# Cost of Elitist Selection in Discrete Black-Box Optimization

**Carola Doerr**

CNRS and Université Pierre et Marie Curie, Paris, France

**PGMO project "Towards a Complexity Theory for Black-Box Optimization" (2014 --- 2016)**

Anne Auger[1], Dimo Brockhoff[2], Benjamin Doerr[3], Carola Doerr[4], Nikolaus Hansen[1], Timo Koetzing[5], Johannes Lengler[6], Jonathan Rowe[7]

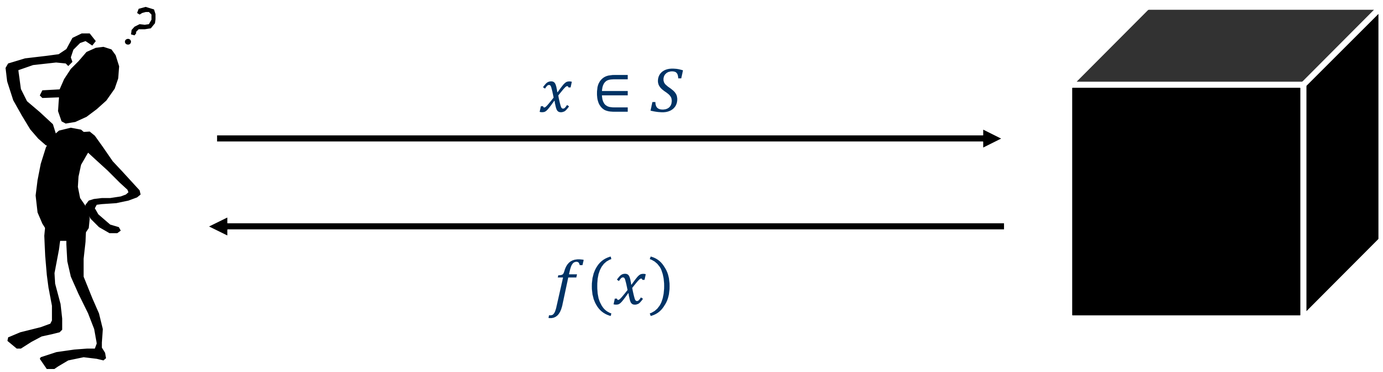
([1]INRIA Saclay, [2]INRIA Lille, [3]LIX, Ecole Polytechnique, [4]CNRS and LIP6, UPMC, [5]HPI Potsdam, [6]ETH Zürich, [7]U. of Birmingham)

# Goals of the Talk

- Give you an idea of what our project is about
- At the end of the talk, you should have an idea
    - what black-box complexity is
    - how it relates to runtime analysis
    - that both ideas complement each other well
  - about what kind of research questions we are working on in the project

# Black-Box Optimization

- Black-Box Setting: Goal is to optimize a function $f: S \rightarrow \mathbb{R}$




- Learn about $f$ only through queries
- Performance measure: worst-case (among all $f \in \mathcal{F}$) expected # of queries needed until an optimal search point is evaluated for the first time
- Motivation:
  - Large optimization problems with lots of data,
  - privacy issues,
  - …

