The expanding search ratio of a graph

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**Rutgers Business School and LSE

Background

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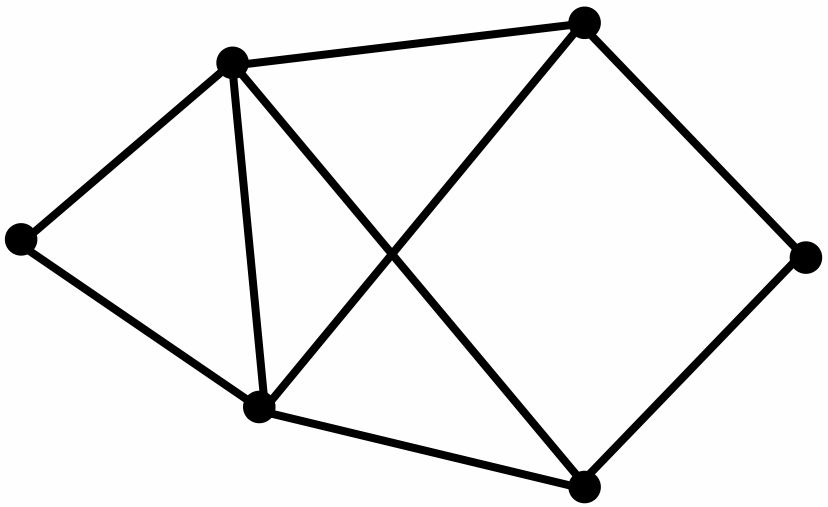

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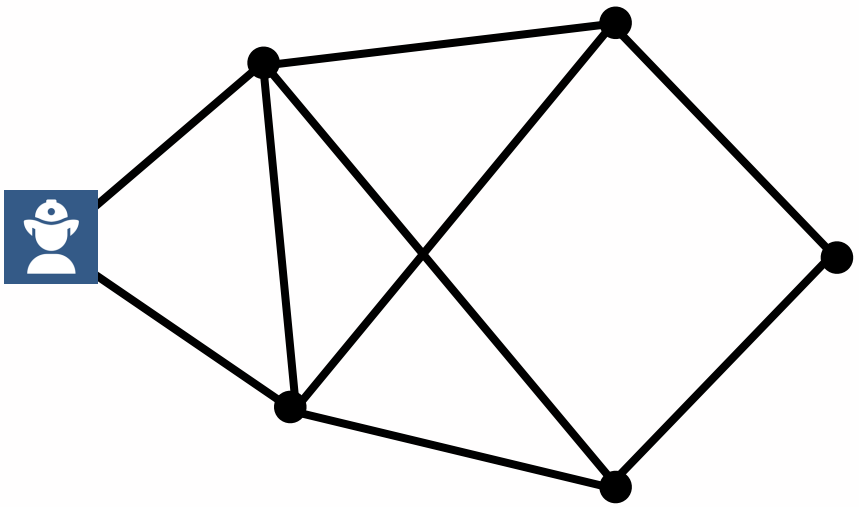

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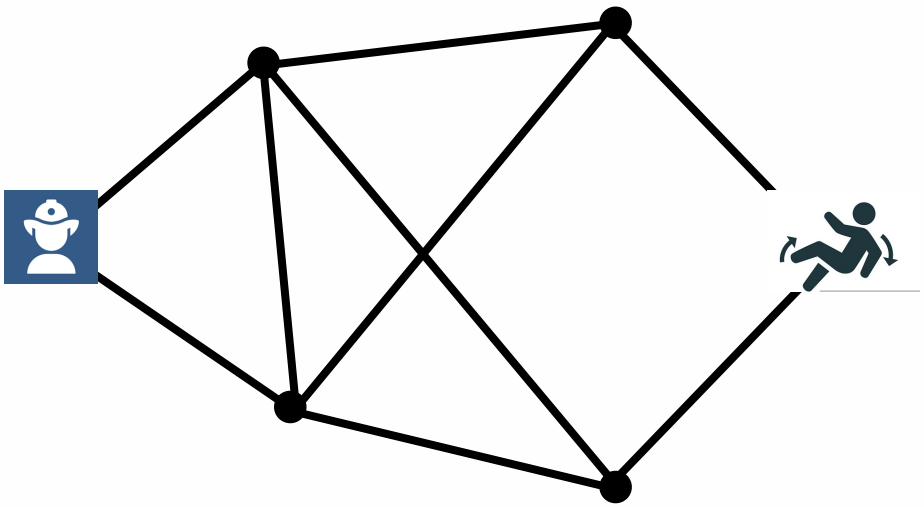

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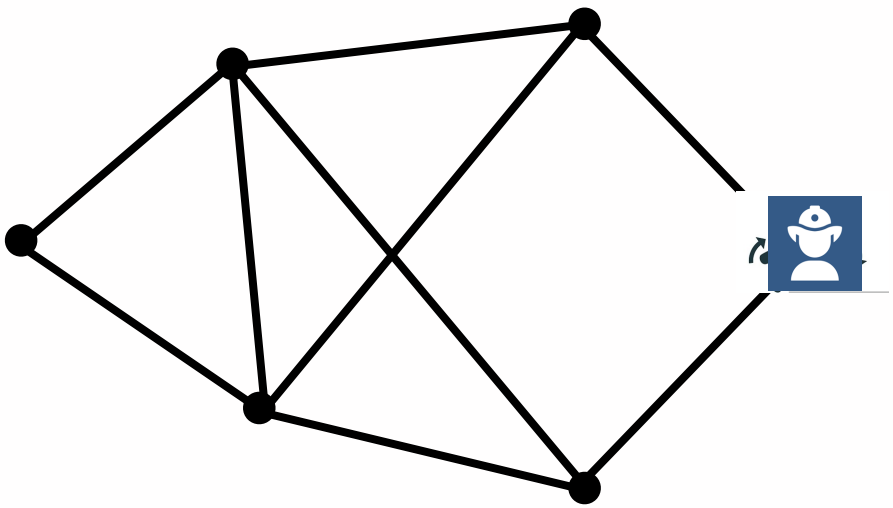

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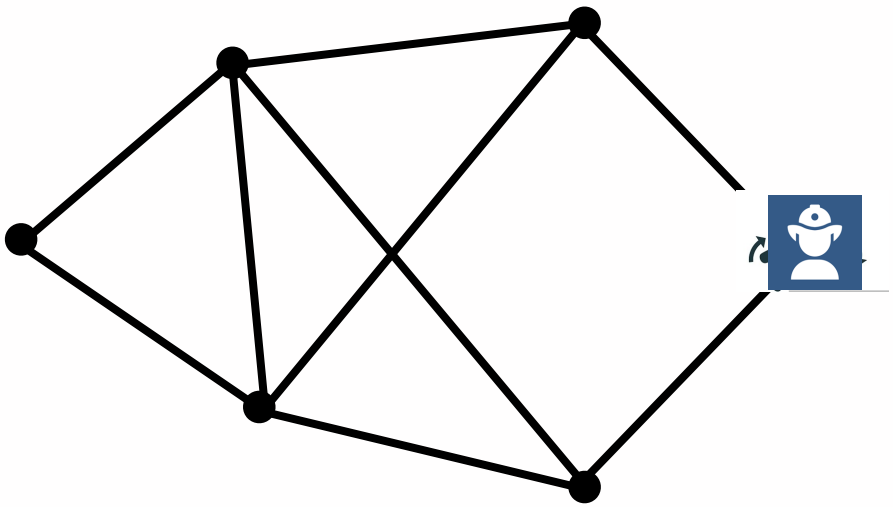

How do we define the search cost?

What is the performance measure?

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Searching a Fixed Graph
[Koutsoupias, Papadimitriou, Yiannakakis 1996]

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Pathwise search

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Expanding search

Sum of (weighted) search times

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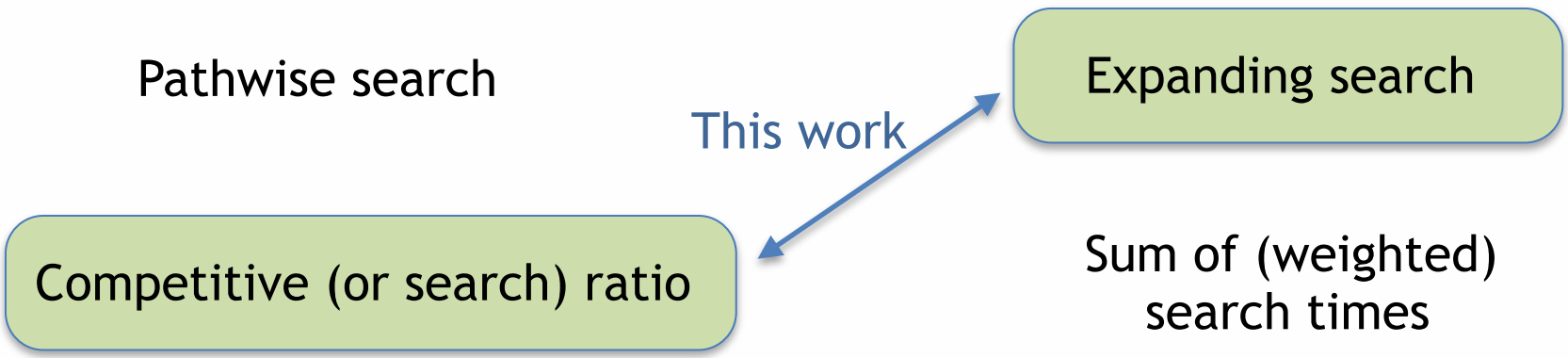
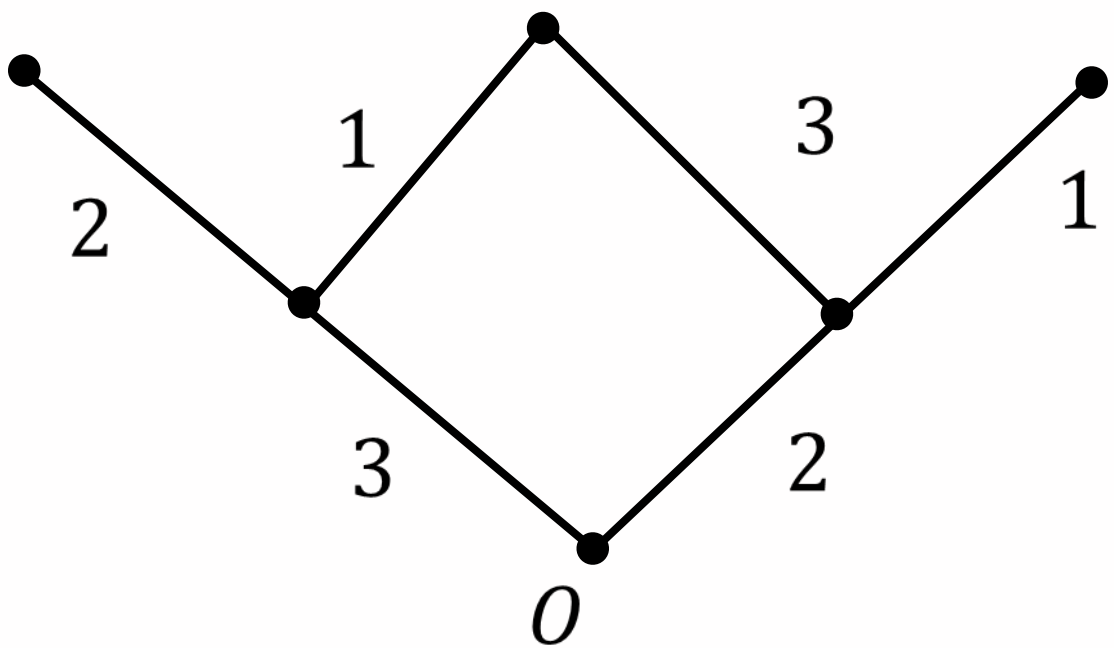
Pathwise search



Sum of (weighted) search times

Expanding search

An expanding search of a (weighted, connected) graph with origin O is a sequence of edges each one of which is incident to a previously searched vertex



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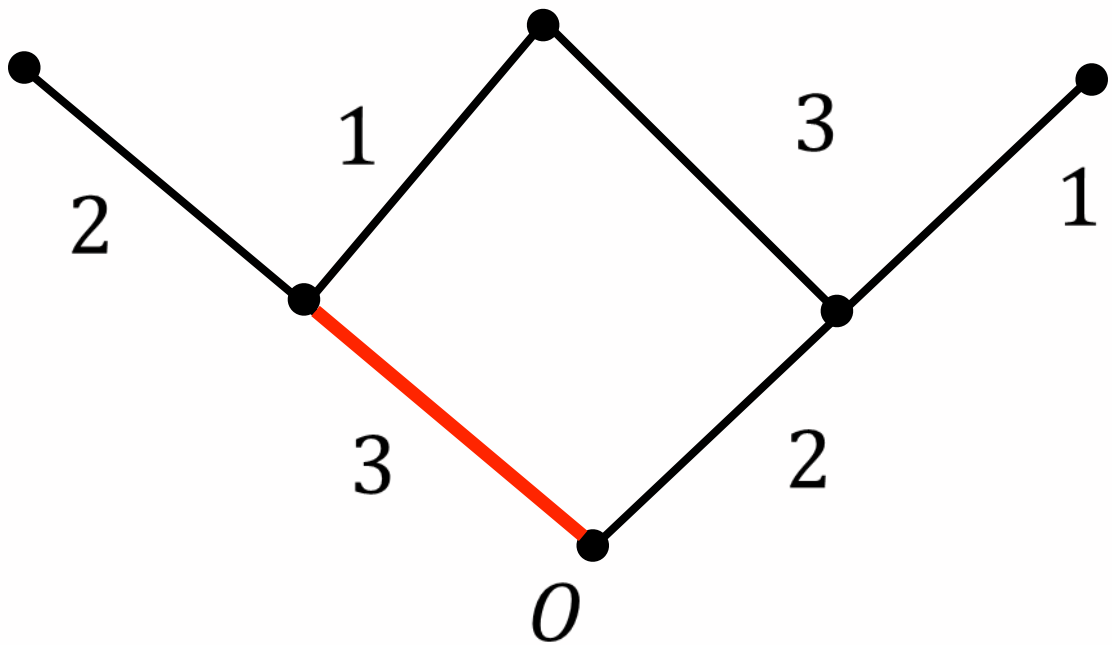

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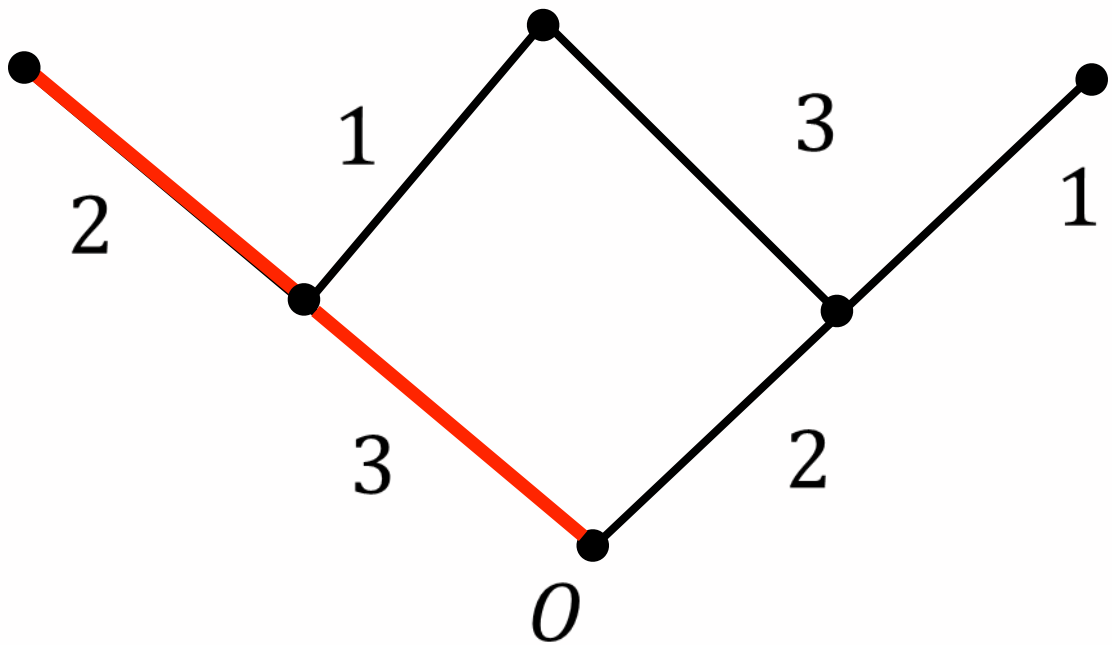

