Stability versus Optimality in Optimization over Time

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PGMO days

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A PGMO project

Stability versus Optimality in Dynamic Environment Algorithmics (2016-2017)
Stability versus Optimality in Optimization over Time (2017-2018)

Resource allocation problems

- A set of resources, a set of demands
- Allocate resources to demands
- Constraints: availability of resources, compatibility between resources/demands,...
- Facility location problems, matching problems, scheduling problems,...

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Planning

- A time horizon: $t = 1, 2, \dots, T$
- At each time t: demands, available resources, constraints,... $\rightarrow$ an instance I_t.

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$\rightarrow$ General question: can we find a sequence of solutions $S_1, \ldots, S_t$ that both (1) are good ones (2) induce small modification costs (stability) ?

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Planning

$\rightarrow$ General question: can we find a sequence of solutions $S_1, \ldots, S_t$ that both (1) are good ones (2) induce small modification costs (stability) ?
$\rightarrow$ Central question: how far do I know the 'future'? Blind? Limited lookahead? Full knowledge?

Introduction

- Facility location problem (D. Eisenstat, C. Mathieu, N. Schabanel: Facility Location in Evolving Metrics. ICALP (2) 2014: 459-470.)
- Matching and spanning trees (A. Gupta, K. Talwar, U. Wieder. Changing bases: multistage Optimization for Matroids and Matchings. ICALP 2014: 563-575)
- Here: Santa Claus problem, with E. Bampis (PR LIP6,UPMC), and S. Mladenovic (Master 2 student, UPMC).

Santa Claus!

- A set of gifts (resources), a set of children (agents)
- Each gift has a value/utility
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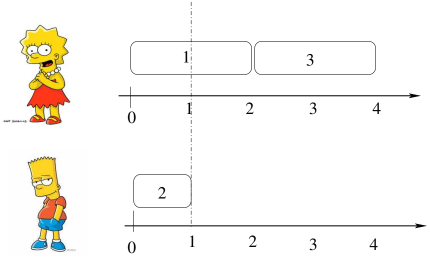
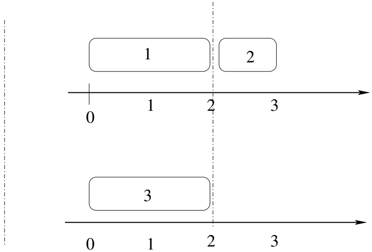
Santa Claus, Max-min fair allocation, maximize min makespan (scheduling)

Santa Claus!

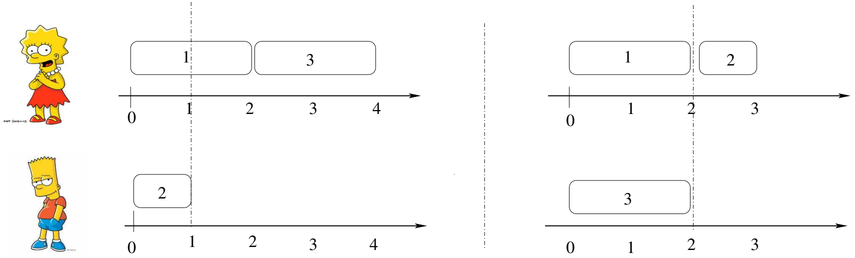


Solution with value 1



Solution with value 2

Santa Claus!



Solution with value 1

Solution with value 2

Complexity and approximation

r-appximation algorithm A: on every instance I:

$$val(A(I)) \geq r \times opt(I)$$

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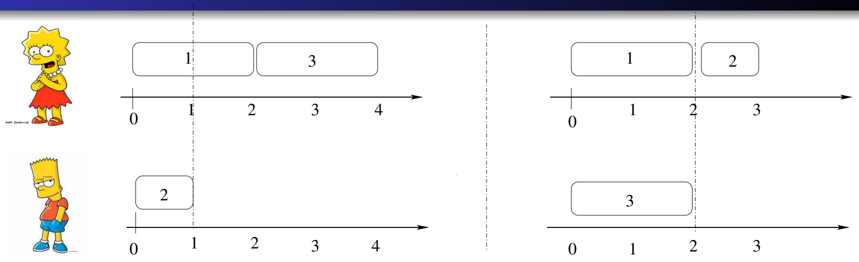

