Bidding on day-ahead markets: a dynamic programming approach

Jérôme De Boeck[1], M. Labbé[1],
É. Marcotte[2], P. Marcotte[3], G. Savard[4]

1 : Graphes et Optimisation Mathématique, Université Libre de Bruxelles - INRIA Lille
2 : ENGIE, 3 : Université de Montréal, DIRO
4 : École Polytechnique de Montréal, Département de mathématique et génie industriel

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Outline

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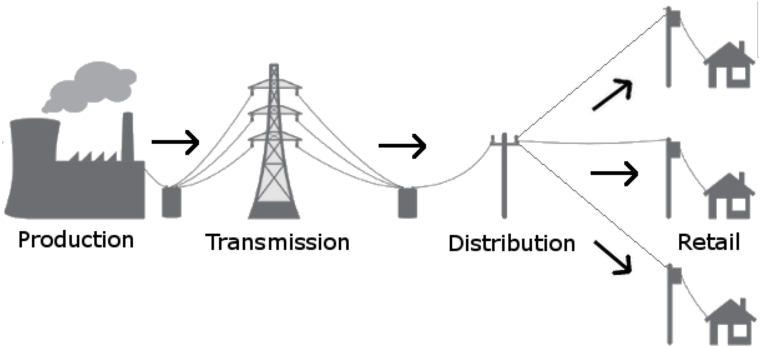
Electricity supply chain

Vertical structure in the 20^{th} century.

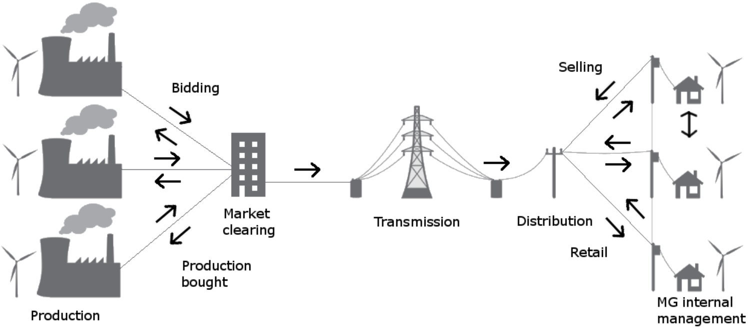
- National production and transmission companies
- Fossil fuel



Electricity supply chain

Horizontal structure appearing in the 21^{st} century.

- Renewable energy sources
- Deregulation of electricity markets
- Local production units
- Organization of customers into micro-grids



Deregulated electricity markets

- Competition between Generation Companies (GCs)
- The competition is regulated by a Market Operator
- The competition takes place in a day-ahead market
- The spot-price is influenced by bids made by the GCs to the MO
- Multi-period problem involving Unit Commitment for GCs
- Generally tackled through heuristic methods
- Studied through exact methods considering linear costs

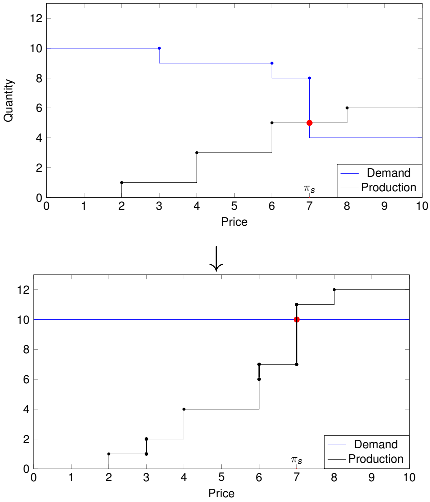
Outline

Bidding problem (BP)

- Day-ahead electricity market
  - Price-allocation for electricity π by the MO
  - Generation companies :
    - Production units J with linear cost c_j and capacity $\overline{q}_j$
    - Price-quantity bids (λ_j, q_j) for each generator
    - Maximize profit $\sum_{j \in J} (\pi - c_j) q_j^{MO}$
- NP-hard problem
- Different interests for actors with uncertainty
- Aim in considering an important number of scenarios