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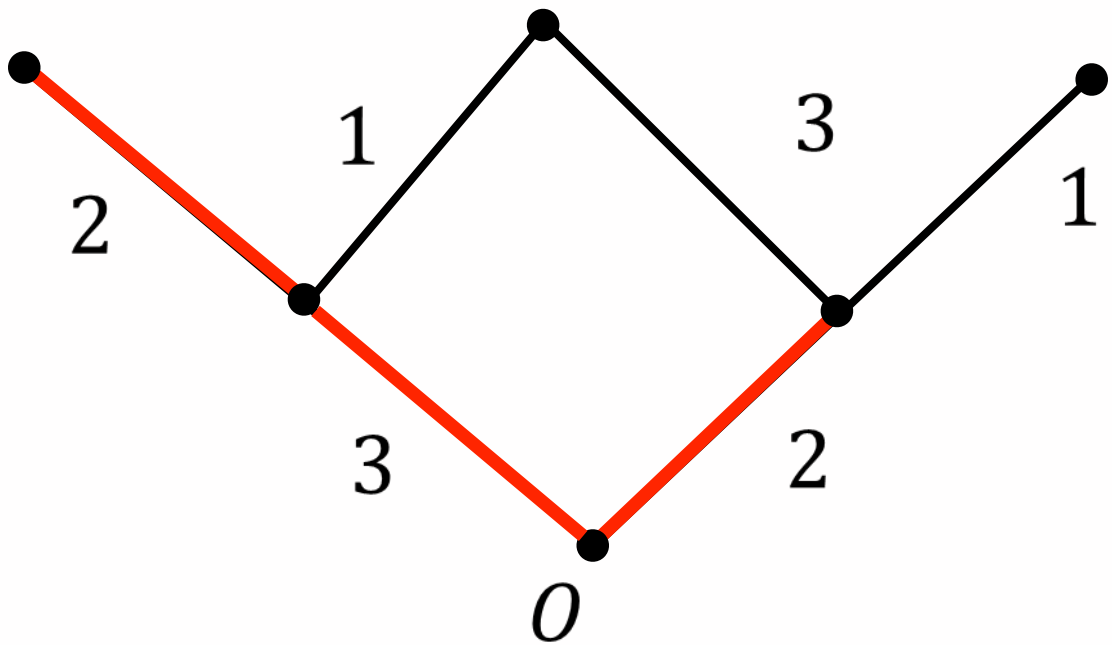

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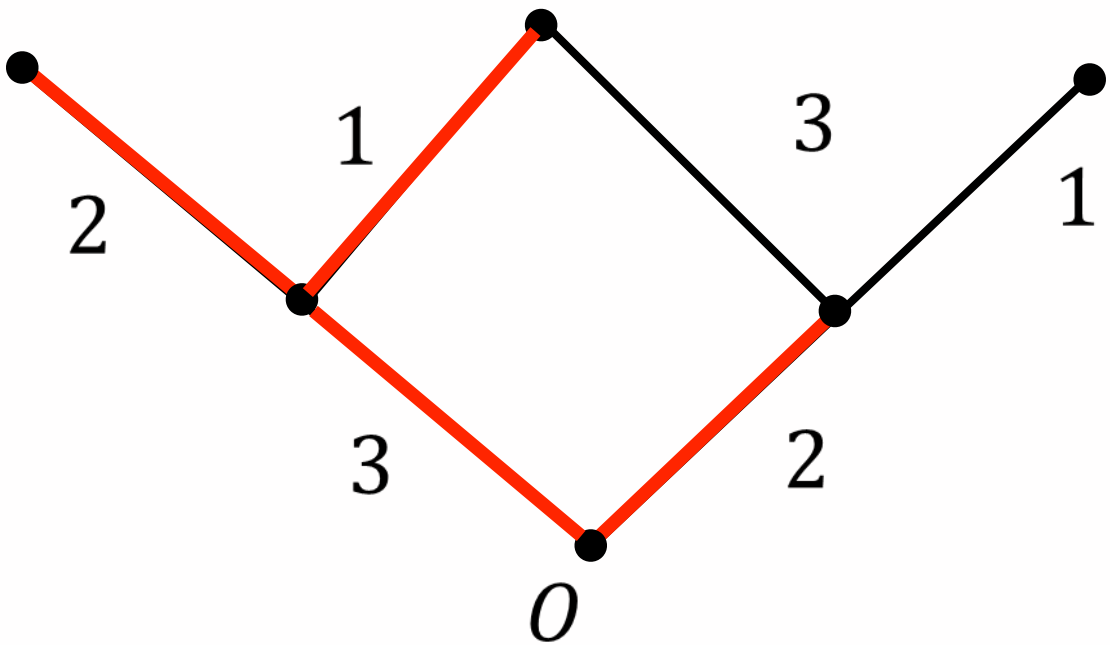

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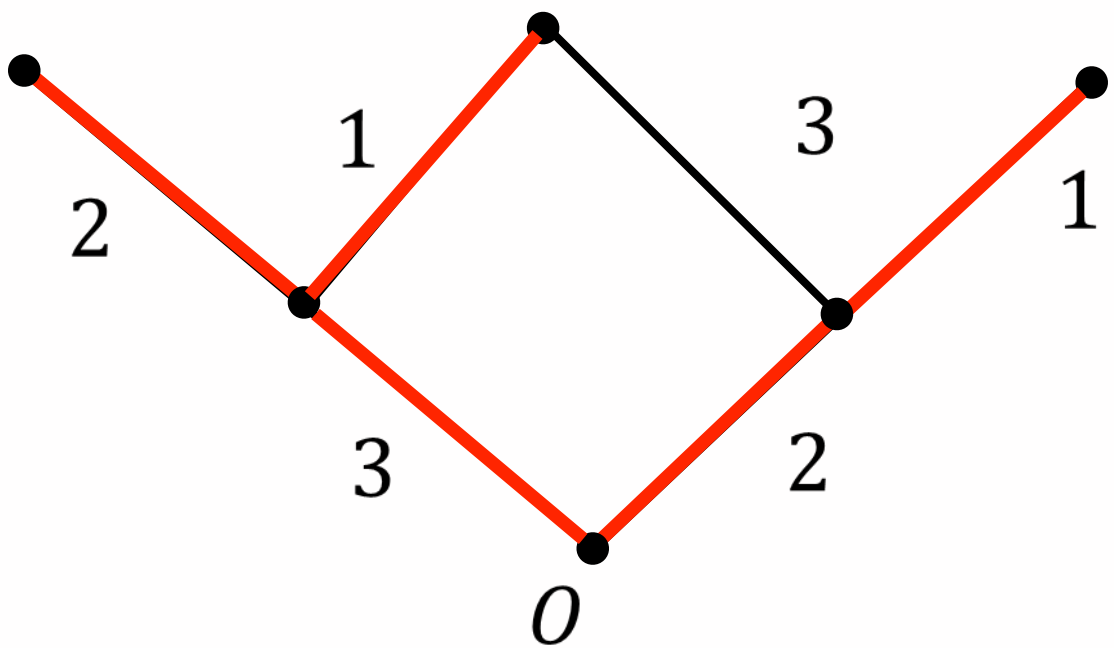
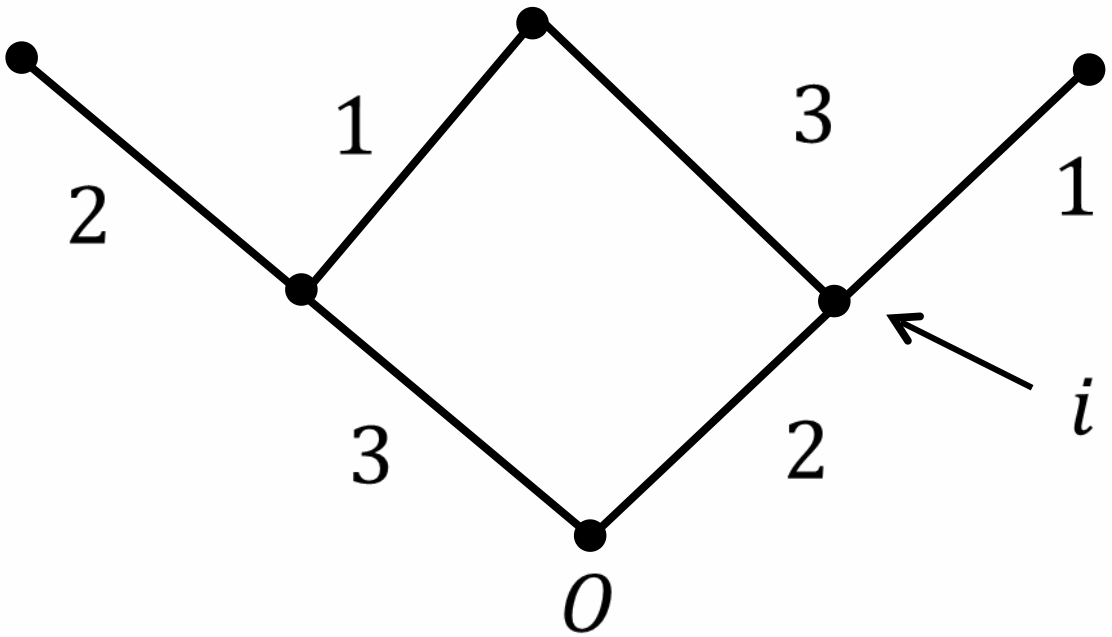
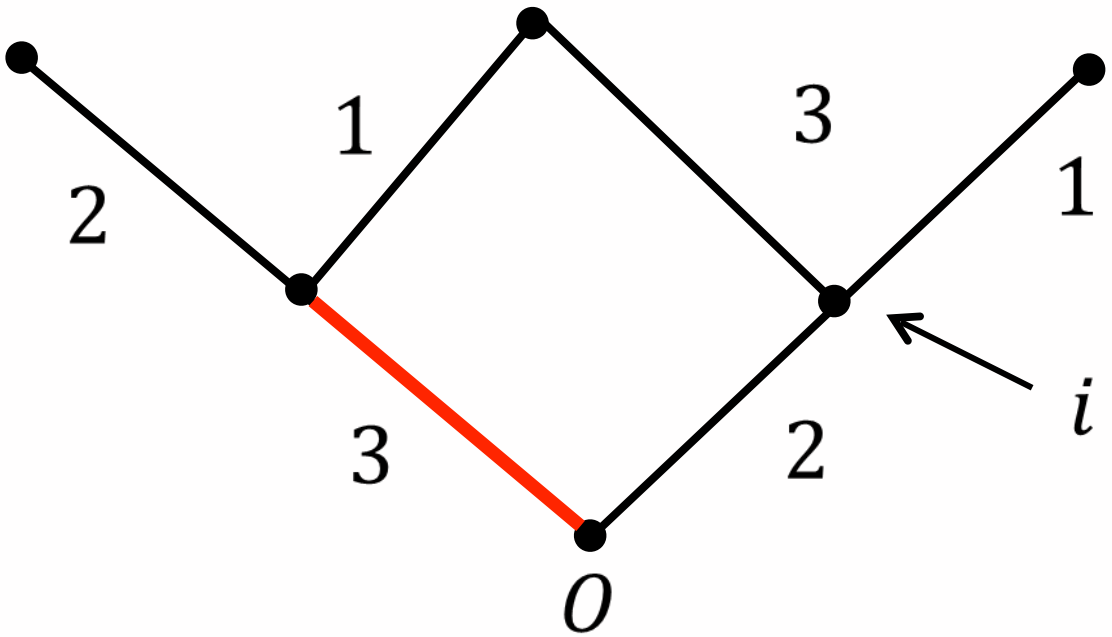


Search time



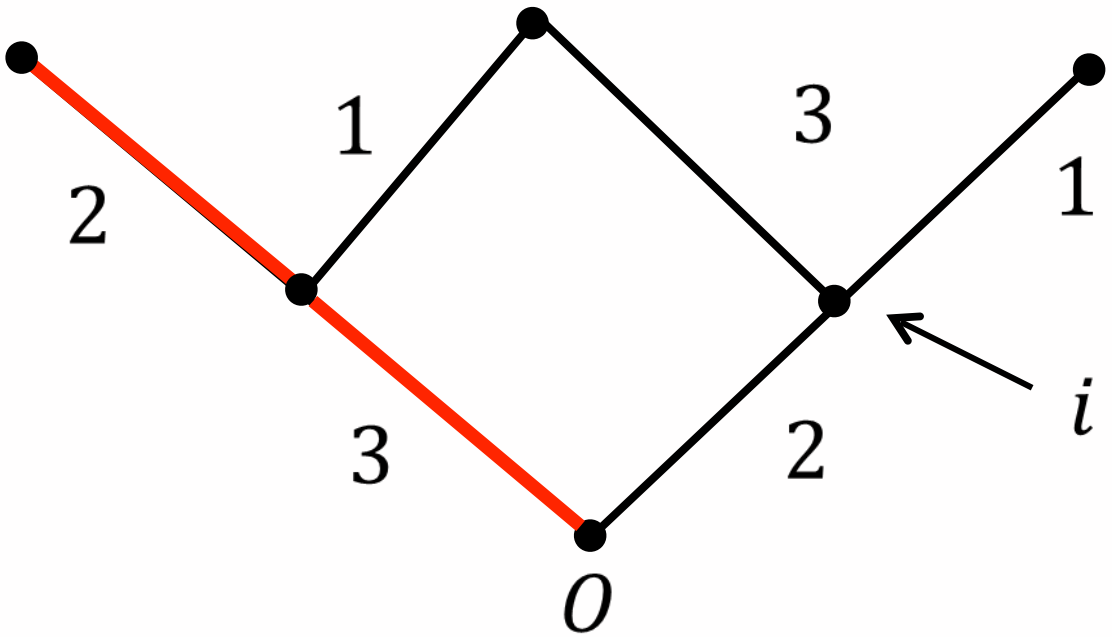
For a search S and vertex i, the search time $T(S, i)$ is the time i is first discovered E.g., $T(S, i) =$

Search time



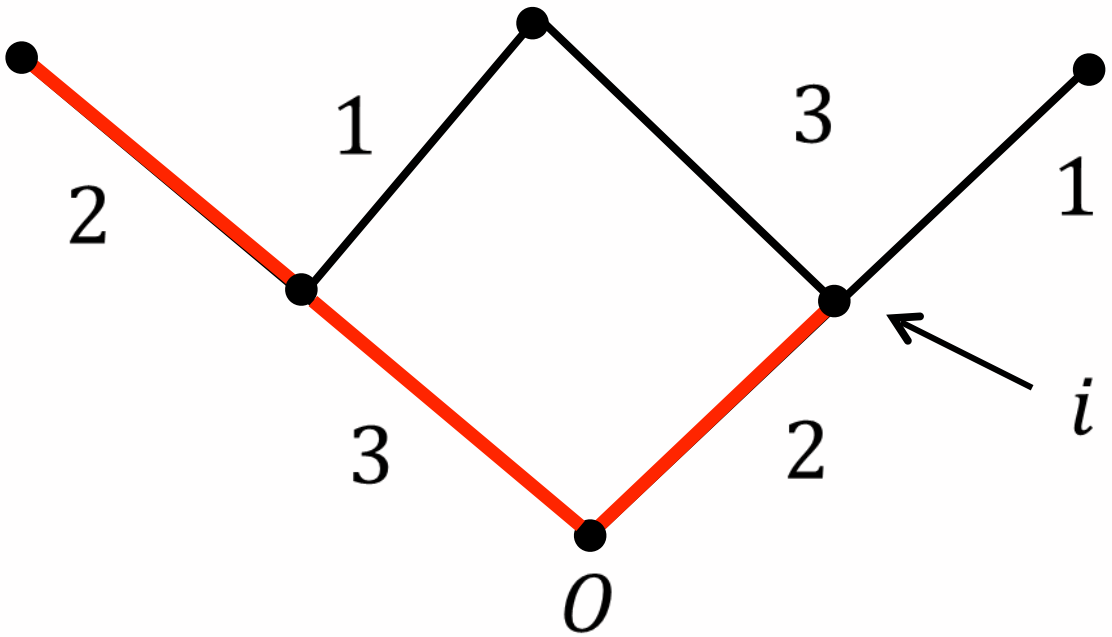
For a search S and vertex i, the search time $T(S, i)$ is the time i is first discovered E.g., $T(S, i) = 3$

Search time



For a search S and vertex i, the search time $T(S, i)$ is the time i is first discovered E.g., $T(S, i) = 3+2$