# Runtime Analysis vs. Black-Box Complexity

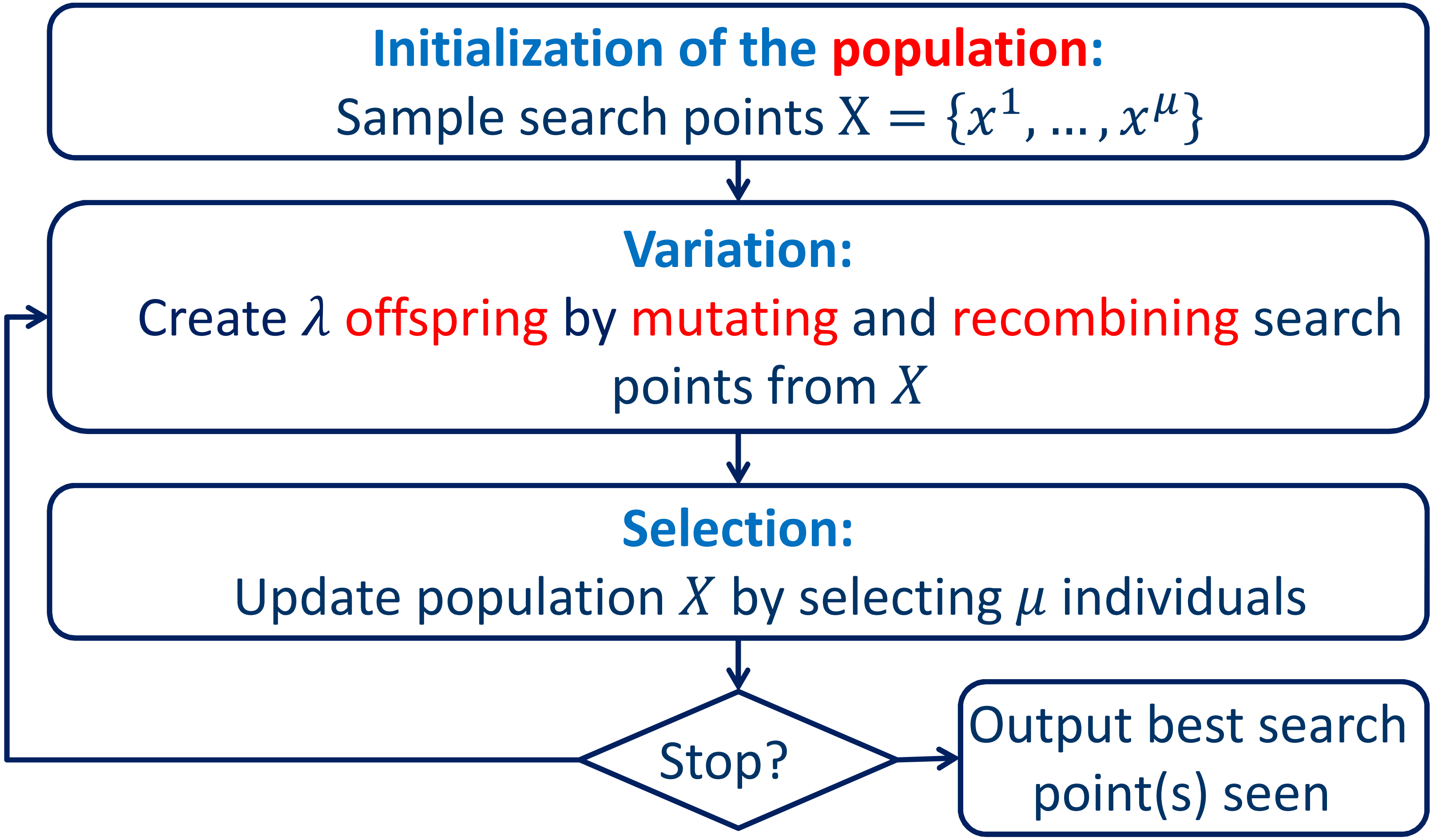

How long does Algorithm A need (in expectation) to optimize problem P?

How long does it take the fastest algorithm (in expectation) to optimize problem P?

- Objective of black-box complexity: understand how certain algorithmic choices influence the performance of black-box optimizers
- Algorithmic choices:
  - Size of the memory
  - Usage of absolute ("$f(x) = 17$")
    or only relative ("$f(x) > f(y)$") fitness information
  - …