Bidding problem

- Step-wise demand function
- Transformable to constant demand problem



Bidding problem

- Scenarios $s \in S$:
  - Probability p_s
  - Demand d_s
  - Competitor generators J^c
  - Competitor bids $(\tilde{\lambda}_j^s, \tilde{q}_j^s)$
- Maximize expectation of profit with bids $(\lambda_1, q_1), ...(\lambda_m, q_m)$

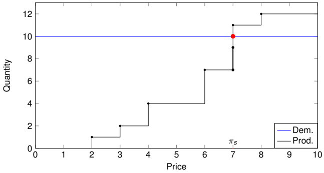
Bi-level Formulation

- Leader : our GC
- Follower : the Market Operator

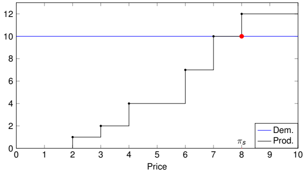
$$
\begin{aligned}
\max \quad & \sum_{s\in S} p_s \sum_{j\in J} (\pi^s - c_j) q_j^s \\
\text{s.t.} \quad & 0 \leq q_j \leq \overline{q_j}, \quad \forall j \in J \\
& 0 \leq \lambda_j \leq \overline{\lambda}, \quad \forall j \in J \\
& (q_j^s, \pi^s) \in \arg\min \sum_{s\in S} \Big(\sum_{j\in J} \lambda_j q_j^s + \sum_{j\in J^c} \tilde{\lambda}_j q_j^s \Big) \\
& \qquad \text{s.t.} \quad \sum_{j\in J\cup J^c} q_j^s = d_s, \quad \forall s \in S \quad (\pi_s) \\
& \qquad\qquad 0 \leq q_j^s \leq q_j, \quad \forall j \in J, s \in S \\
& \qquad\qquad 0 \leq q_j^s \leq \tilde{q}_j^s, \quad \forall j \in J^c, s \in S
\end{aligned}
$$

MIP Formulation

- Optimistic assumption :
  - Priority over competitors
  - Maximum possible spot price



$\longrightarrow$



Our bid $(7, 2)$ is sold $\qquad$ $\pi_s = 8$

- Case with fixed quantities (BP-Q) studied by M. Fampa et al. [1]
- Our MILP for BP ... abort ...

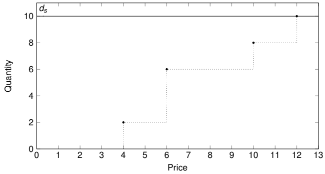
1. M. Fampa, L.A. Barroso, D. Candal and L.Simonetti, *Bilevel optimization applied to strategic pricing in competitive electricity markets*, Computational Optimization and Applications 39 (2008), 121-142

Outline

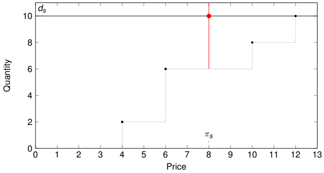
1. Deregulated electricity markets
2. Bidding problem
3. Dynamic programming approach
4. Variants of BP
5. Numerical results

Dynamic programming approach

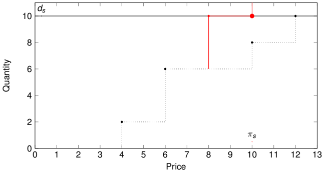
- Consider the following scenario :



- Select good prices → how ?
- Select good quantities → how ?



Bid everything at price 8



Bid 4 units at price 8

Dynamic programming approach

- $\Lambda = \{\lambda_1, ..., \lambda_n\}$, be the orders prices of competitors
- J_i, q_i : the generators/quantity bidden at the i^{th} price the λ_i
- $\mathcal{J}_i, Q_i$: cumulative generators/quantities

Two possible solution representations :

- Bids per generator $\{(\lambda_j, q_j)\}_{j \in J}$
- Bids per price $\{(J_i, q_i)\}_{\lambda_i \in \Lambda}$

Dynamic programming approach

DP based on Lemmas describing an existing optimal solution :