Search time



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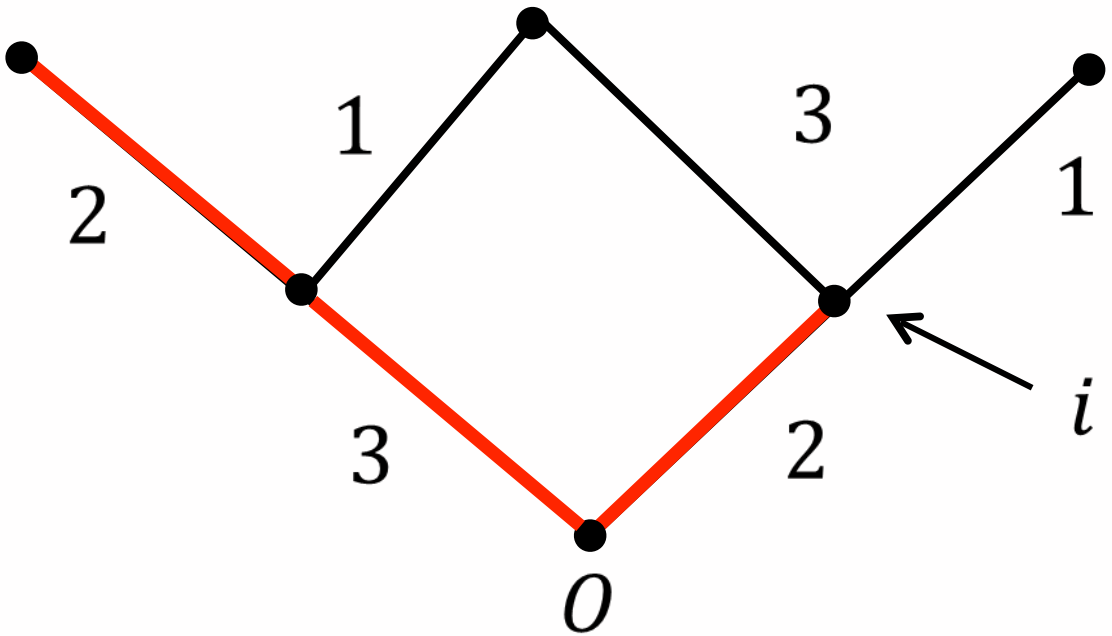
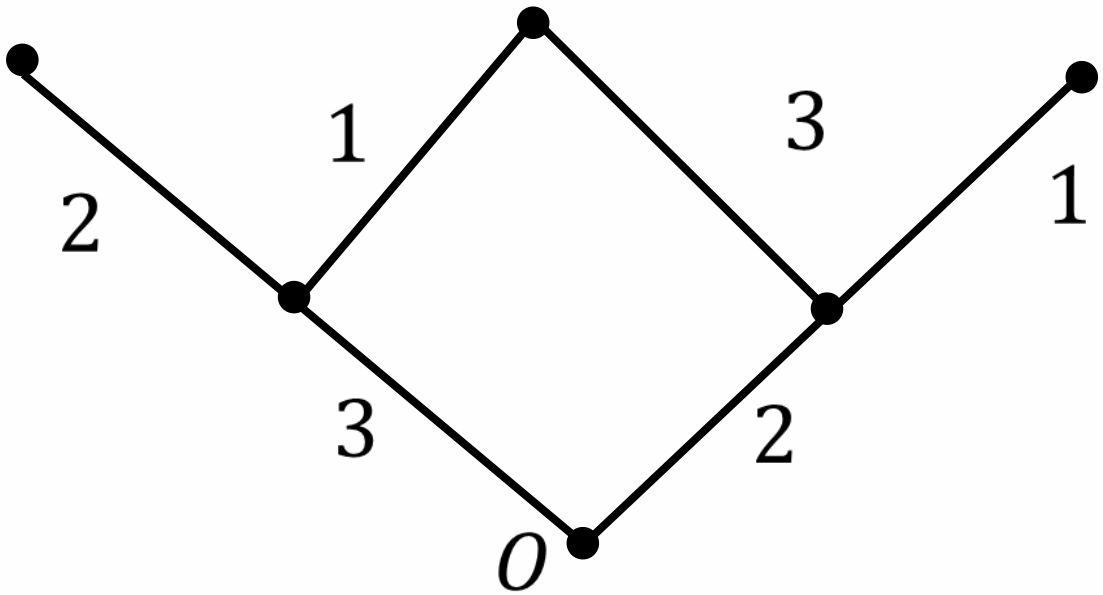


For a search S and vertex i, the search time $T(S,i)$ is the time i is first discovered E.g., $T(S,i) = 3+2+2 = 7$

The normalized search time $\hat{T}(S,i)$ is defined as $T(S,i)/d(O,i)$
E.g., $\hat{T}(S,i) = 7/2$

Search ratio



The search ratio σ_S of S is $\max_i \hat{T}(S,i)$

The search ratio σ of G is $\min_S \sigma_S = \min_S \max_i \hat{T}(S,i)$

If a strategy S minimizes the search ratio it is optimal

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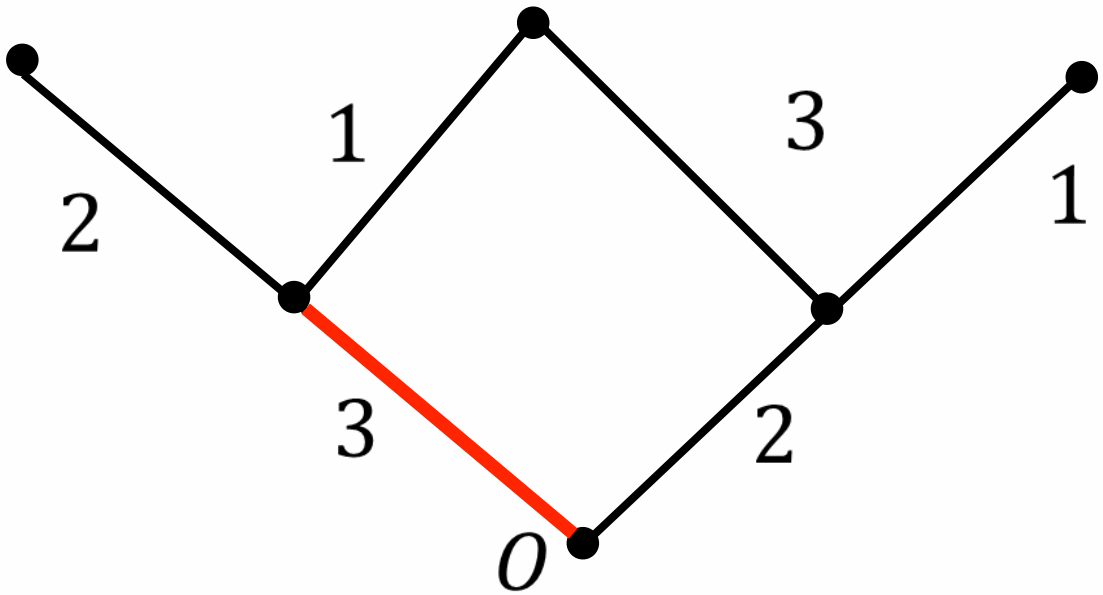

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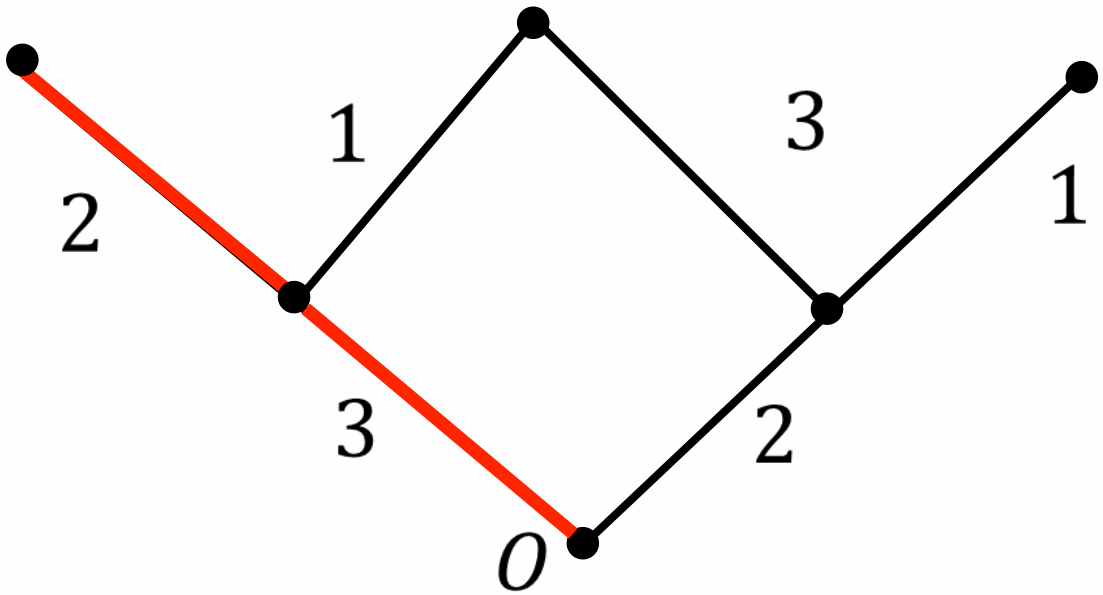

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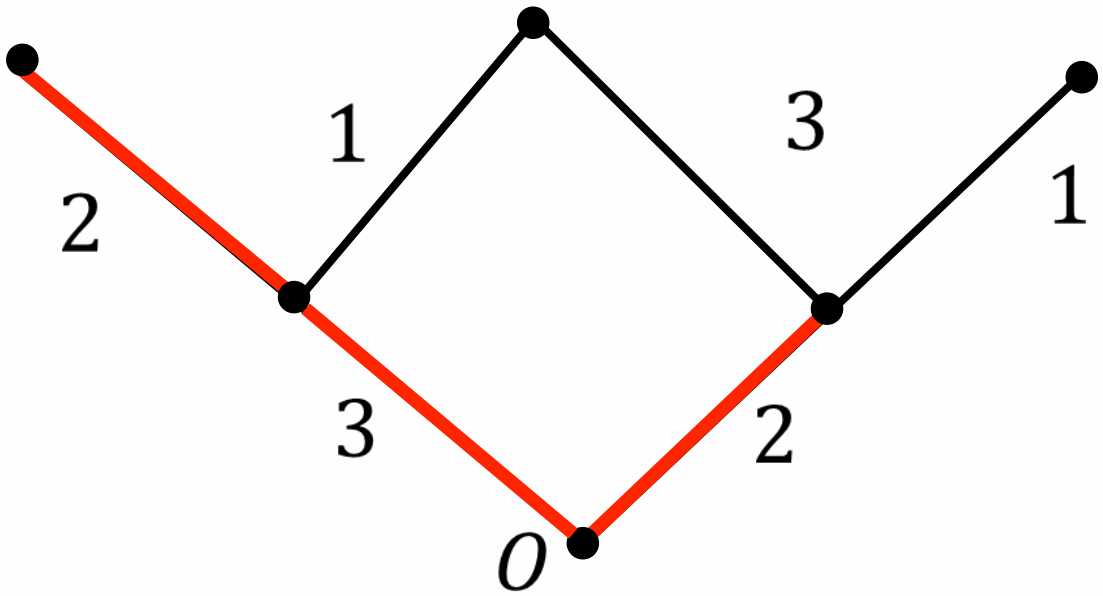

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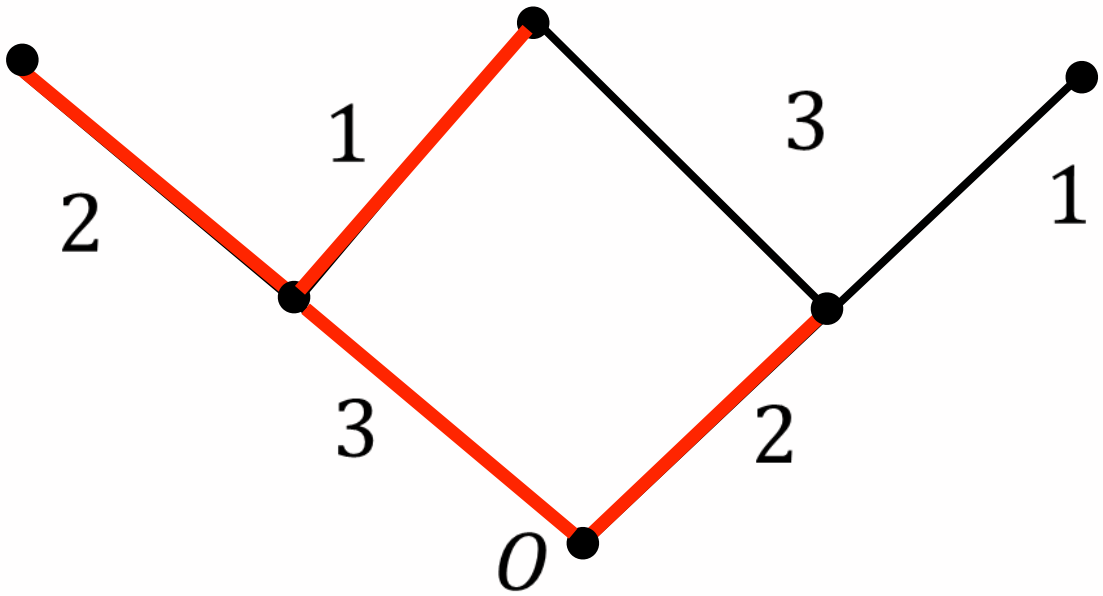

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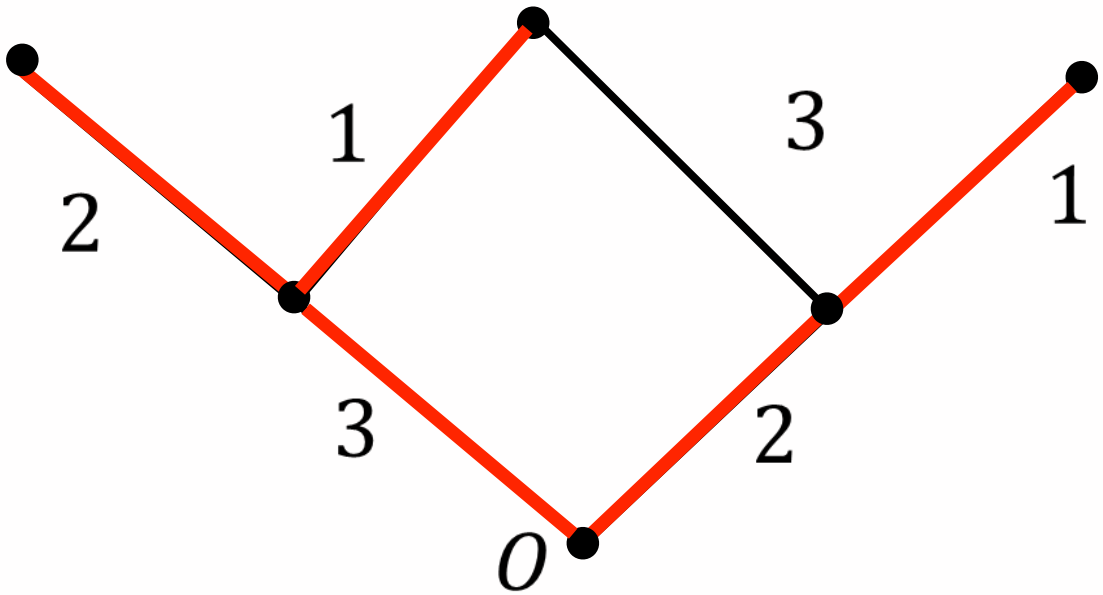

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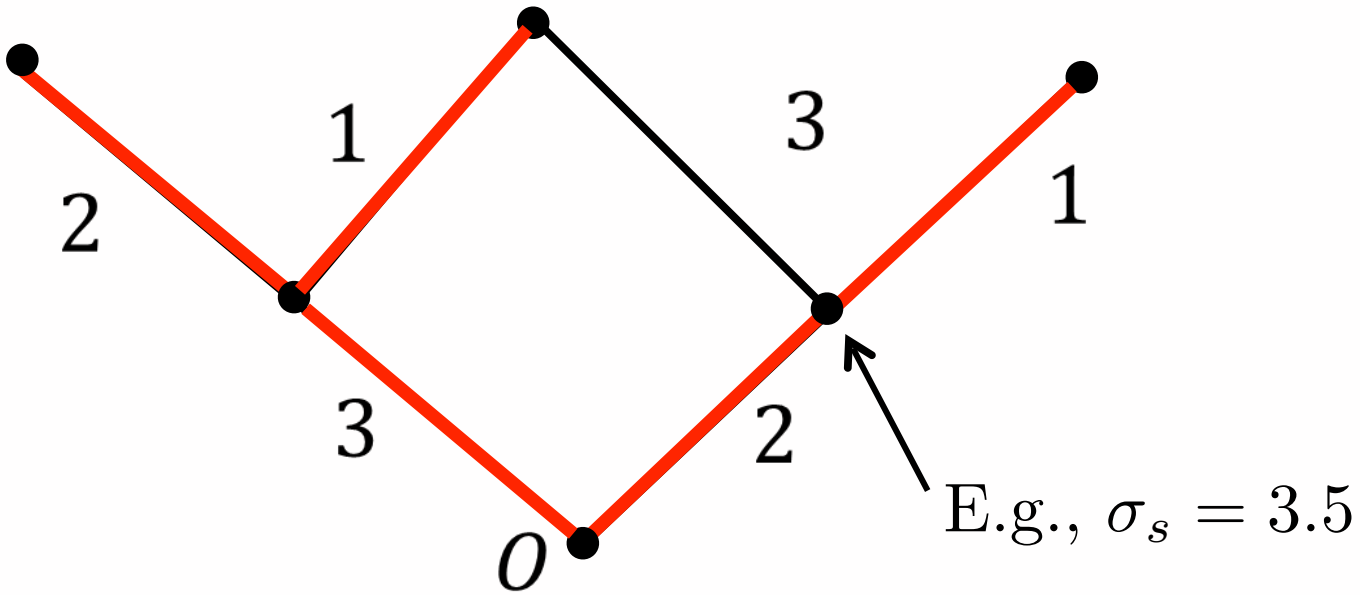

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Easy and hard cases

For trees or graphs with unit edge weights, it is optimal to search the vertices in order of their distance from *O*.