Solution with value 1 Solution with value 2

Complexity and approximation

- NP-hard
- $(1-\epsilon)$-approximation algorithm (PTAS) G. J. Woeginger: A PTAS for maximizing the minimum machine completion time. ORL, 1997
- Constant approximation algorithm for Santa Claus with restriction: a child can refuse (gives value 0) some gifts. C. Annamalai, C. Kalaitzis, O. Svensson: Combinatorial Algorithm for Restricted Max-Min Fair Allocation. ACM TALG, 2017

Santa Claus over Time

The model

- A sequence $I_1, \dots I_T$ of instances. Gifts and children remain, but value and compatibility may change over time.
- A solution sequence $(S_1, \dots, S_T)$. It has:
  - A 'Santa Claus' revenue: sum of values of S_t on I_t.

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  - A 'Santa Claus' revenue: sum of values of S_t on I_t.
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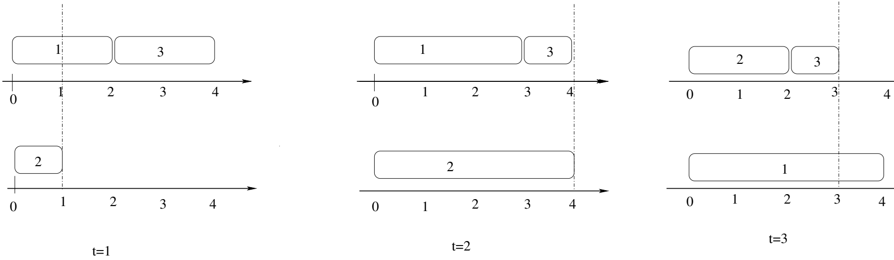
  - Aggregation: maximize Santa Claus revenue + transition revenue.



Santa Claus revenue: $1 + 4 + 3 = 8$
Transition revenue: $K(3 + 1) = 4K$
$\rightarrow$ value $8 + 4K$, to be maximized.

Simplest case

- No restriction: each child may receive any gift
- Full knowledge on the sequence of instances.

Approximation algorithm

- Can I find a solution sequence with good Santa Claus revenue?
  $\rightarrow$ Apply an algorithm on each instance: $(S_1, S_2, \ldots, S_t)$

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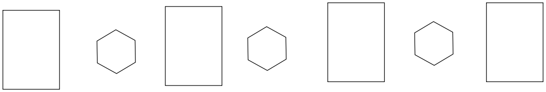
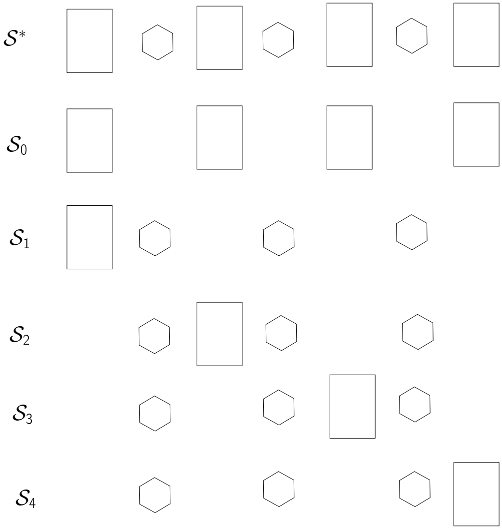
- Can I 'combine' them? $\rightarrow$ Full knowledge, I can compute and take the best one.

