# Reoptimization algorithms for combinatorial problems

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PGMO days

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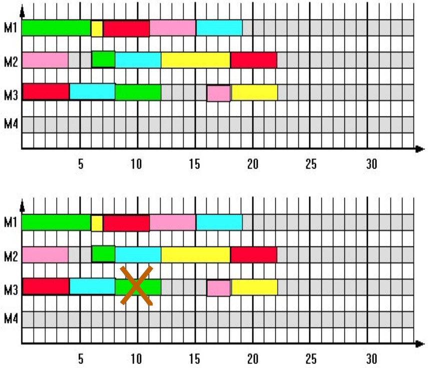
# A PGMO project

Stability versus Optimality in Dynamic Environment Algorithmics

# Reoptimization




$\rightarrow$ How to rebuild a good/optimal solution?

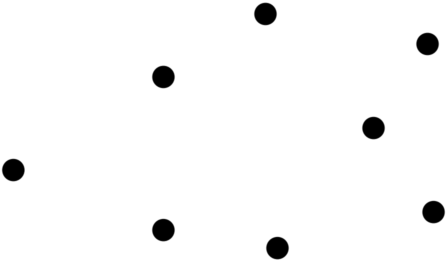
# Reoptimization?

Framework

- An instance $I$, a (optimal) solution on $I$
- A (slightly) modified instance $I'$
- How can I efficiently rebuild a solution on $I'$?

- An example: reoptimizing TSP
- Some other results
- Stability concerns

# An exemple: TSP



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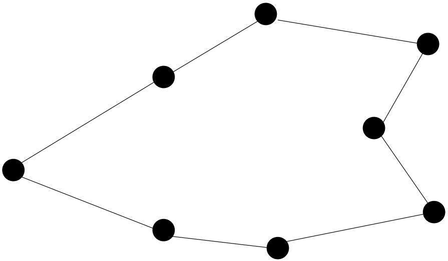

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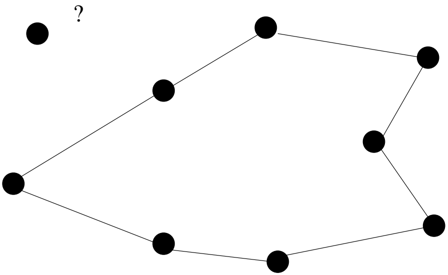
TSP+: one node insertion



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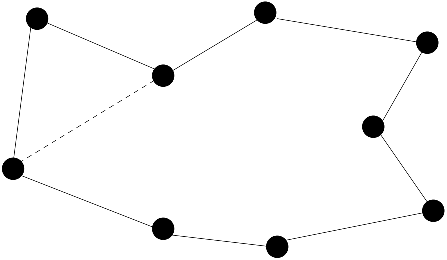

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**Remember...**

Metric TSP is *NP*-hard.
It is approximable with ratio 3/2 by Christofides' algorithm ($val(S) \leq 3/2OPT$ on any instance)

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Approximation algorithm?

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**Good news**

Best insertion is a 3/2-approximation algorithm (reoptimization) [ABS03]

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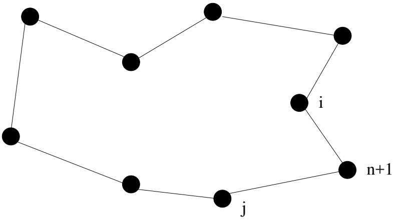


$OPT(I')$

$$OPT(I') \geq OPT(I) + d(i, n+1) + d(j, n+1) - d(i,j)$$

$$val(S) \quad \leq \quad OPT(I) + 2d(i, n+1)$$

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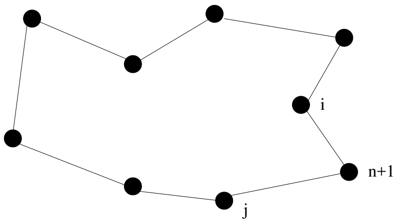


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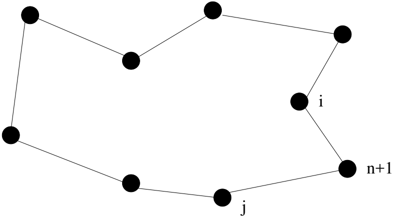


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# An exemple: TSP

**First attempt**

Best insertion: insert the new vertex at the best position in the current tour.

**Good news**

Best insertion is a 3/2-approximation algorithm (reoptimization) *in time* $O(n)$. [ABS03]

**Bad news**

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**Better ratio and better running time?**

Yes: 4/3-approximate solution in $O(n^2)$ [M15]

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- Some other results
- Stability concerns

# Some other results

- Many problems tackled from the point of view of reoptimization
  $\rightarrow$ Other network design problems: Steiner trees, variants of TSP (Max, with deadlines) ([BKK09,BZ12,KT15,...])
  $\rightarrow$ Graph subset: independent set [BMP13],...
  $\rightarrow$ Scheduling problems [BC14]

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- Negative results?