For general graphs deciding whether $\sigma \leq R$ is NP-hard

Proof: Reduction from 3-SAT

General graphs

There is a polynomial-time algorithm that approximates the search ratio within a factor of $4 \ln 4 + \epsilon$

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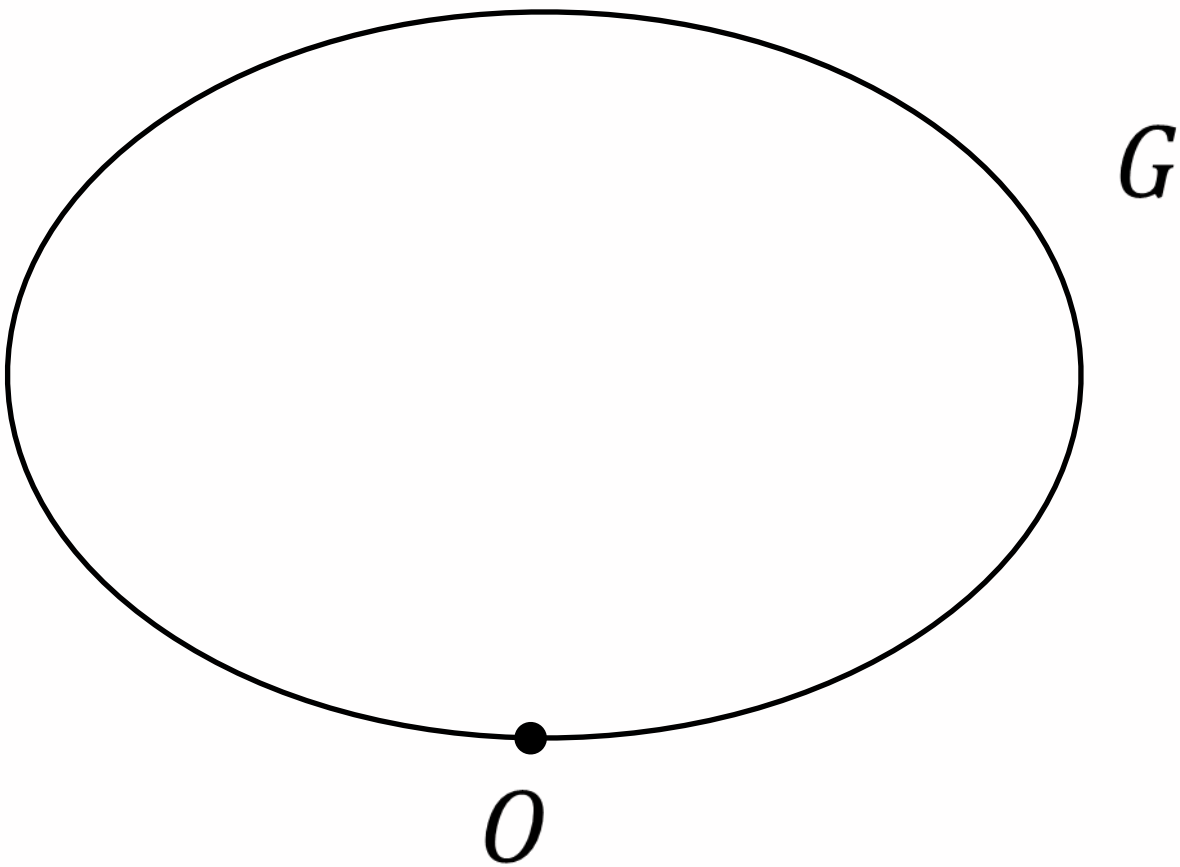

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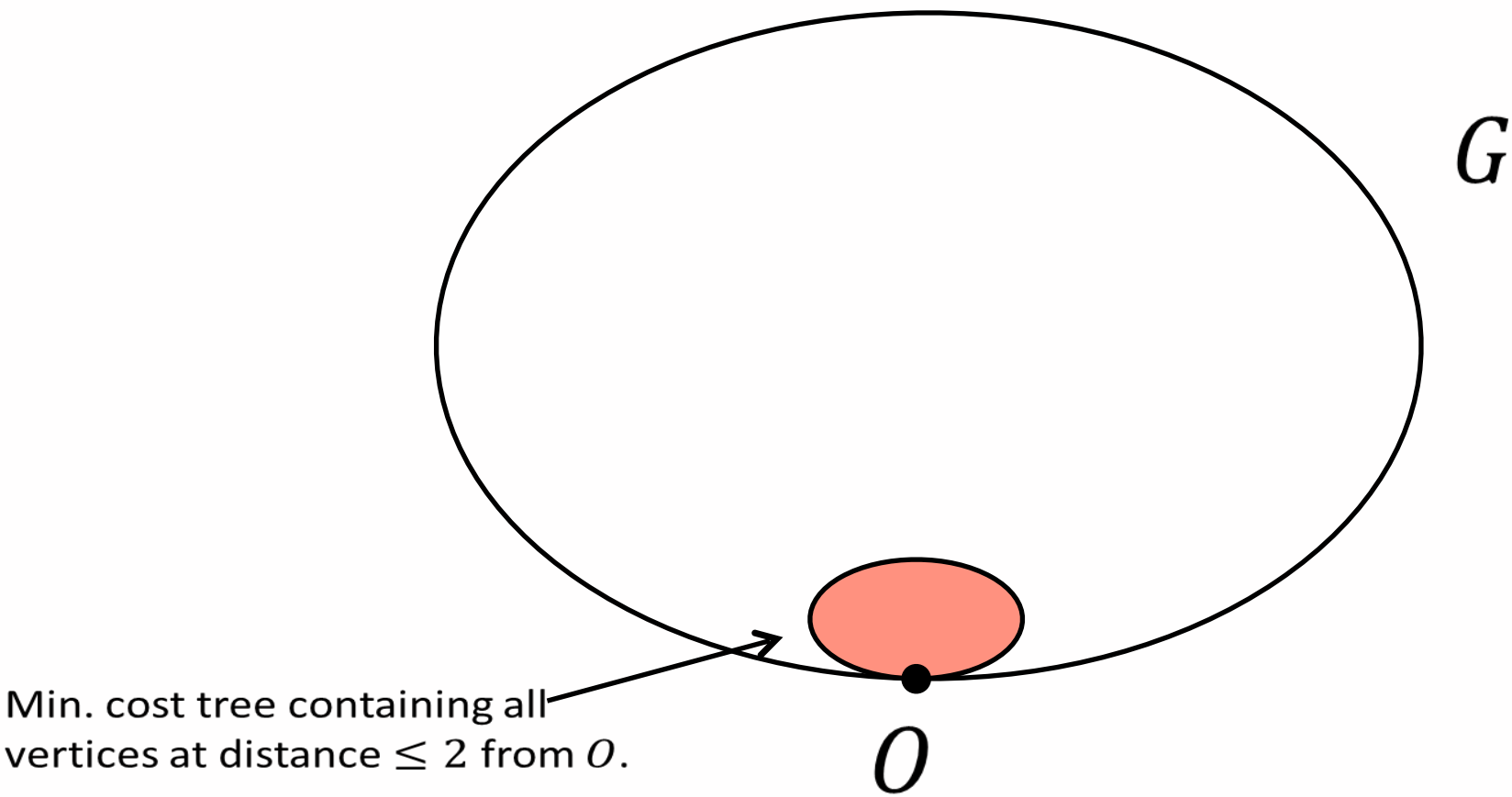

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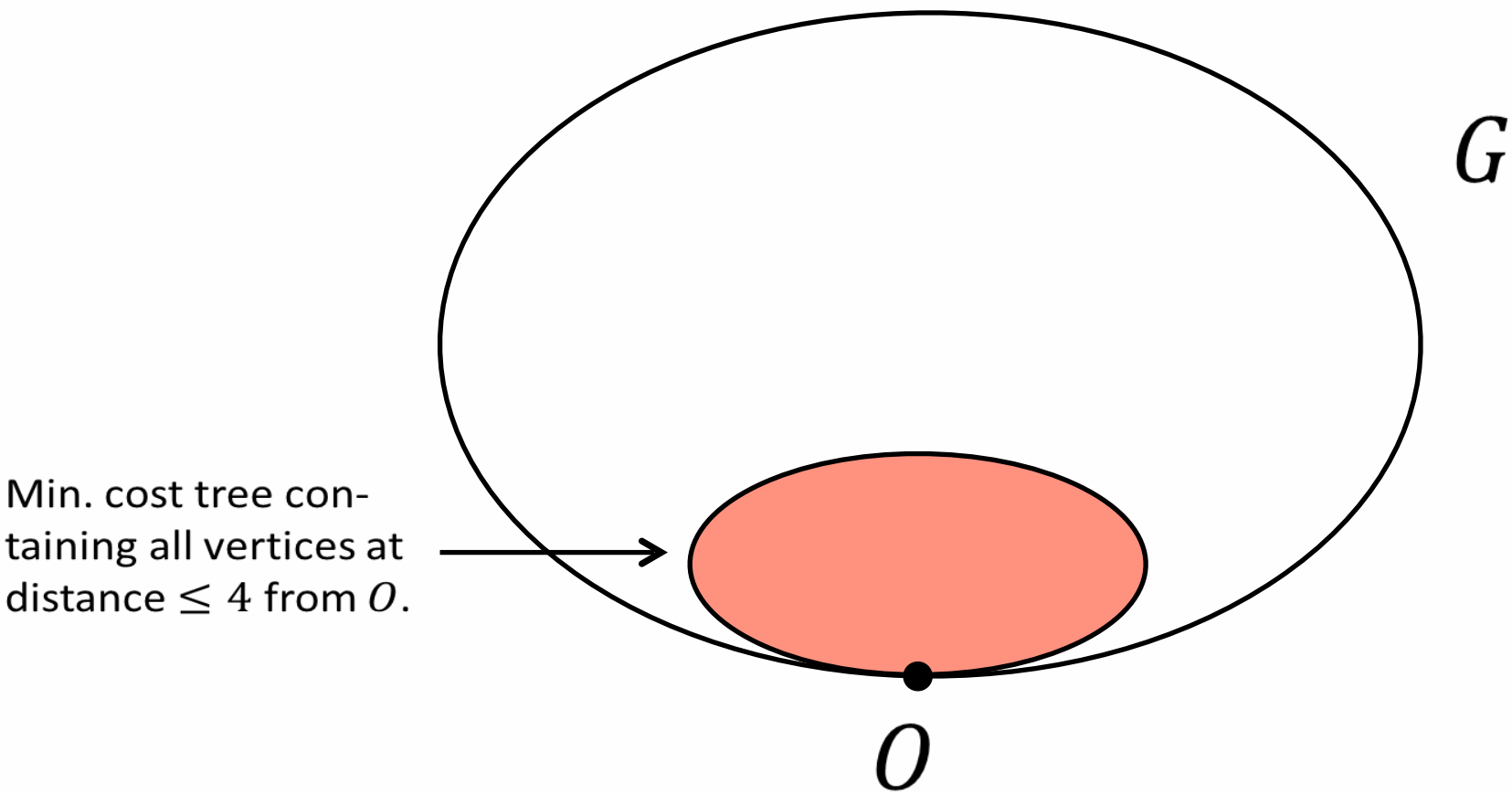

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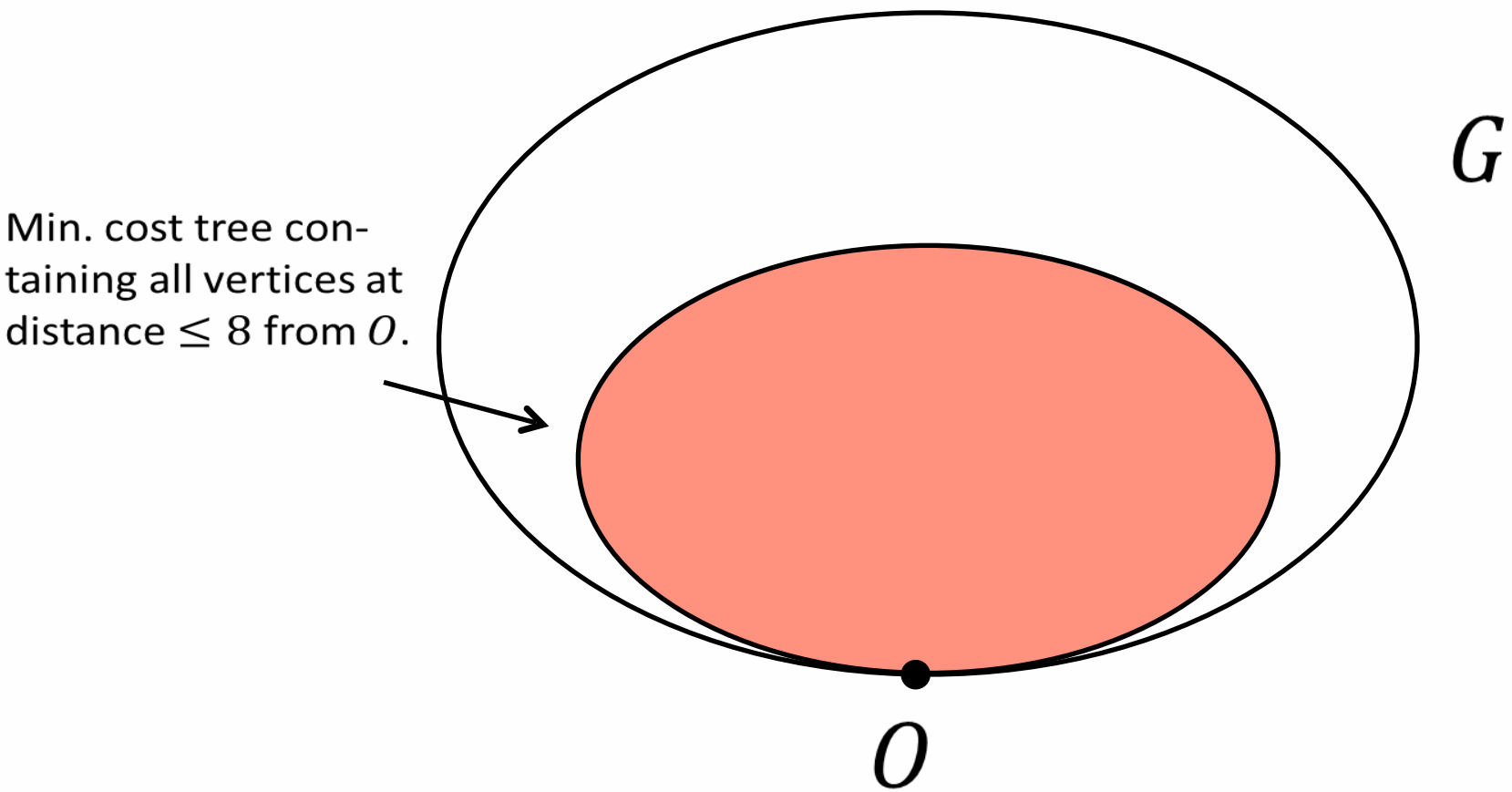


Randomized search ratio

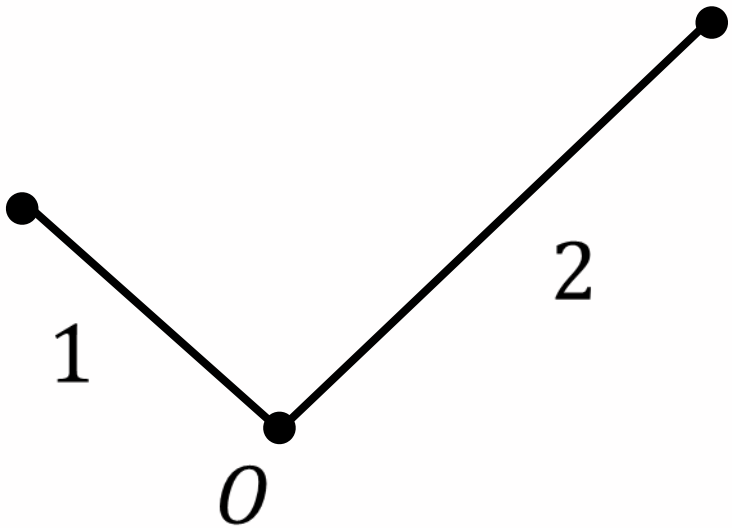
For a randomized search s and a vertex i, the expected search time and expected normalized search time are denoted by $T(s,i)$ and $\hat{T}(s,i)$

The randomized search ratio ρ_s of a random search s is $\max_i \hat{T}(s,i)$

The randomized search ratio ρ of a graph is $\min_s \rho_s = \min_s \max_i \hat{T}(s,i)$

Game theoretic interpretation

Finding the optimal randomized search is equivalent to finding the optimal strategy in a zero-sum search game between a Searcher and a Hider.



