ([Ber+17])



# Inexact Proximal Bundle Progressive Hedging (work in progress)

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Energy  
optimization  
problem

Dual-primal  
view

Primal-dual view

References

Primal-dual approach: Inexact Progressive Hedging *à la bundle*.

- Relax the nonanticipativity constraints  $\rightarrow$  Progressive Hedging approach.

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- Solving the dual problem with a proximal bundle method is equivalent to minimize an **Augmented Lagrangian** in primal variables:

$$w^+ \in \arg \min_{w \in \mathcal{N}^\perp} \left\{ \check{\phi}(w) - \frac{1}{2t} \|w - \hat{w}\|^2 \right\} \iff \begin{cases} \hat{\alpha} \in \arg \min_{\alpha} L_{\xi}(\alpha, \hat{w}(\xi)) \\ \hat{g}(\xi) = \hat{x}(\xi) - \mathbb{E}[\hat{x}] \\ w^+(\xi) = \hat{w}(\xi) - t\hat{g}(\xi) \end{cases}$$

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- We can introduce a **stopping test** measuring the estimates of primal feasibility and duality gap.
  - Price to pay: dual serious step sequence  $\rightarrow$  subsequential convergence to solutions.



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- Call the (inexact) oracle at  $w^+$ , and check **sufficient decrease** of the dual function ([OSL14]).



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