- Prices of bids are in Λ
- Thresholds for Q_i only depend on residual demands and production capacities
- Impact on profit adding a bid (J_i, q_i) to a solution with bids up to price λ_{i-1} only depends on J_i, Q_{i-1} and q_i

Dynamic programming approach

- $R^*_i(\mathcal{J}_i, Q_i) = \max_{(\mathcal{J}_{i-1}, Q_{i-1}) \in \Theta_{i-1}(\mathcal{J}_i, Q_i)} R^*_{i-1}(\mathcal{J}_{i-1}, Q_{i-1}) + \Delta((\mathcal{J}_{i-1}, Q_{i-1}), (J_i, q_i))$
- Optimum value of BP is $R^*_n(J, Q_n)$, for an unknown Q_n

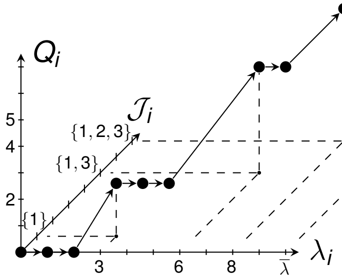
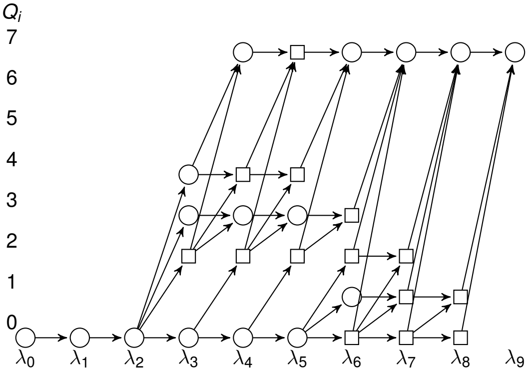


FIGURE – BP solution with bids $(3, 2), (7, 1)$ and $(6, 2)$

Outline

Bidding Problem Relaxation (BP-R)

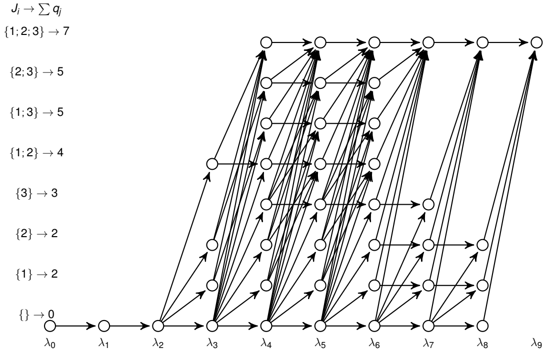
- Consider each generator can split its capacity over several bids
- Polynomial number thresholds for Q_i
- Optimal value can be found computing $R_n^*(J, \overline{q}^J)$



- Complexity in $O(n|S|^3|J^c|^2)$
- Upper bound for BP

Bidding Problem with fixed Quantities (BP-Q)

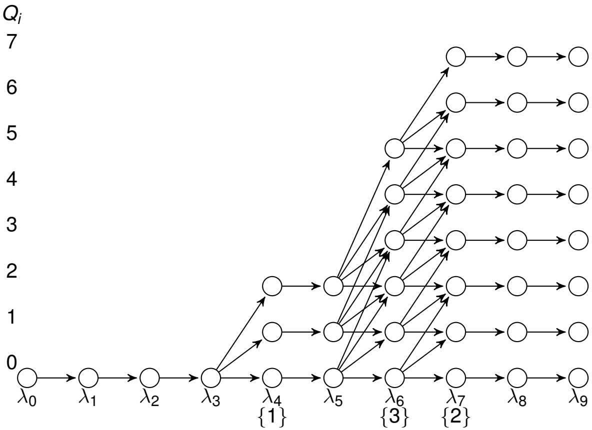
- Determine best prices for fixed q_j, studied by Fampa et al.
- Much more dense graph
- Optimal value can be found computing $R_n^*(J, \overline{q}^J)$



- Complexity in $O(n|S|2^{2m} \log m)$
- Provides feasible solutions for BP

Bidding Problem with fixed Prices (BP-P)