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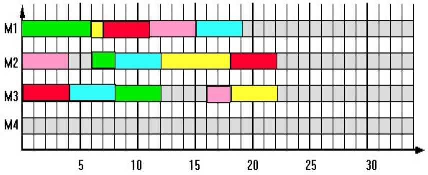
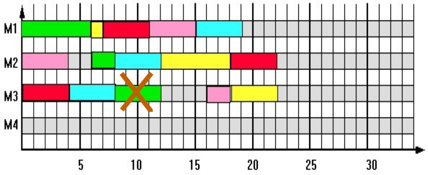
# Stability concerns




→ How to rebuild a (optimal) solution . . .

**without destroying all my initial solution??**

→ Transition costs, stability of solutions

# Example of TSP

Best insertion: 3/2-approximation, in $O(n)$, removal of only one edge!
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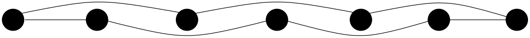
Best insertion: 3/2-approximation, in $O(n)$, removal of only one edge!
Best Insertion + Christofides: 4/3, but may change everything.
Can I do better than 3/2 (Best Insertion) without changing a lot of edges?

# Example of TSP

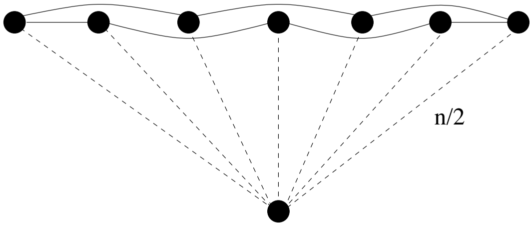
$n$ vertices, $d(i, i+1) = 1$

# Example of TSP



$opt = 2(n-1)$

# Example of TSP



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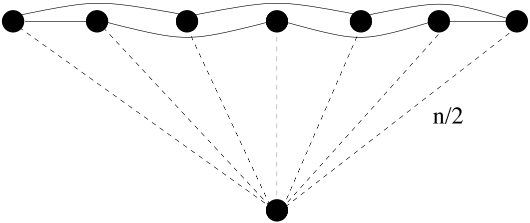


Best Insertion: roughly $3n$

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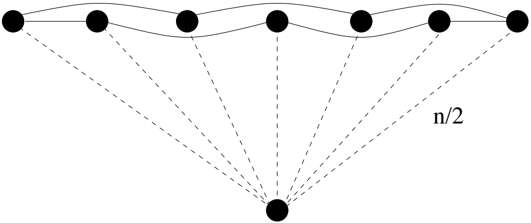
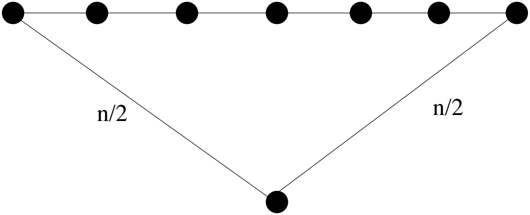



Best Insertion: roughly $3n$

If I remove at most $\alpha n$ edges: at best
$2n/2 + 2(1-\alpha)n + \alpha n = (3-\alpha)n$

# Example of TSP

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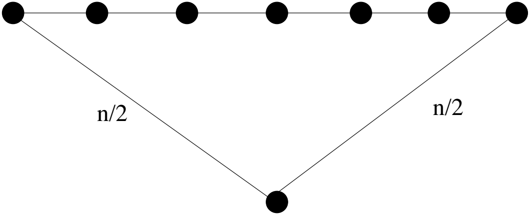


OPT: roughly $2n$

If I remove at most $\alpha n$ edges: ratio no better than $3/2 - \alpha/2$

# Two recent approaches

- Bound $k$ on the number of changes in the solution
  $\rightarrow$ Can I compute the best solution $S'$ for $I'$ *at distance at most* $k$ from $S$?

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  $\rightarrow$ Can I compute the best solution $S'$ for $I'$ *at distance at most* $k$ from $S$?
  Typically bruteforce is $n^k$: can I do it in FPT time?
  *Parameterized complexity*
  [AEFRS15]

# Two recent approaches

- Find good tradeoffs between number of modifications and quality of solution.
  [STT12]: if an optimal solution on $I'$ is at distance $d$ from $S$, can I compute a solution $S$ such that:
  $\rightarrow val(S) \leq r_1 OPT(I')$
  $\rightarrow d(S, S') \leq r_2 d$

How to combine:
→ efficiency (low complexity)
→ quality of solution (approximation ratio)
→ stability of solution (transition costs)

*Hope to come back in one year with some answers...*

*Thanks for your attention*

# References

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