Hider/	1,2	2,1
1	1	3
2	3/2	1

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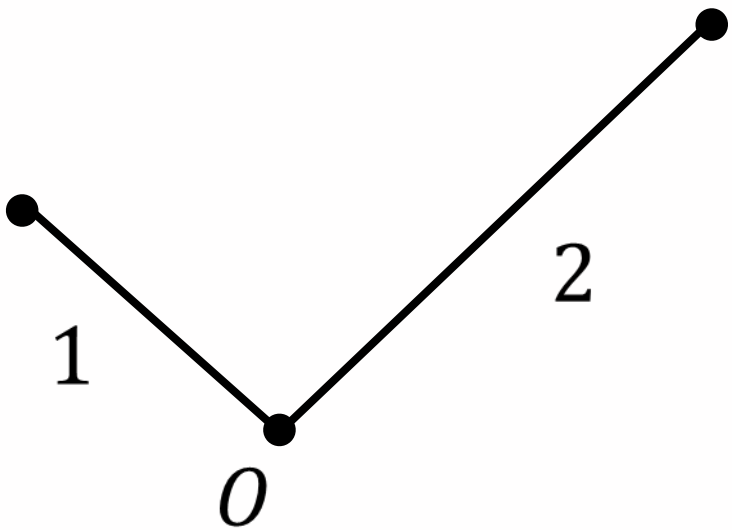

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Optimal randomized search: start with short edge with probability $4/5$ and long edge with probability $1/5$.

Randomized search ratio, $\rho = 7/5$.

2-approximate strategy

For trees or graphs with unit-length edges, the optimal deterministic strategy is a 2-approximation of the optimal randomized strategy

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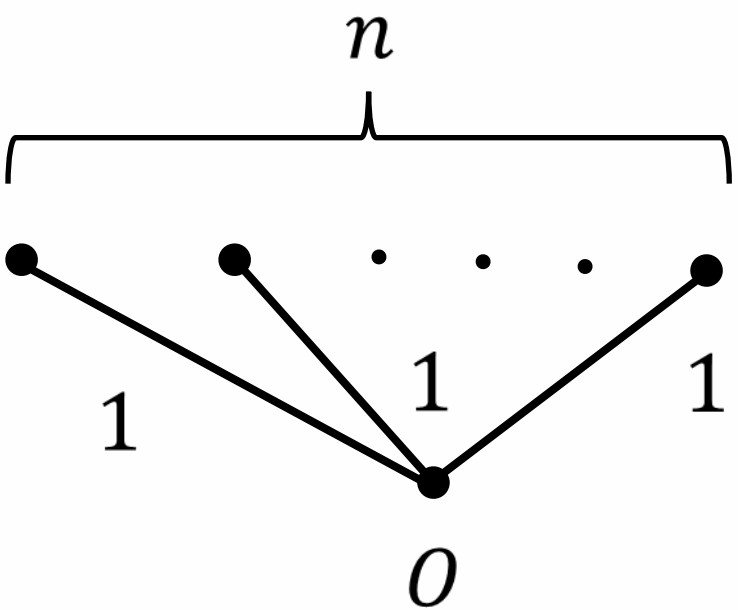
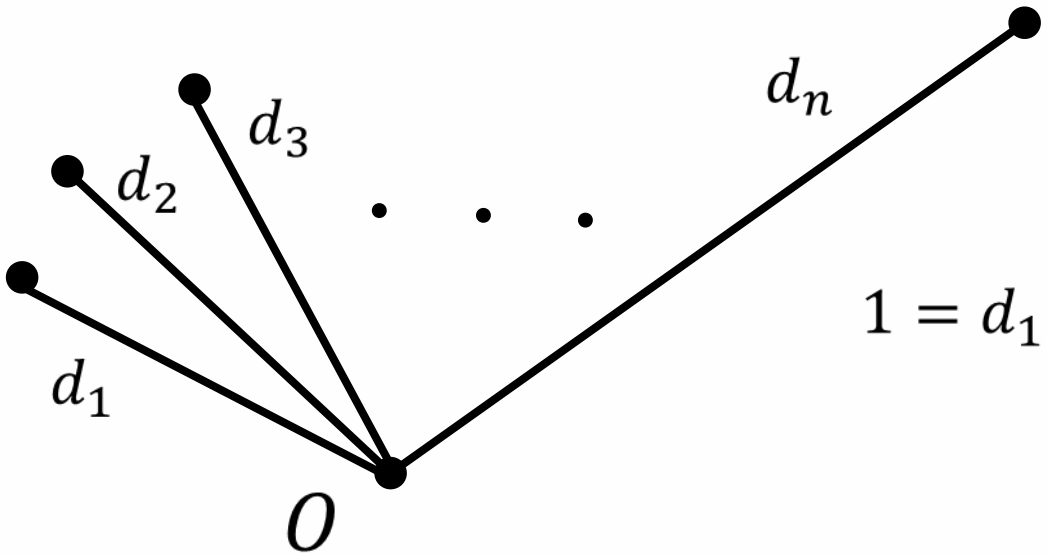


$$\sigma = n \text{ and } \rho \approx n/2$$

Randomized star search



$$1 = d_1 \leq d_2 \leq \cdots \leq d_n$$

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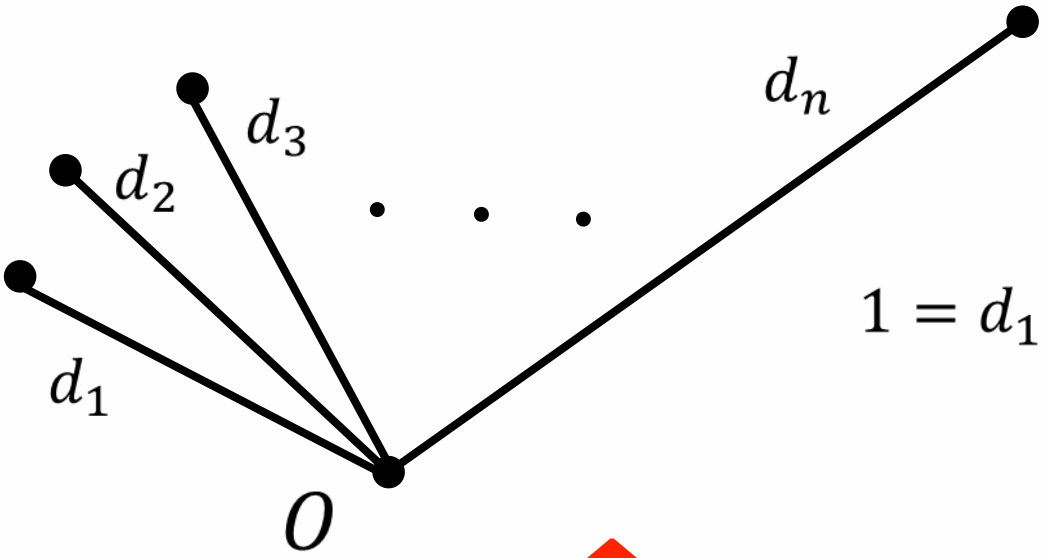

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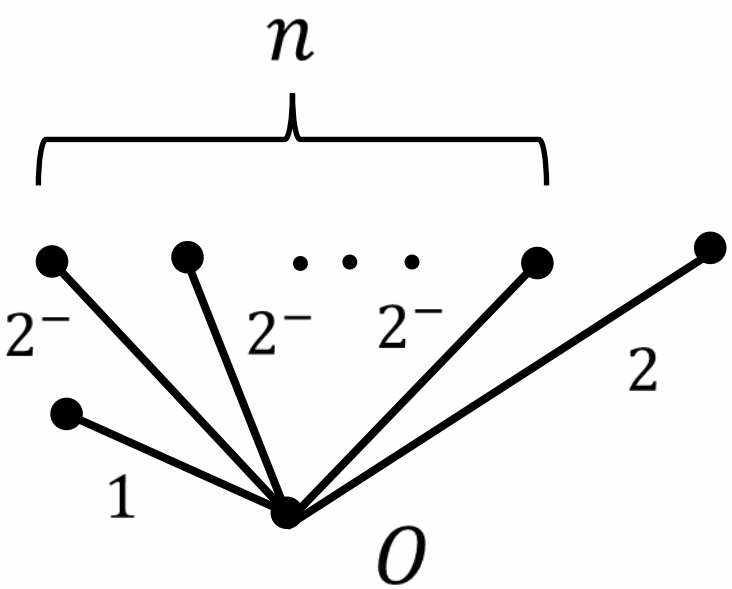
First approach: Randomize between all edges with length l satisfying $2^j \leq l < 2^{j+1}$ for $j = 0, 1, \ldots$

Unfortunately it does not work.....



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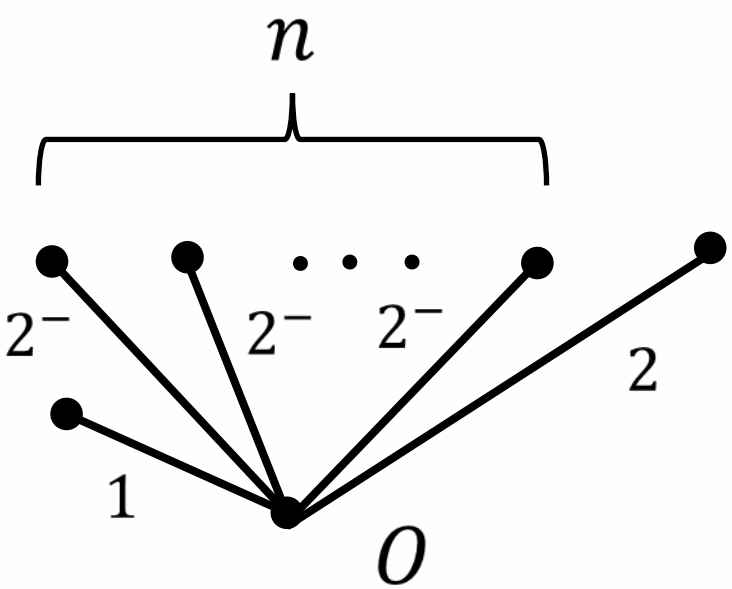
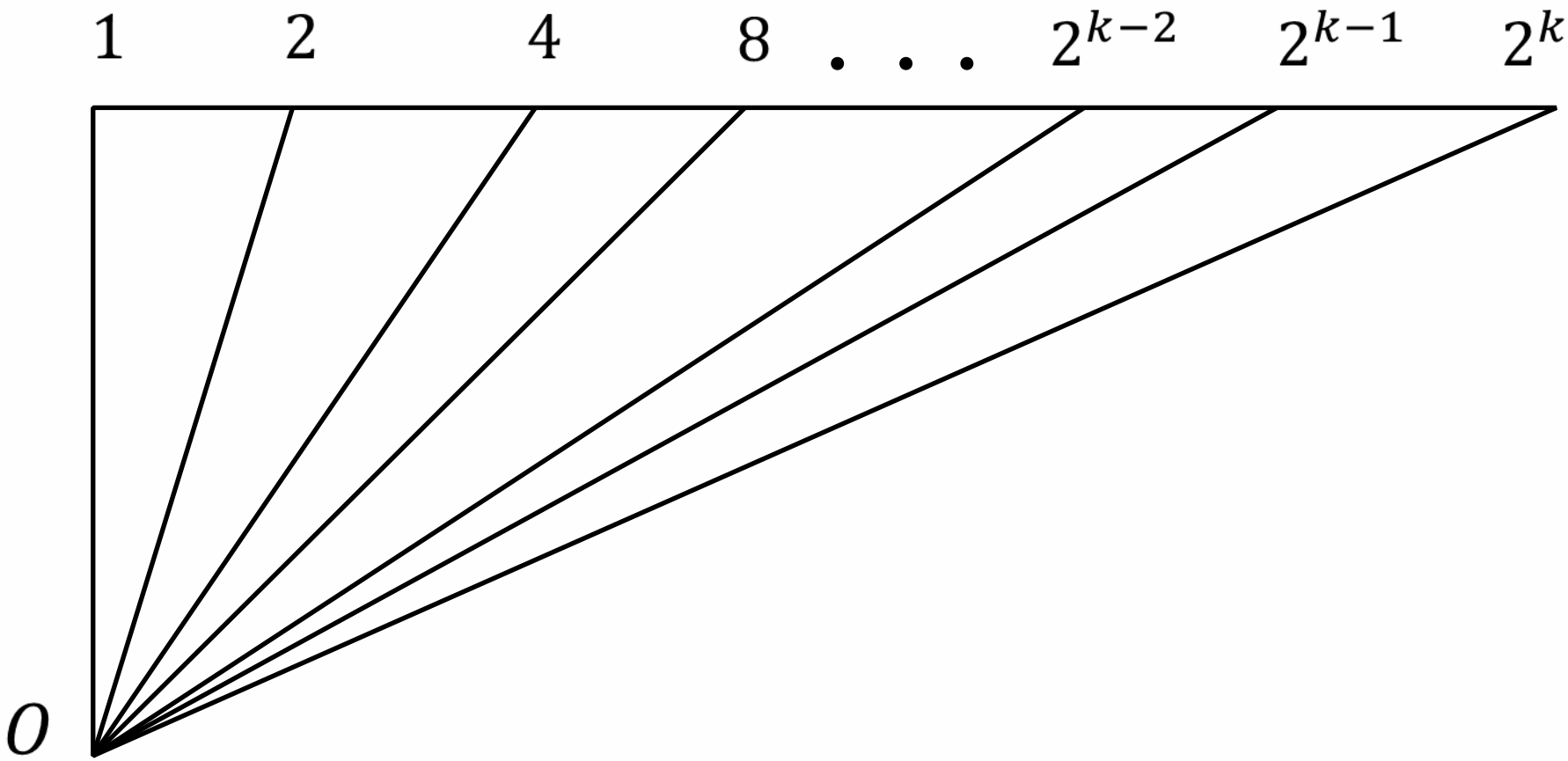


This has search ratio $\approx \frac{2n}{2} = n.$

But $\rho \approx \frac{n}{2}$.