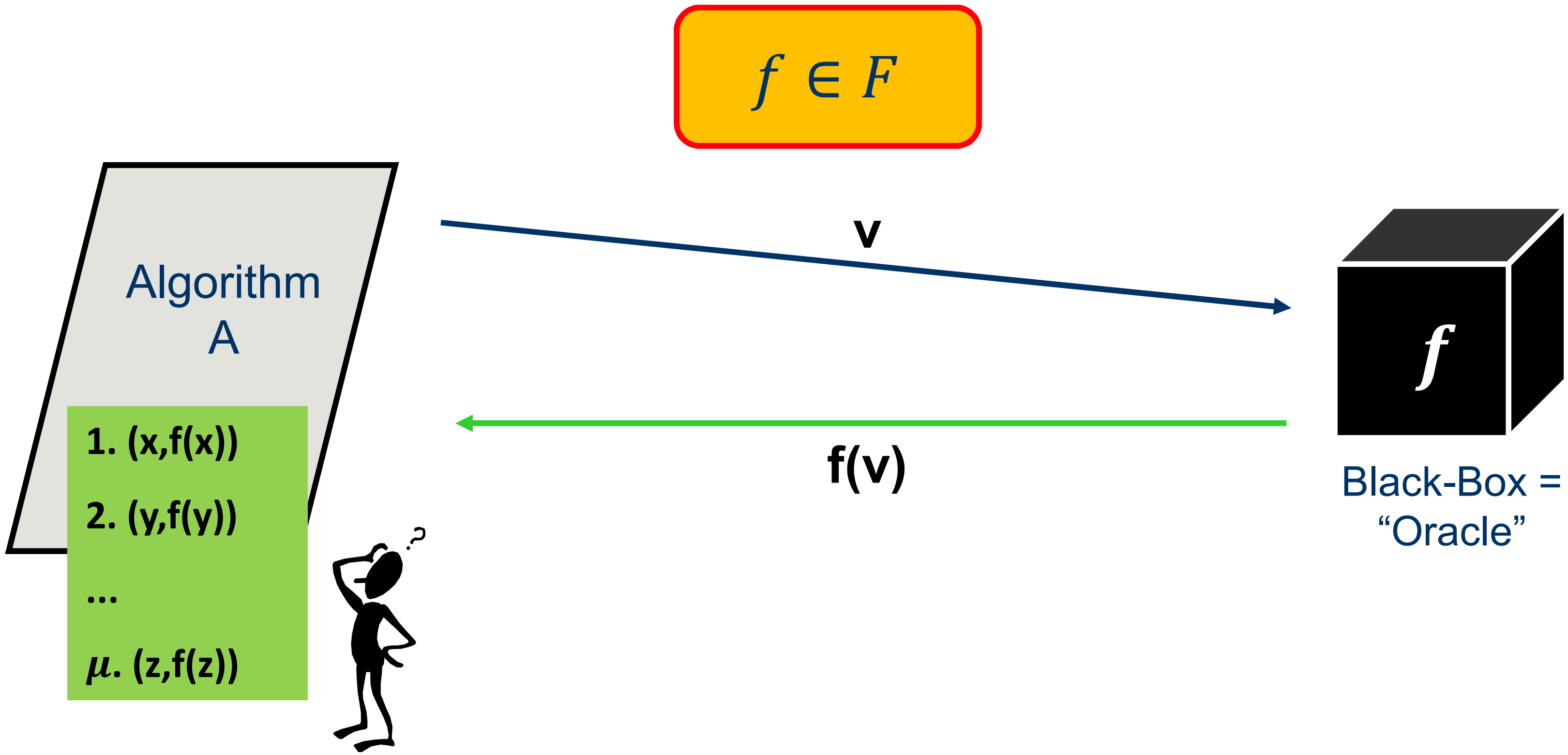
# Example: Memory Restriction

- Motivation: many black-box optimizers do not store full search history
- Example: Evolutionary Algorithms



# Memory-Restricted Black-Box Model

- Suggested in [Droste/Jansen/Wegener ToCS 2006]




- Research question in black-box complexity: how does the performance of memory-restricted algorithms compare to that of unrestricted ones?

# The Hamming Distance Problem

- Unknown target vector $z \in \{0,1\}^n$
- Goal is to learn $z$
- "Fitness" of a search point $x$ is "closeness of $x$ to $z$", i.e., $n - H(x,z)$
- Mastermind game from the 70s


$z$ = 1 1 1 1 1 0 0 0 0

$x$ = 0 1 0 1 1 0 1 0 1

$f_z(x) = 9 - 4 = 5$

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- How long do you need to solve this problem?

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# The Hamming Distance Problem

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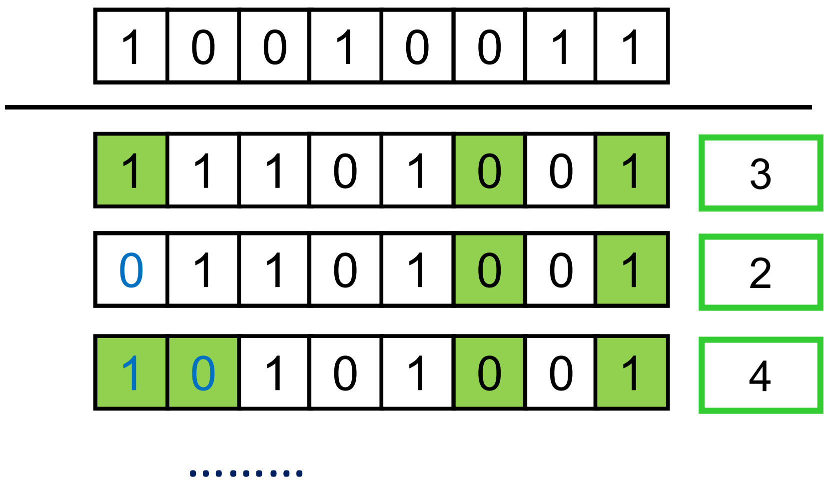
- How long do you need to solve this problem?
- …
- Hopefully, less than $n$+1 queries:



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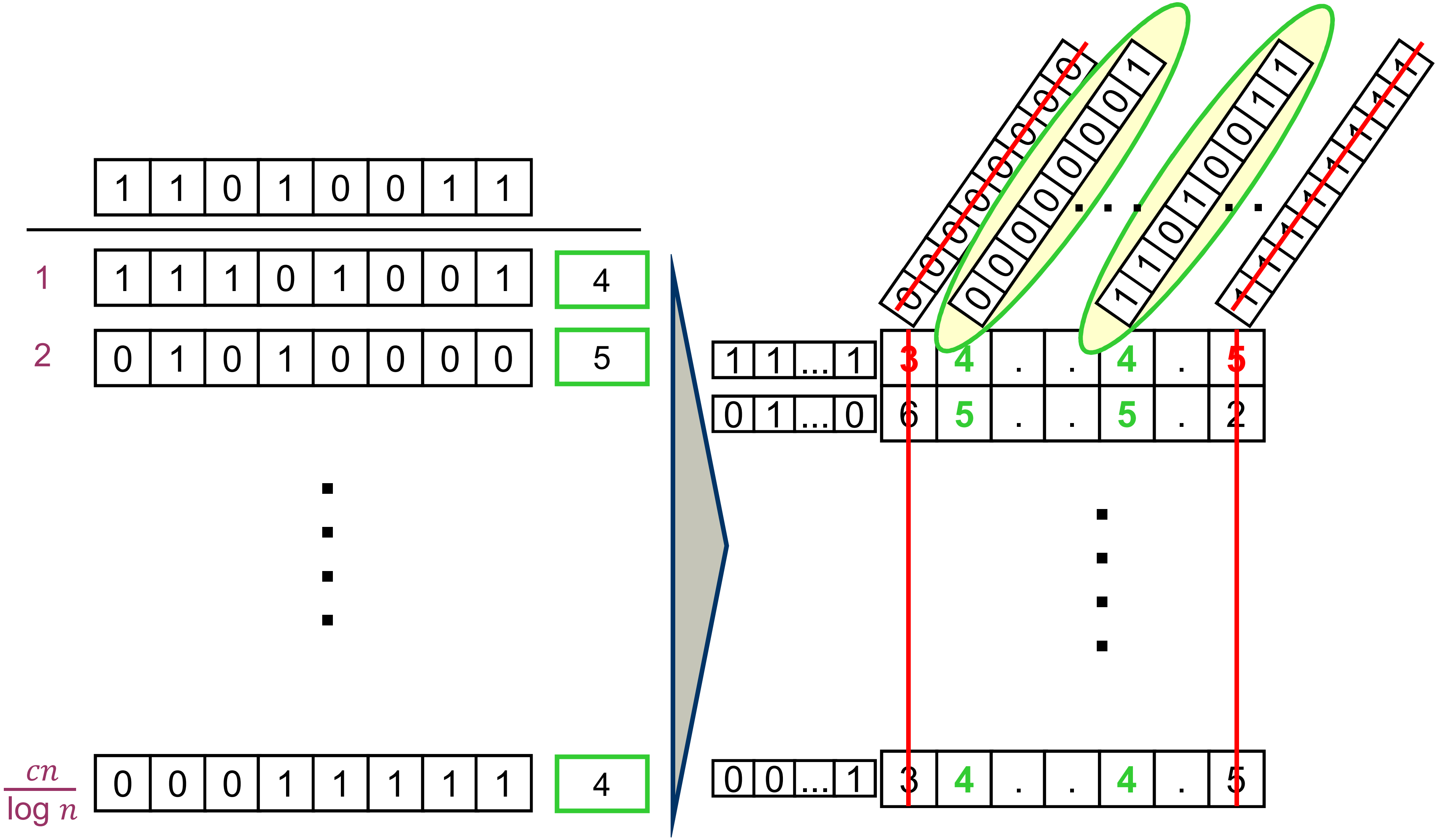
- [Erdős & Rényi, 1963]: one can do better: $O(n/\log n)$ strategy
- Their algorithm needs a lot of memory:

# The Algorithm by Erdős & Rényi / Chvátal



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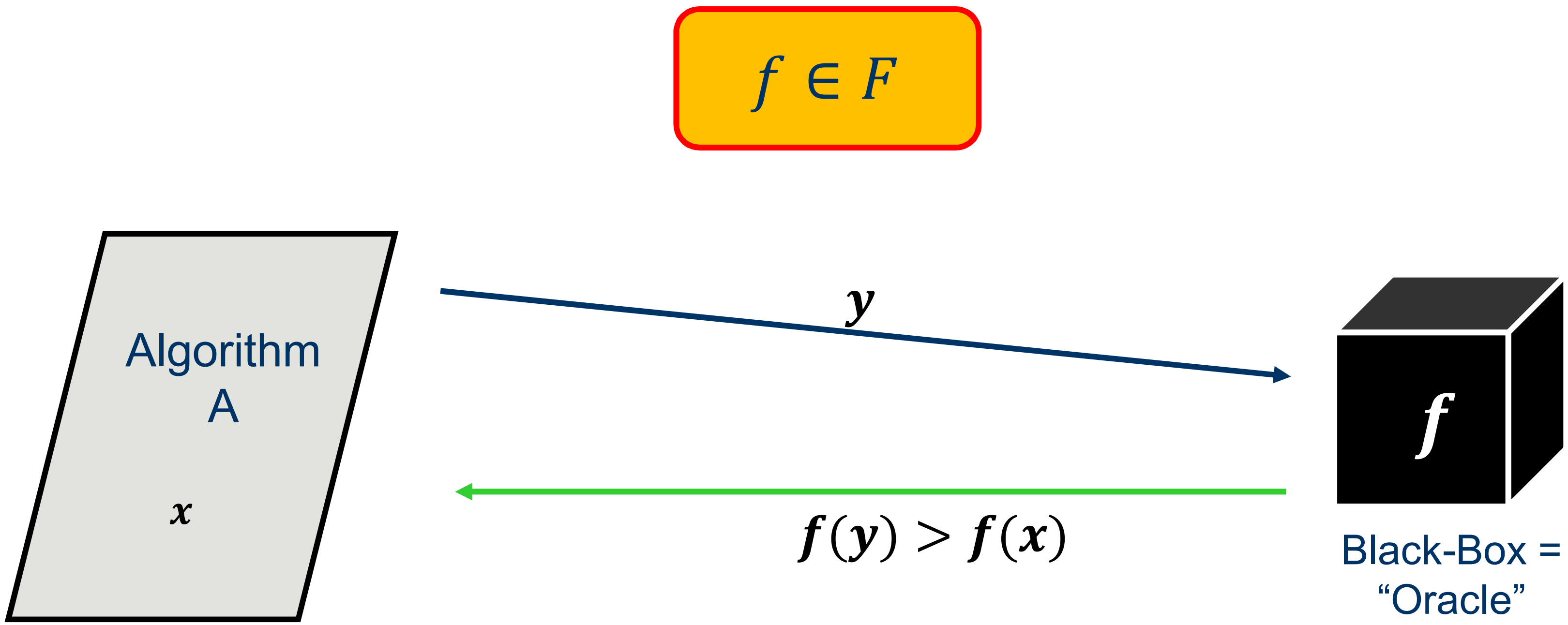
- How long do you need to solve this problem?
- …
- Hopefully, less than $n$+1 queries
- [Erdős & Rényi, 1963]: one can do better: $O(n/\log n)$ strategy
- Their algorithm needs a lot of memory
- Conjecture [Droste/Jansen/Wegener ToCS 2006]: when algorithm can only story one search point and its fitness, $\Omega(n \log n)$ queries are needed to solve this problem
- [Doerr/Winzen ToCS 2014]: falsified conjecture: $O(n/\log n)$ still possible

# The Comparison-Based Black-Box Model

- Suggested in [Teytaud/Gelly PPSN 2006] and [Doerr/Winzen Algorithmica 2014]
- Motivation: many black-box optimizers do not use *absolute* but rather *relative* fitness values (e.g., elitist selection, truncation selection,...)