$\mathcal{S}^* = (S_1^*, \ldots, S_t^*)$ an optimal solution sequence:

$\mathcal{S}^*$ 

Approximation algorithms for fair resource allocation



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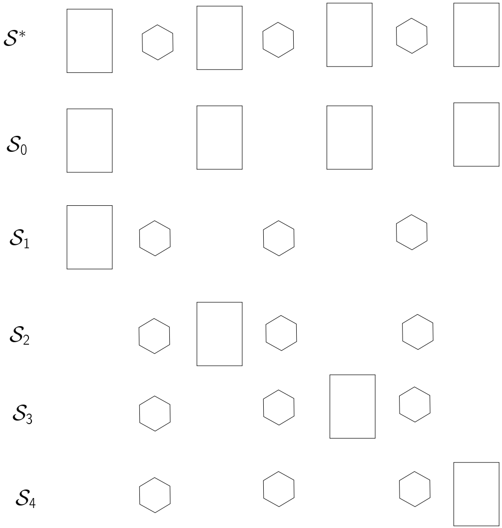

$\rightarrow$ ratio 7/4.

Approximation algorithms for fair resource allocations

Analysis

- If I use an exact algorithm for subroutine on Santa Claus: $2/3$ if $T = 2$ time steps, decreasing down to $1/2$ if no bound on time steps
- If I use a ρ-approximation algorithm for subroutine on Santa Claus:
  $2\rho/(\rho + 2)$ if $T = 2$ time steps, decreasing down to $\frac{\rho}{\rho+1}$ if no bound on time steps

Corollary

Santa Claus over Time is $1/2 - \epsilon$ approximable in polynomial time (full knowledge, no restriction).

Approximation algorithms for fair resource allocations

	No restriction gifts to children	Restriction gifts to children
Full knowledge (off-line)	$\frac{\rho}{\rho+1}$	(A)?
Blind (on-line)	(B)?	(C)?

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Yes. With a greedy algorithm I can still find a solution which maximizes the transition revenue.

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Santa Claus with Restriction over Time is constant approximable in polytime.

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Case (B)

Yes. I can manage to build on-line a solution combining good quality and transition revenue.
At each time step t: compare the current Santa Claus revenue, and the transition revenue if we keep the previous solution.

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Corollary

Santa Claus over Time is $1/2 - \epsilon$ approximable in polynomial time in a blind setting (on-line).

Approximation algorithms for fair resource allocations

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Case (C)

No! Impossible to get any approximation ratio.

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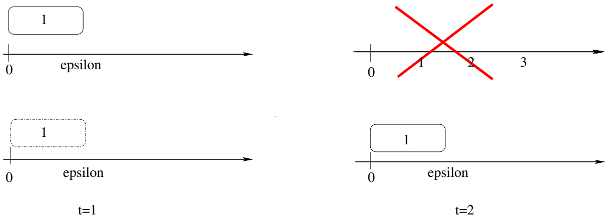

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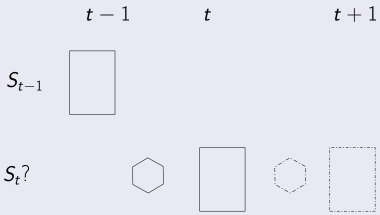

- Compare the three potential revenues ...

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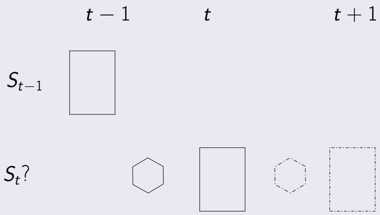

- Compare the three potential revenues ...
- But bet on the future only if it gives much more money!

Some results

	No restriction tasks on machines	Restriction tasks on machines
Full knowledge (off-line)	$\frac{\rho}{\rho+1}$	$\frac{\rho}{\rho+1}$
Blind (on-line)	$\frac{\rho}{\rho+1}$	No ...

Case (C)

But: with lookahead only 1 (I see I_t and I_{t+1}) I can do something. Ratio $\frac{\rho}{4\rho+2}$.

Corollary

Santa Claus with Restriction over Time is constant approximable in polytime even in a blind setting (on-line).

Conclusion

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- Inapproximability results?
- Other ways to aggregate quality and stability of solutions
- Model on instances evolution over time?
- Tailored model for a specific application?

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Thanks PGMO !