Better idea: randomize in "random stages"



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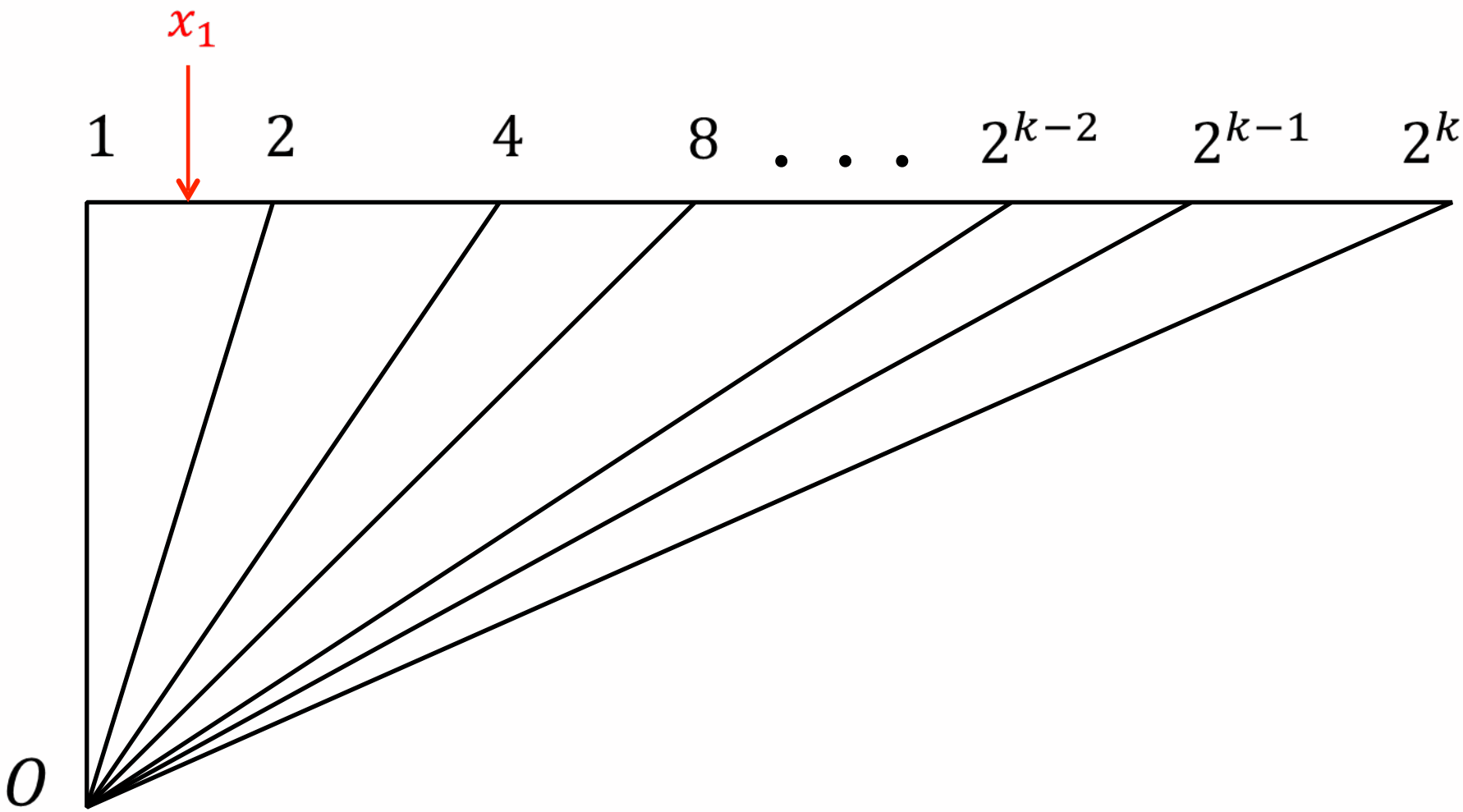

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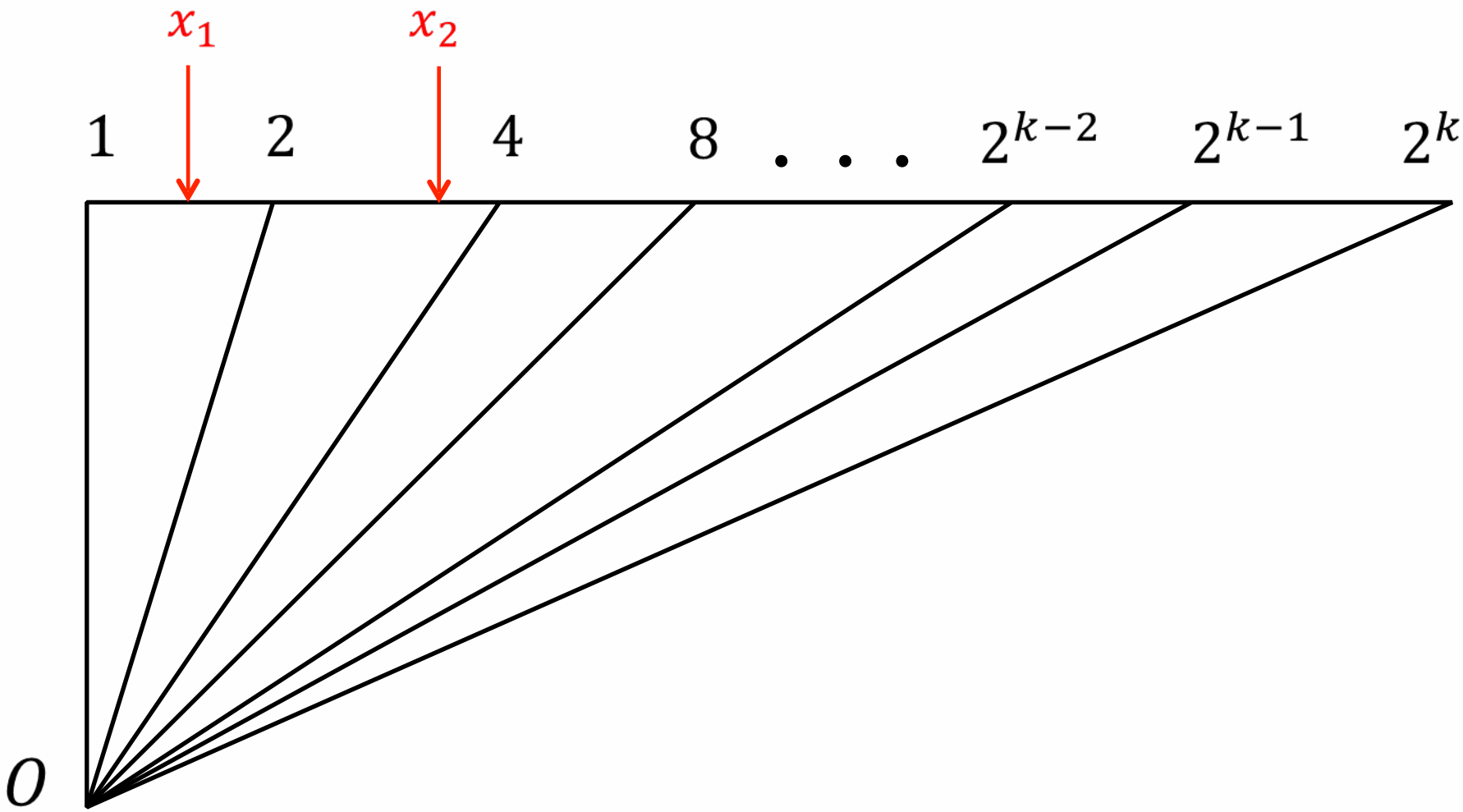

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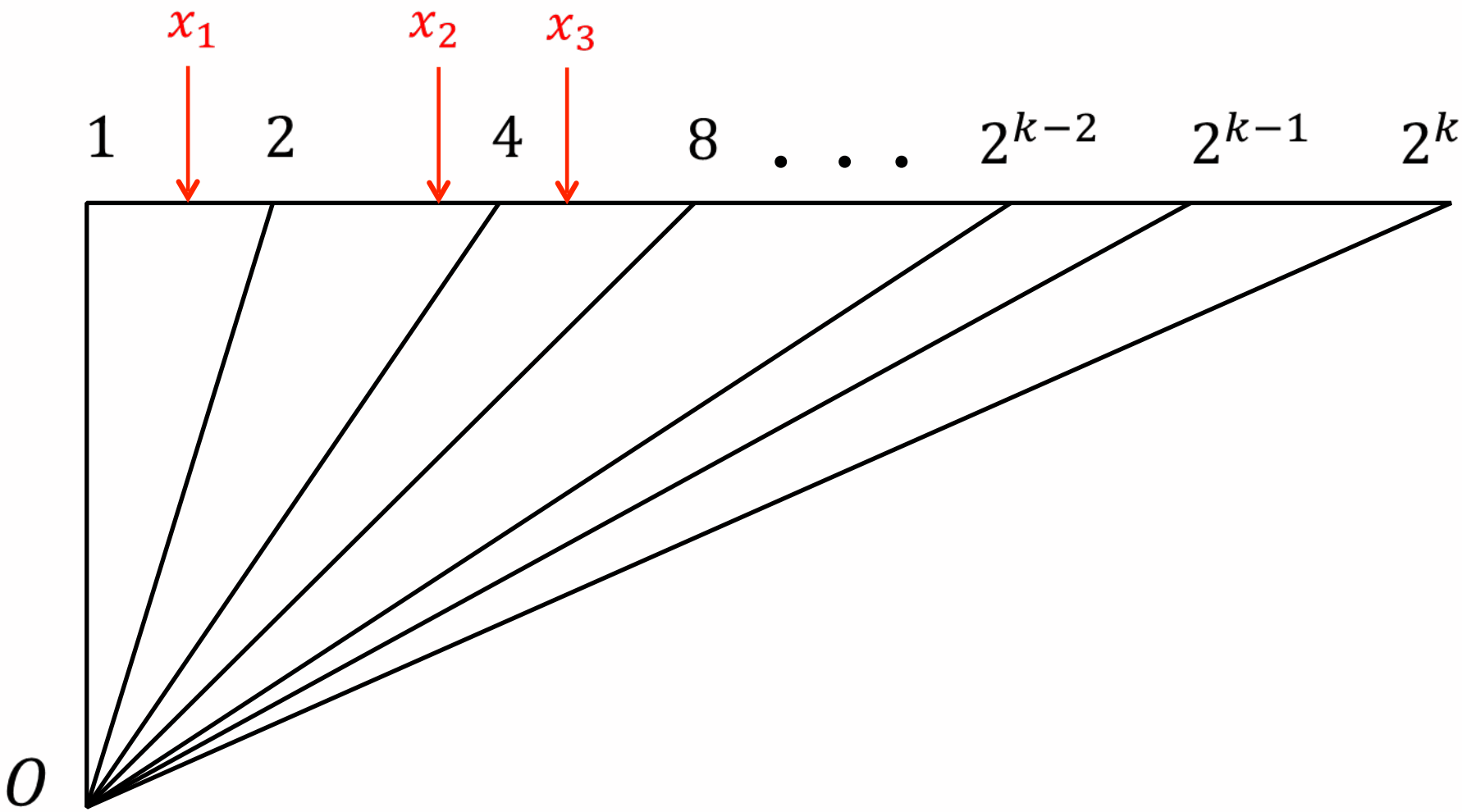

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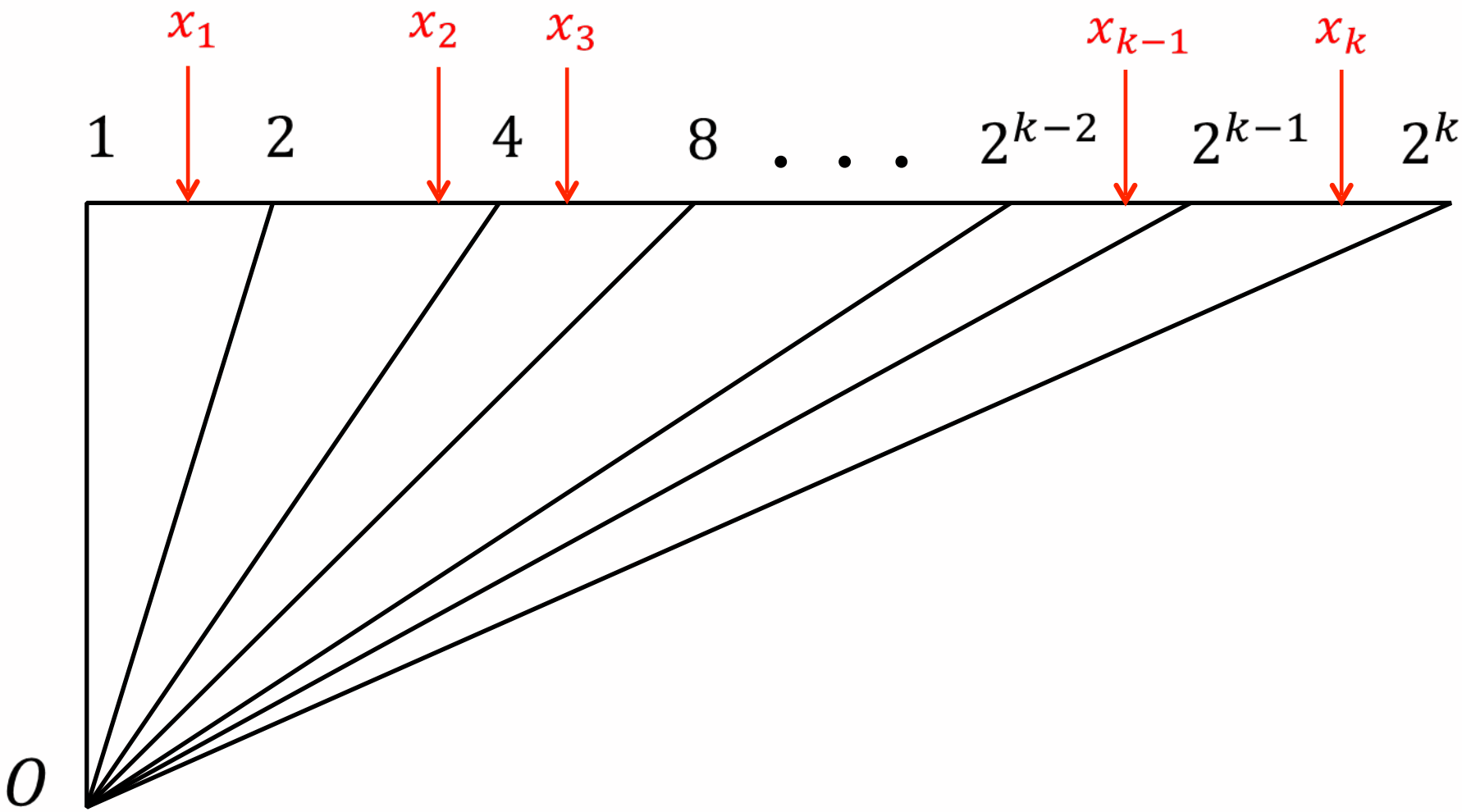

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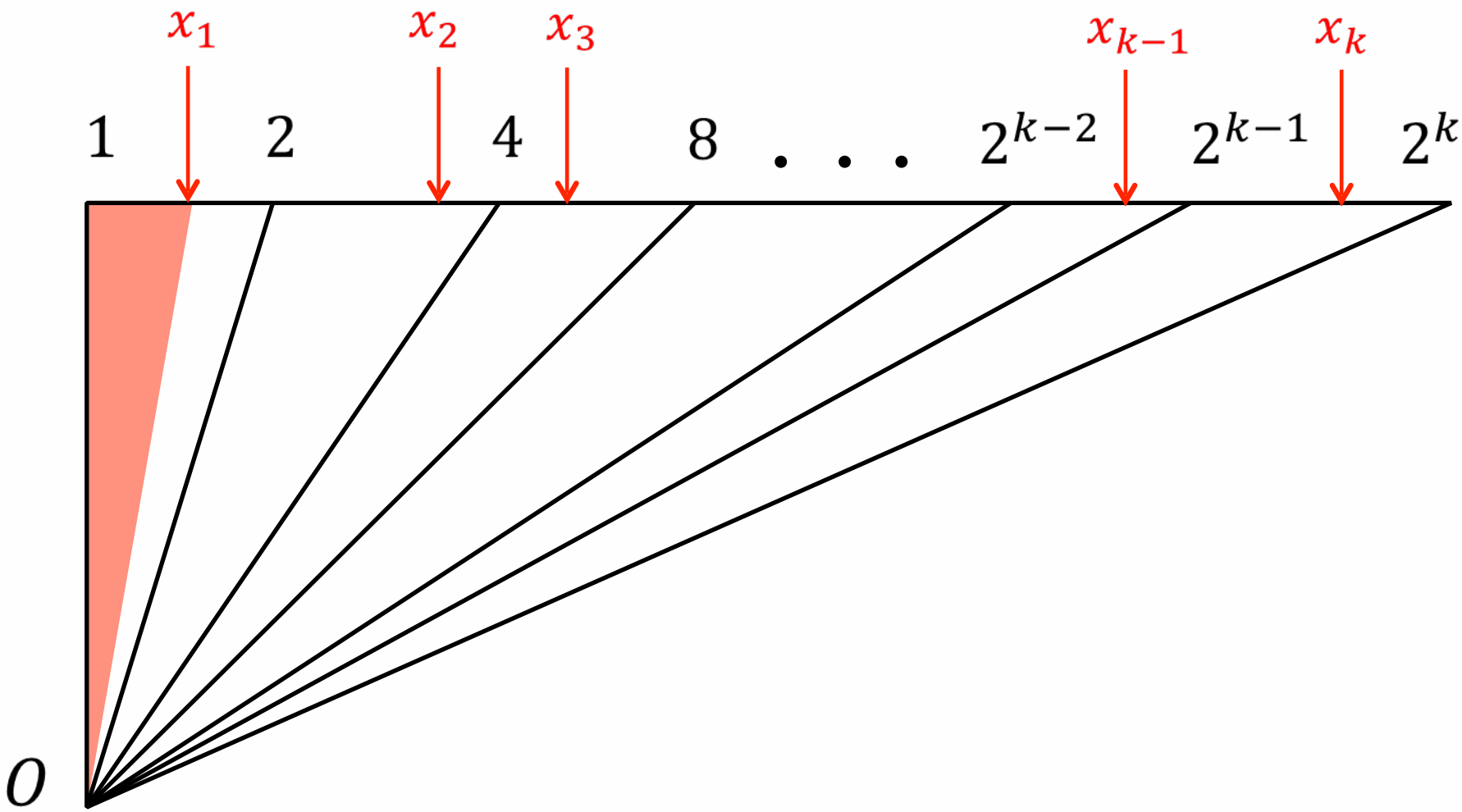

