Abstract:

In this 250th anniversary year of the birth of Joseph Fourier, it behoves us to talk of frequency and spectral analysis! The lectures shall visit a number of different techniques that have been developed and applied in the last 30 years, to carry out what engineers and applied mathematicians commonly call time-frequency analysis; in different settings, this approach also goes by the name micro-local analysis. The goal is to decompose signals, functions and operators in ways that preserve, isolate or emphasize local features in both time (or space) and frequency (or momentum). Decompositions of this type can be viewed as analogous to standard music notation, which tells the musician which notes (= frequency information) to play when (= localization in time).

Tools used for time-frequency localization include, for instance, the so-called short-time Fourier transform as well as wavelets and curvelets; both tools have applications that range widely, and that include, to name a few, semiclassical approximations and estimates in quantum mechanics, image compression, new tools for art conservators, and filters used for gravitational wave detection. We will also discuss the important role of sparsity, a central concept not only in signal analysis (compressed sensing) but also in inverse problems and large-scale computation./////