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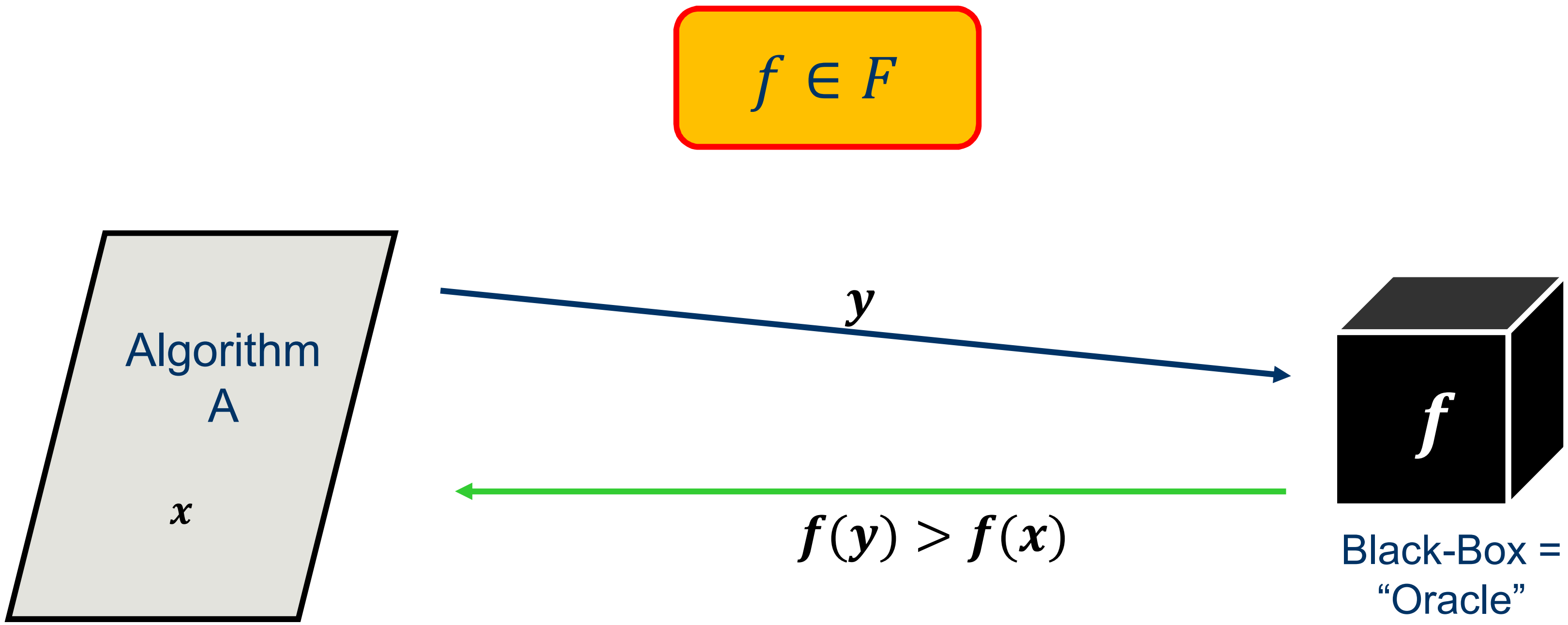
- Motivation: many black-box optimizers do not use *absolute* but rather *relative* fitness values (e.g., elitist selection, truncation selection,...)
- In the combined comparison-based and memory-restricted model (memory = 1 previous query), $\Theta(n)$ queries are needed to solve the Hamming distance problem

  → in this model, the simple $n$+1 strategy is asymptotically optimal!
  (implementing it in a way that uses only memory 1 is not so trivial)

# The Elitist Black-Box Model

- Suggested in [Doerr/Lengler GECCO 2015]
- Motivation: (on top of being memory-restricted and comparison-based) many black-box optimizers continue search in the most promising regions ("greedy" behavior)




- How does this influence the performance?

# Results for the Elitist Black-Box Model

- Hamming-distance problem:
  - algorithms that with high probability solve any instance in $\mathrm{O}(n)$ queries
  - we do not know any algorithm that needs $O(n)$ queries in expectation
    (rather philosophical question as $O(n)$ can be achieved by using restarts)
- For many other problems, elitist selection can cause huge performance gaps, e.g., $O(n^2)$ for non-elitist strategies but $\Omega(2^n)$ for elitist ones

# Our Project

- Analysis and development of black-box models
  → understand influence of algorithmic choices on performance
- Development of mathematical tools supporting the analysis
- Promotion of black-box complexity as research topic

- New project (2016-17): Parameter Optimization via Drift Analysis