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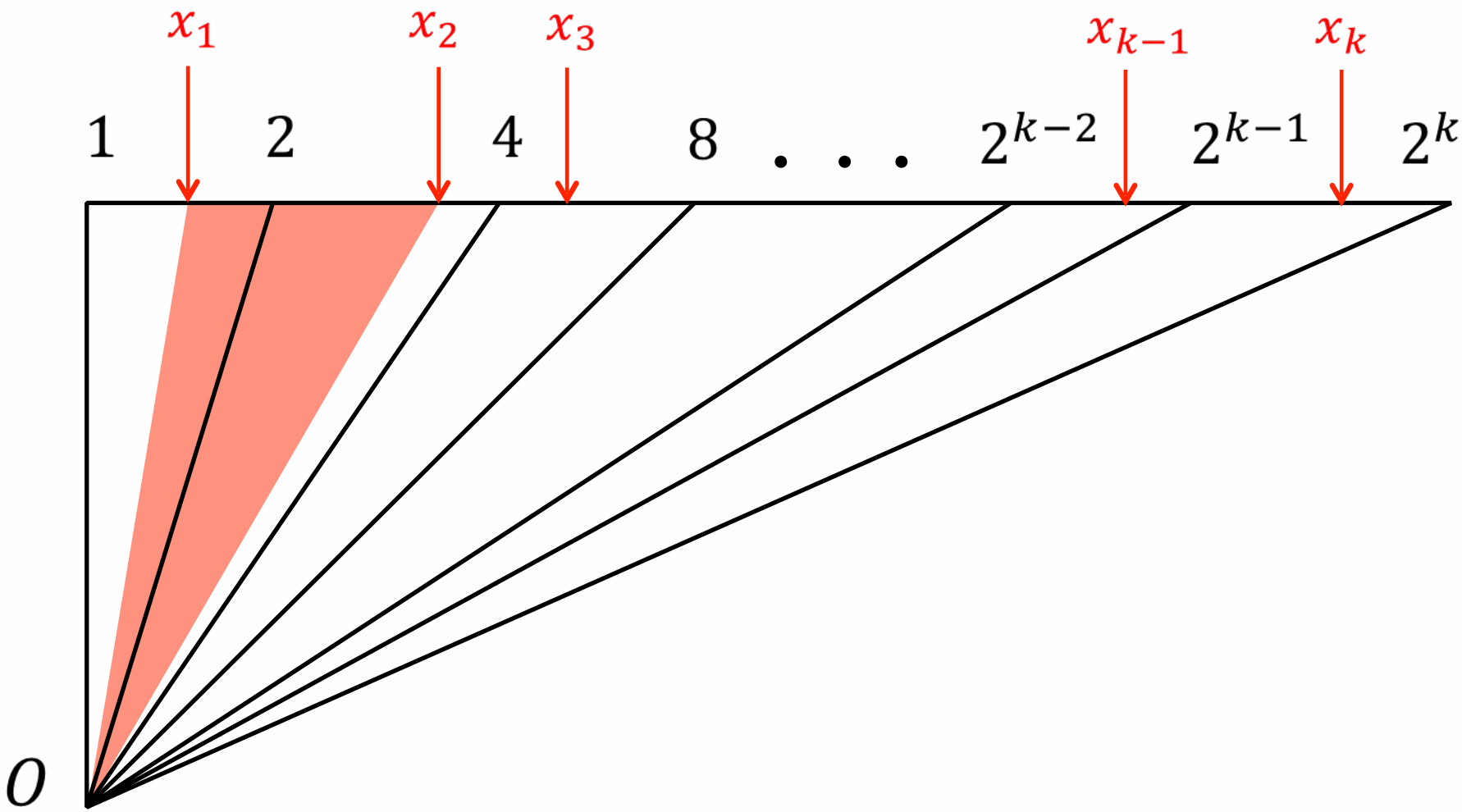

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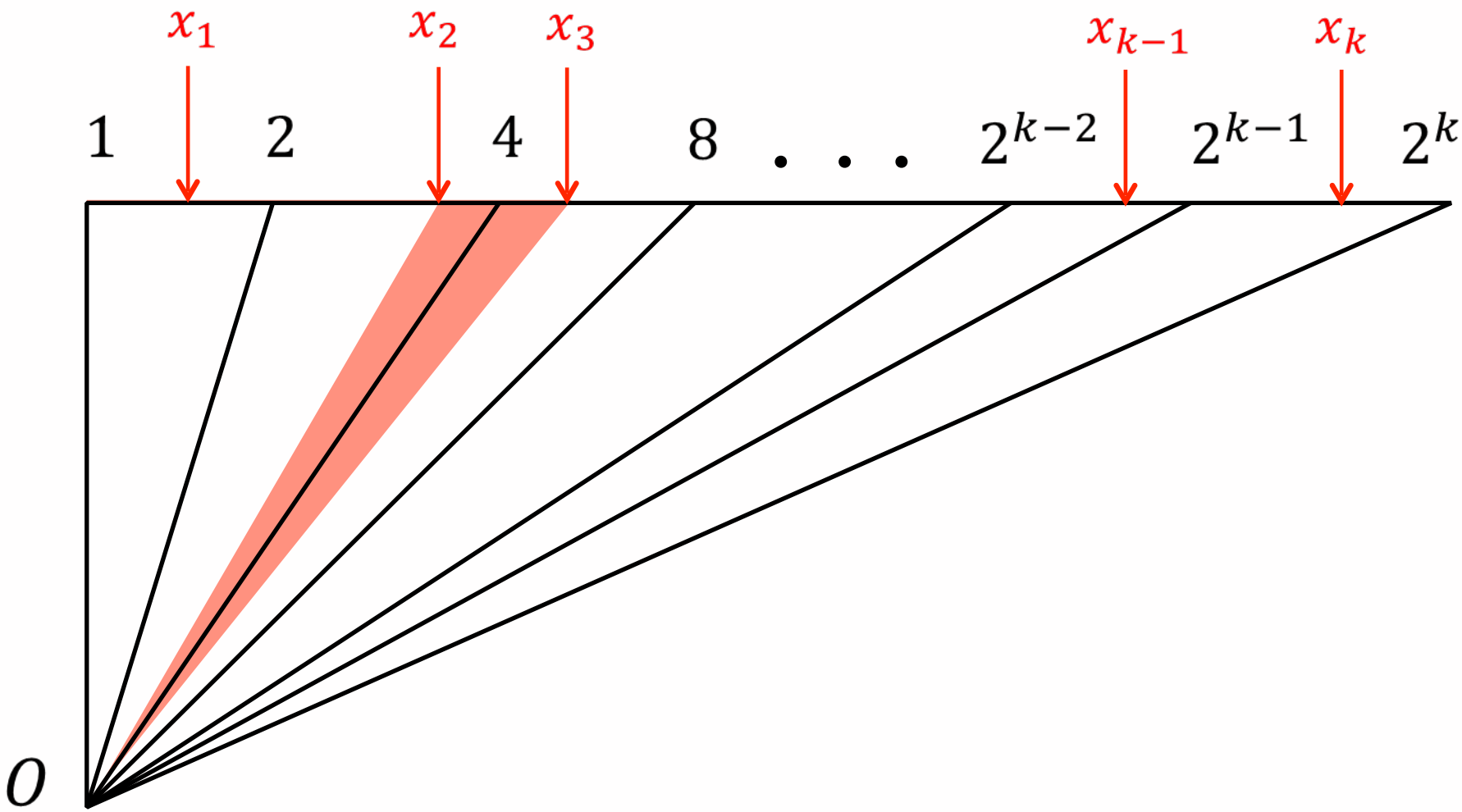

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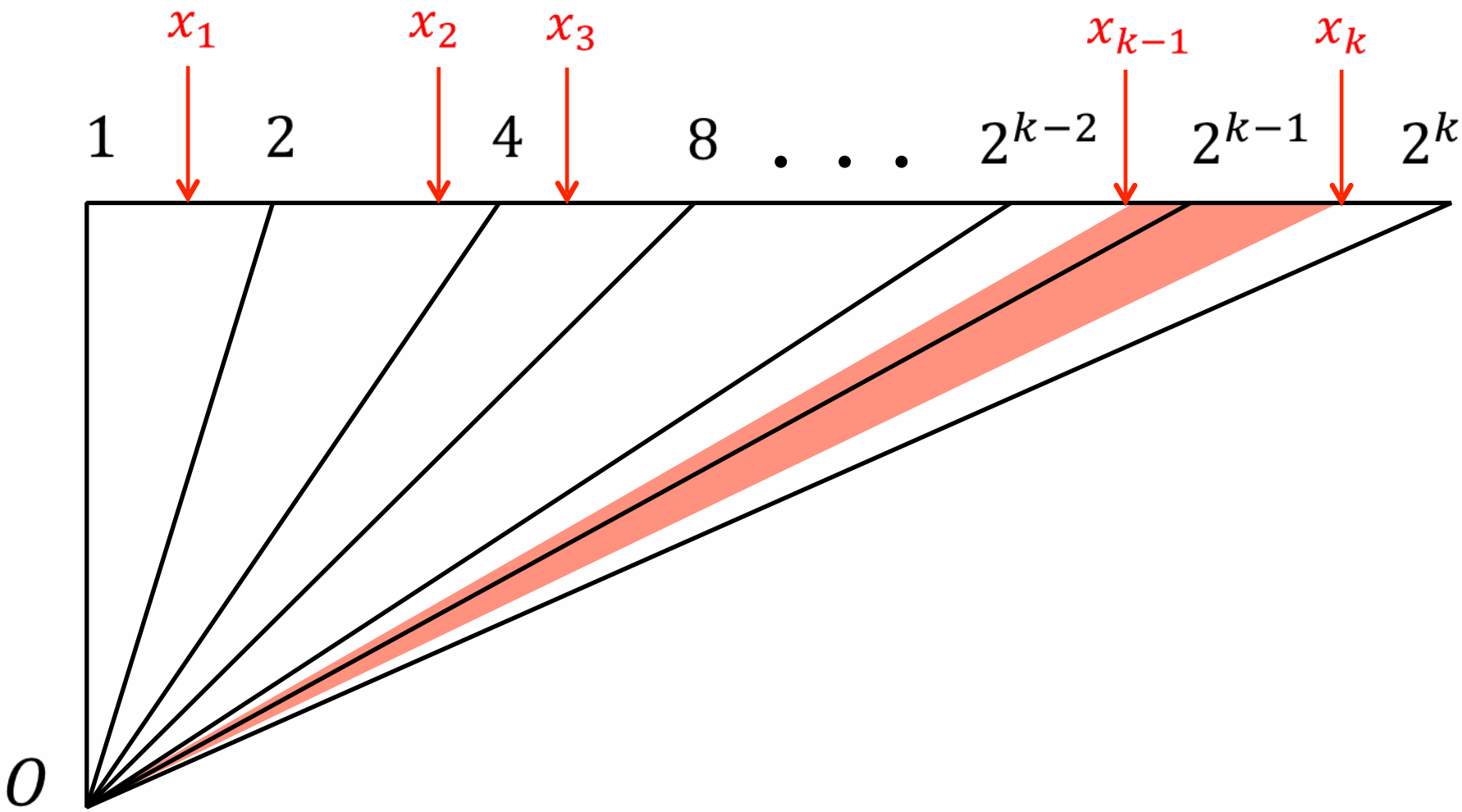

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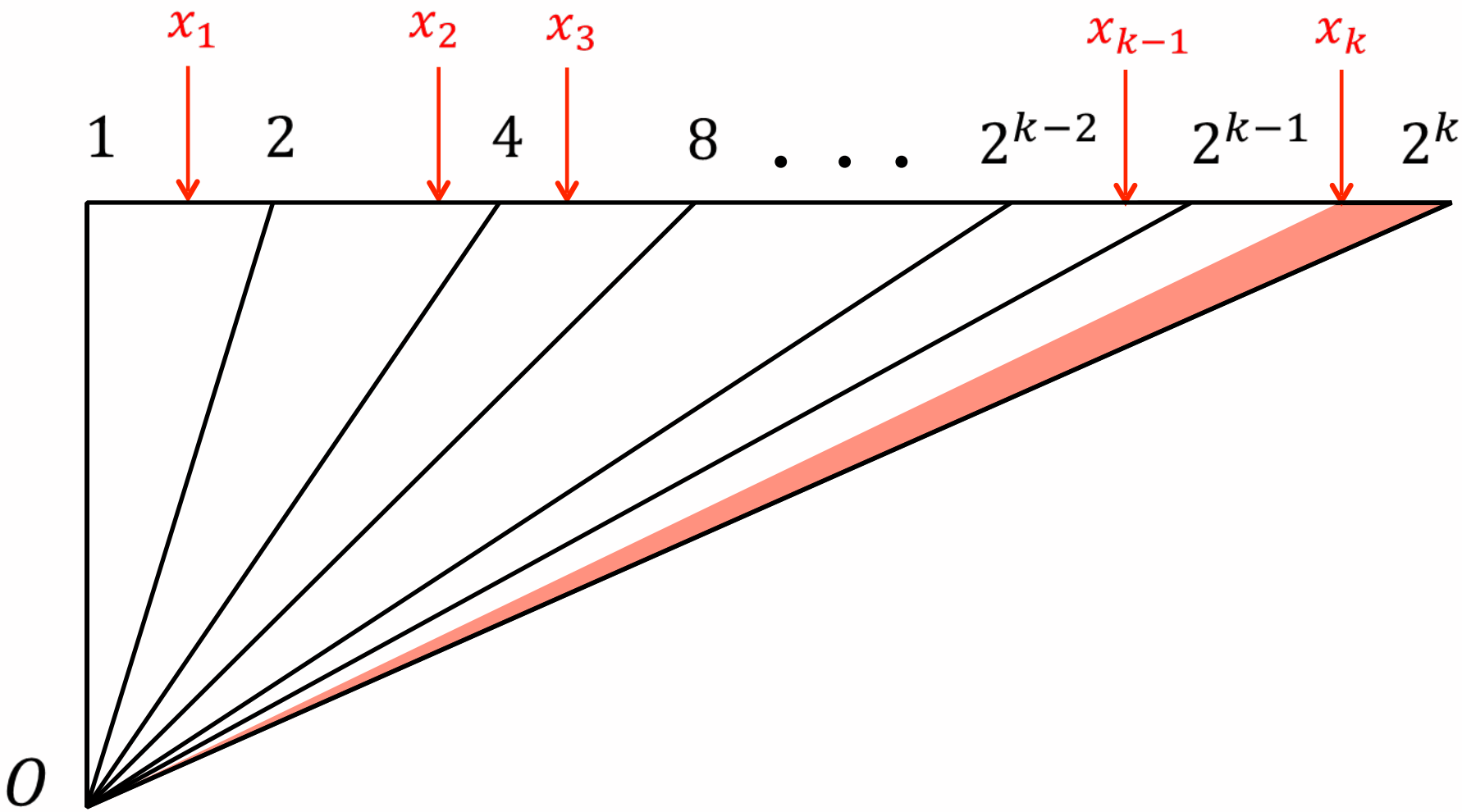

