Reoptimization algorithms for combinatorial problems

Bruno Escoffier, LIP6, UPMC

PGMO days

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Stability versus Optimality in Dynamic Environment Algorithmics
How to rebuild a good/optimal solution?
Framework

- An instance $I$, a (optimal) solution on $I$
- A (slightly) modified instance $I'$
- How can I efficiently rebuild a solution on $I'$?
▶ An example: reoptimizing TSP
▶ Some other results
▶ Stability concerns
An exemple: TSP
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TSP+: one node insertion

?
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An exemple: TSP

Remember…

Metric TSP is $NP$-hard.
It is approximable with ratio $3/2$ by Christofides’ algorithm
($val(S) \leq 3/2OPT$ on any instance)

Reoptimization version: TSP+
Easier (knowledge of an optimal solution on $I$) …
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### Reoptimization version: TSP+$+$

Easier (knowledge of an optimal solution on $I$) … … but still $NP$-hard. Proof: otherwise insert vertices one by one. Approximation algorithm?
First attempt

Best insertion: insert the new vertex at the best position in the current tour.

Good news

Best insertion is a 3/2-approximation algorithm (reoptimization) \cite{ABS03}.
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Best insertion is a 3/2-approximation algorithm (reoptimization) [ABS03]
\[ \text{OPT}(I') \geq \text{OPT}(I) + d(i, n+1) + d(j, n+1) - d(i, j) \]

\[ \text{val}(S) \leq \text{OPT}(I) + 2d(i, n+1) \]
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\[ \leq \text{OPT}(I) + d(i, n + 1) + d(j, n + 1) \]
\[ \leq \text{OPT}(I') + d(i, j) \leq \frac{3}{2}\text{OPT}(I') \]
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Bad news

Best insertion is a $3/2$-approximation algorithm (reoptimization)!
First attempt
Best insertion: insert the new vertex at the best position in the current tour.

Good news
Best insertion is a 3/2-approximation algorithm (reoptimization) \textit{in time }$O(n)$. [ABS03]

Bad news
Best insertion is a 3/2-approximation algorithm (reoptimization)!
An exemple: TSP

Better ratio?

→ Apply Best Insertion
→ Apply Ch. algorithm on the final instance
→ Output the best solution

This gives a $\frac{4}{3}$-approximate solution

[AEMP09]

Better ratio and better running time?

Yes: $\frac{4}{3}$-approximate solution in $O(n^2)$

[M15]
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[AE09]

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▶ Some other results
▶ Stability concerns
Some other results

- Many problems tackled from the point of view of reoptimization
  - Other network design problems: Steiner trees, variants of TSP (Max, with deadlines) ([BKK09, BZ12, KT15, ...])
  - Graph subset: independent set [BMP13], ...
  - Scheduling problems [BC14]
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- Negative results?
An example: reoptimizing TSP
Some other results
Stability concerns
→ How to rebuild a (optimal) solution ... without destroying all my initial solution??
→ Transition costs, stability of solutions
Example of TSP

Best insertion: $3/2$-approximation, in $O(n)$, removal of only one edge!
Best Insertion + Christofides: $4/3$, but may change everything.
Best insertion: 3/2-approximation, in $O(n)$, removal of only one edge!
Best Insertion + Christofides: 4/3, but may change everything. Can I do better than 3/2 (Best Insertion) without changing a lot of edges?
Example of TSP

$n$ vertices, $d(i, i + 1) = 1$
Example of TSP

\[ \text{opt} = 2(n - 1) \]
Example of TSP

Best Insertion: roughly $3^n$

If I remove at most $\alpha n$ edges: at best $\frac{2n}{2} + 2(1 - \alpha)n + \alpha n = (3 - \alpha)n$
Example of TSP

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Best Insertion: roughly $3n$
If I remove at most $\alpha n$ edges: at best
$2n/2 + 2(1 - \alpha)n + \alpha n = (3 - \alpha)n$
Example of TSP

If I remove at most $\alpha n$ edges: ratio no better than $\frac{3}{2} - \frac{\alpha}{2}$

OPT: roughly $2n$
Example of TSP

OPT: roughly $2n$

If I remove at most $\alpha n$ edges: ratio no better than $\frac{3}{2} - \frac{\alpha}{2}$
Two recent approaches

- Bound $k$ on the number of changes in the solution
  → Can I compute the best solution $S'$ for $I'$ at distance at most $k$ from $S$?
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- Bound $k$ on the number of changes in the solution
  - Can I compute the best solution $S'$ for $I'$ at distance at most $k$ from $S$?
  - Typically bruteforce is $n^k$: can I do it in FPT time?

*Parameterized complexity*

[AEFRS15]
Two recent approaches

- Find good tradeoffs between number of modifications and quality of solution.

[STT12]: if an optimal solution on $I'$ is at distance $d$ from $S$, can I compute a solution $S$ such that:

$\rightarrow val(S) \leq r_1 OPT(I')$

$\rightarrow d(S, S') \leq r_2 d$
How to combine:
→ efficiency (low complexity)
→ quality of solution (approximation ratio)
→ stability of solution (transition costs)

Hope to come back in one year with some answers. . .

Thanks for your attention