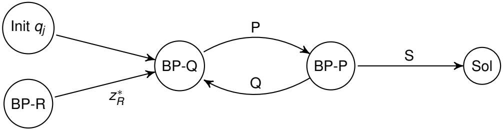
- Determine best quantities for fixed prices λ_j
- Heuristic threshold oracle considered
- Provides optimal solutions in specific cases



- Provides feasible solutions for BP

Combining BP-Q and BP-P

- Heuristic for BP solving BP-Q and BP-P iteratively (HBP)



- Aim at finding a Bertrand-Cournot equilibrium

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BPR : 350566.5
BPQ : 338074.4; 3.56%
(166.0 67.0), (164.0 529.0), (165.0 1300.0), (366.0 22.0), (366.0 1261.0), (367.0 2557.0)
0-BPP : 349989.03
(166.0 55.0), (164.0 529.0), (165.0 1182.5), (366.0 22.0), (366.0 1261.0), (367.0 546.0)
0-BPQ : 349989.06
(165.0 55.0), (166.0 529.0), (165.0 1182.5), (366.0 22.0), (366.0 1261.0), (367.0 546.0)
1-BPP : 349991.0
(165.0 67.0), (166.0 518.0), (165.0 1181.5), (366.0 22.0), (366.0 1261.0), (367.0 546.0)
1-BPQ : 349991.06
(165.0 67.0), (166.0 518.0), (166.0 1181.5), (366.0 22.0), (366.0 1261.0), (367.0 546.0)
2-BPP : 350013.0
(165.0 67.0), (166.0 529.0), (166.0 1170.5), (366.0 22.0), (366.0 1261.0), (367.0 546.0)
2-BPQ : 350013.03
(166.0 67.0), (166.0 529.0), (166.0 1170.5), (366.0 22.0), (366.0 1261.0), (367.0 546.0)
3-BPP : 350013.03; 0.15%
(166.0 67.0), (166.0 529.0), (166.0 1170.5), (366.0 22.0), (366.0 1261.0), (367.0 546.0)
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Outline

Instances

- Instances from the Brazilian Electric System National Operator (2008)[2]
- Algorithms implemented in JAVA 1.8
- Test performed with 2,3 GHz Intel Core i7 and 16 Gb of RAM
- Time in seconds, time HBP = time of iterations BP-Q, BP-P
- Gaps in %
- Averages made over 5 instances

2. M. Fampa and W. Pimentel, *An application of genetic algorithm to a bidding problem in electricity markets*, International Transactions in Operational Research 22 (2015), 97-111

Numerical results

$\lvert J \rvert = 6$	BP-R		HBP				BP-Q	
$\lvert S \rvert$	z_R^*	time	gap	c. gap	iter.	time	gap	time
25	361736	0.22	0.88	48.05	3.2	1.97	1.8	0.18
50	380260	1.68	1.31	34.51	2.8	4.89	2.1	0.47
100	318915	14.73	0.28	55.15	3.2	16.73	1.03	1.06
150	305972	47.14	0.6	22.9	2.4	12.28	0.77	1.56
200	295788	103.86	0.26	12.32	2.8	39.85	0.49	2.13

$\lvert S \rvert = 50$	BP-R		HBP				BP-Q	
$\lvert J \rvert$	z_R^*	time	gap	c. gap	iter.	time	gap	time
2	392752	1.73	1.74	10.83	2.0	0.68	1.87	0.03
4	402934	1.74	1.02	18.98	2.0	0.65	1.7	0.06
6	380260	1.64	1.31	34.51	2.8	4.97	2.1	0.59
8	380288	1.71	0.55	25.98	3.6	25.27	0.7	4.94
10	383738	1.82	0.81	32.77	3.2	285.63	1.25	65.75

Conclusion and futur directions

- Algorithms based on dynamic programming
- Good upper bound and feasible solution found for BP
- Quite resistant to the number of scenarios
- Integration of HBP to a local search mechanism
- Extension of HBP as a heuristic for multi-period bidding problems
- Adaptation of HBP to quadratic and affine costs

Thank you for your attention.

jdeboeck@ulb.ac.be