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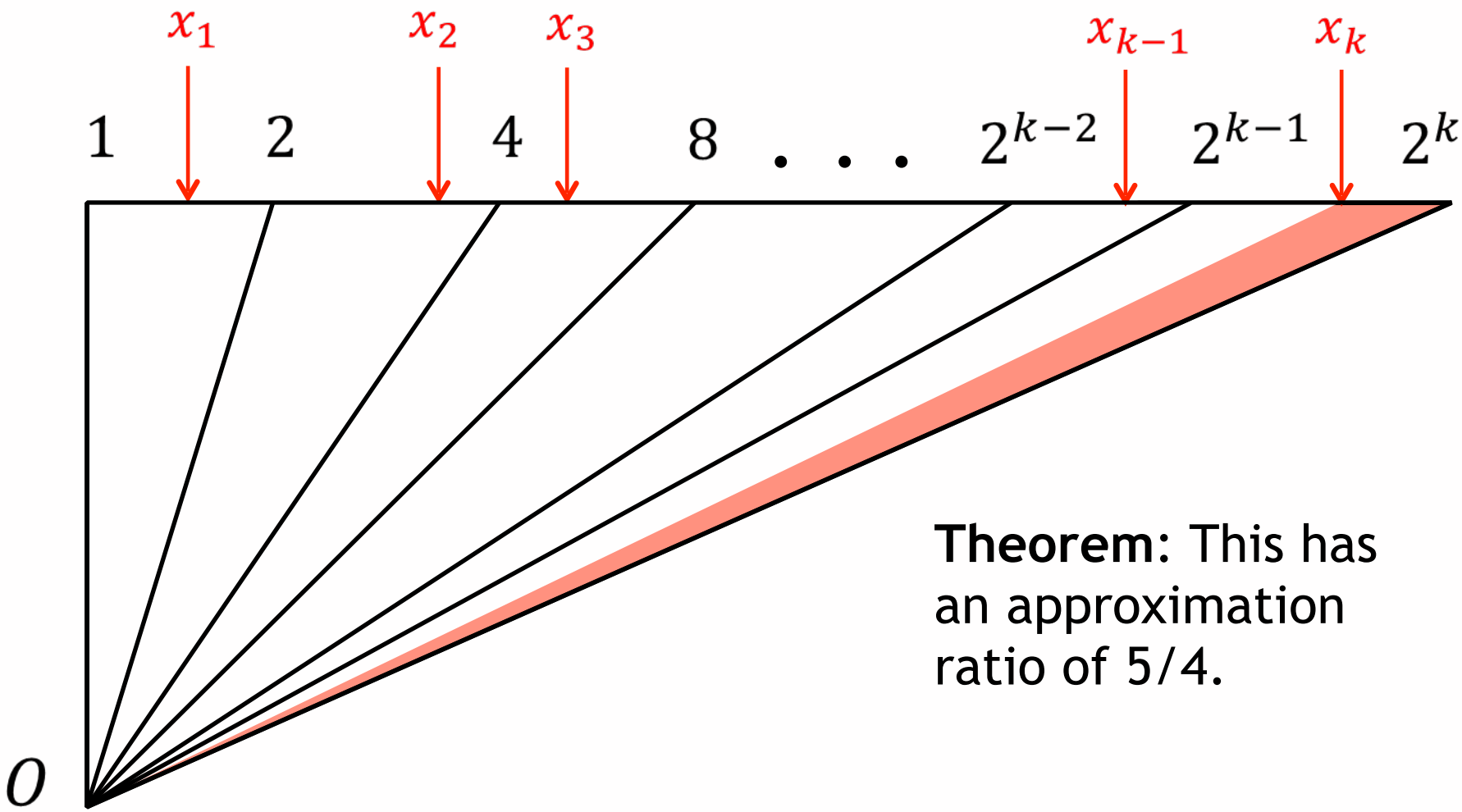
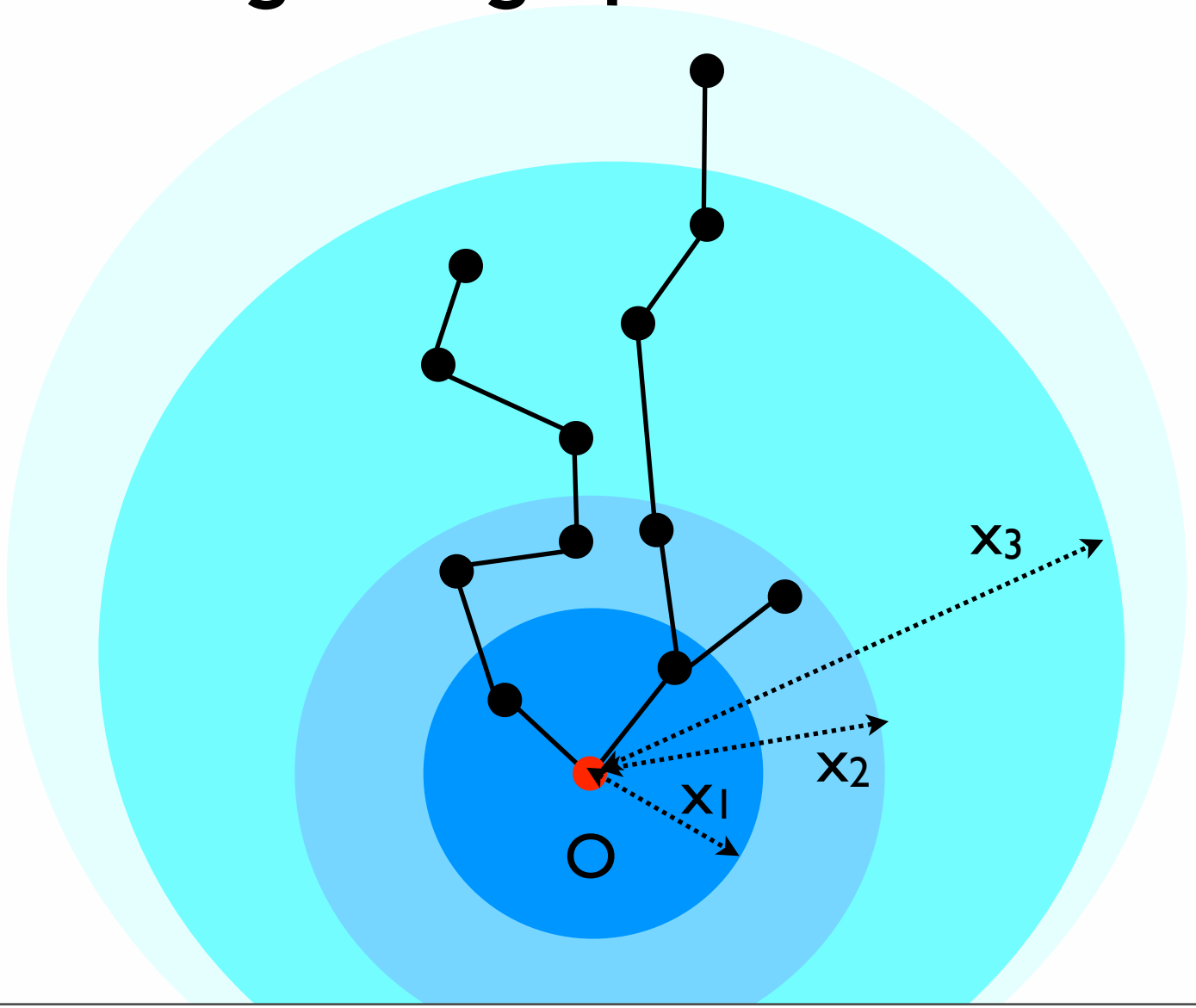


Theorem: This has an approximation ratio of 5/4.

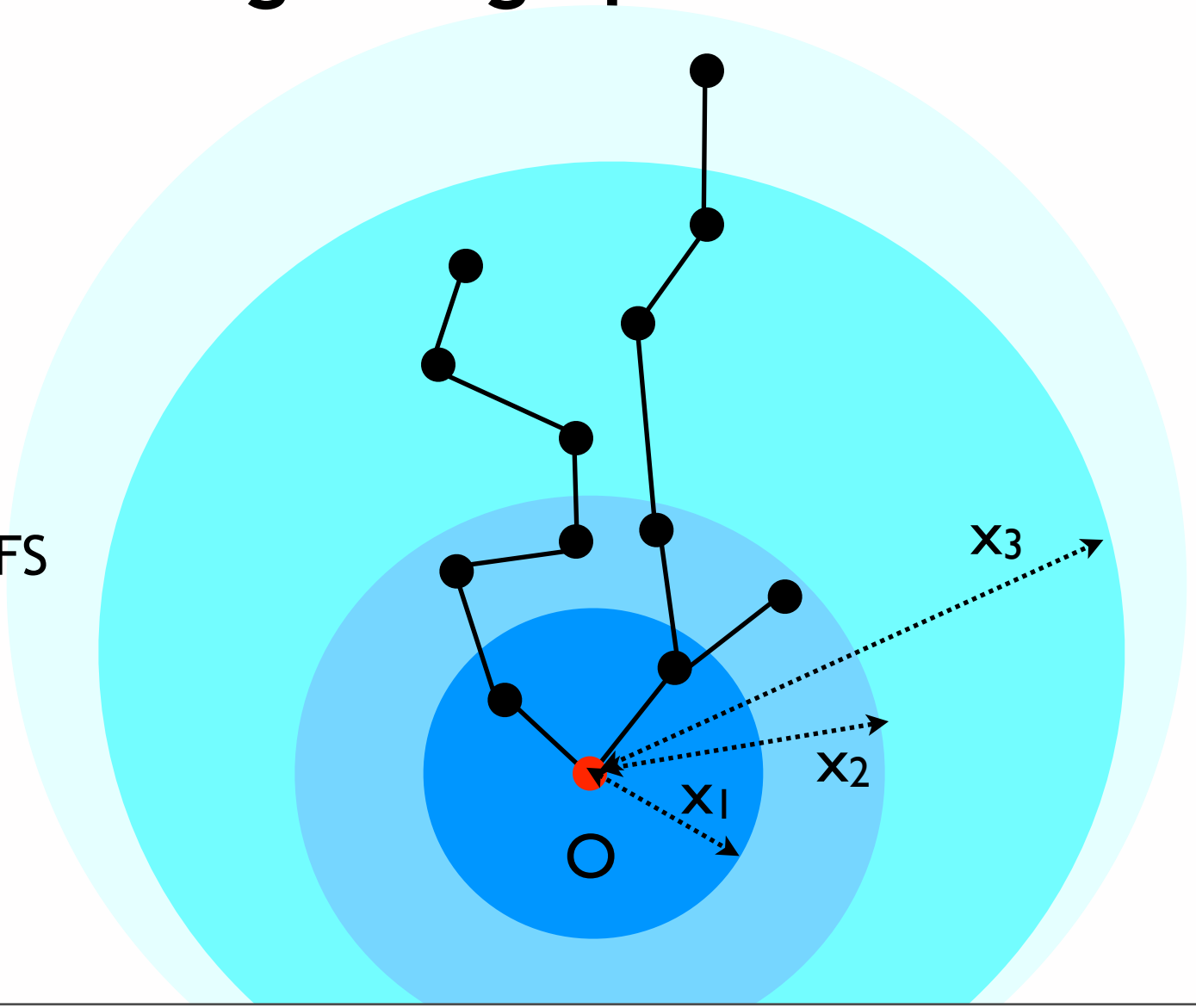
Extension to trees and unweighted graphs



Extension to trees and unweighted graphs

key idea:

Randomized DFS



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On a star graph with n edges $\rho \leq (n+1)/2$,
and this bound is tight

In particular, $\rho = (n+1)/2$ iff all edges have same length

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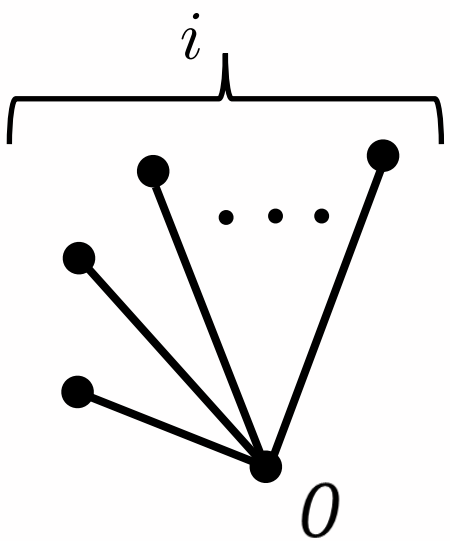

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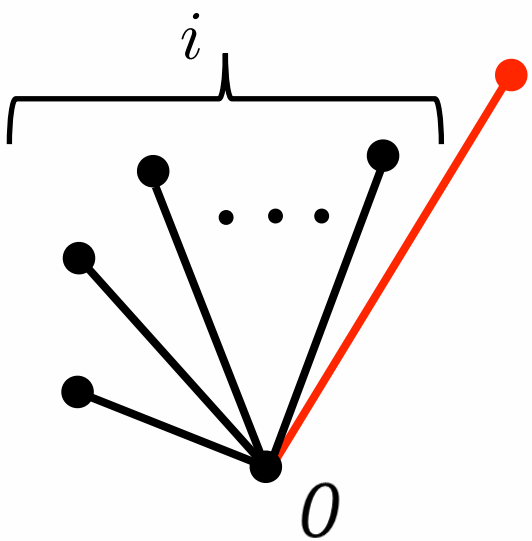

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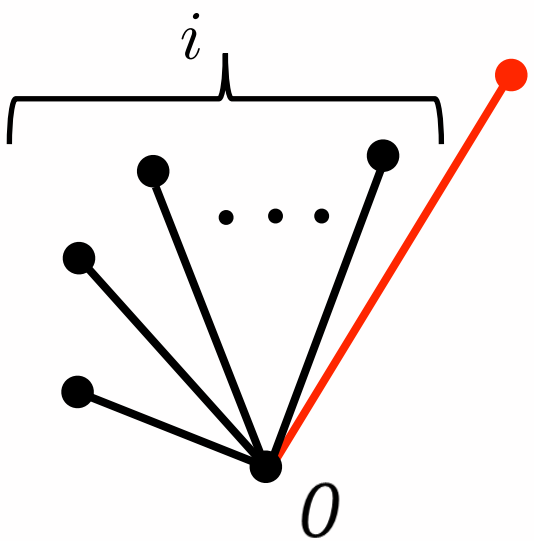

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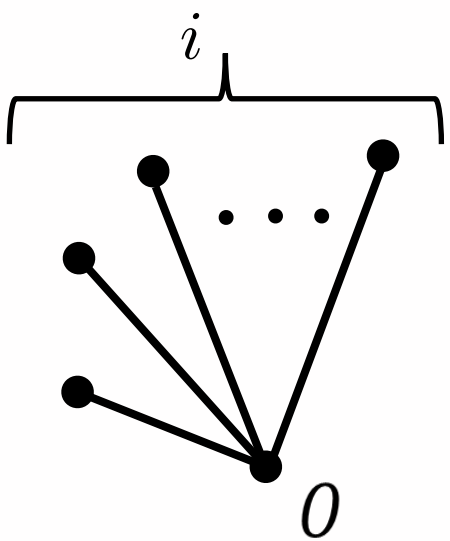

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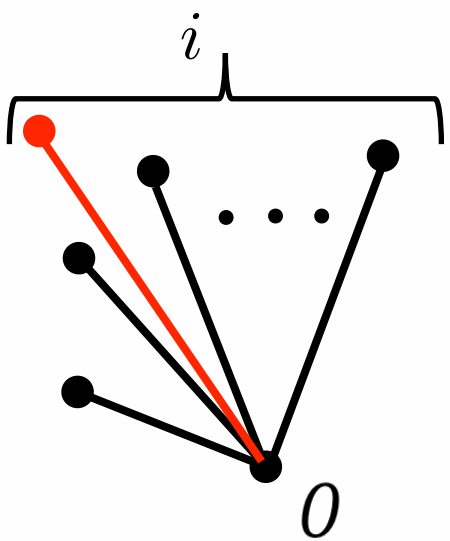

strategy 1: leave it at the end

strategy 2 : place among the i edges u.a.r according to their total length

Outlook

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Continuous setting: Easy for deterministic search, quite hard for randomized search

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What if the graph is revealed on-line?

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Long-term objective: Algorithmic search theory

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Thank you!