Unit Commitment under Market Equilibrium Constraints

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Introduction

Unit Commitment

Unit Commitment problem

Establish the energy output of a set of generation units over a multi-period time horizon, in order to satisfy a demand for energy, while minimizing the cost of generation and respecting technological restrictions of the units

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- Large scale mixed integer program
- Deterministic and robust versions studied in the literature
- Uncertainty models focus on renewable power output
- See Tahanan et al. (4OR 2015) for a survey

Day-ahead Electricity Markets

- Two-sided auctions
- Participants submit orders to buy *(retailers)* or sell *(producers)* electric power during some hours of the following day
- Market clearing: computed prices should ideally support a market equilibrium
- Difficulties with *non-convex bids* (e.g. block bids)
- See Madani and Van Vyve (EJOR 2015) for a survey

Objective

Assuming the producer sells (part of) the energy produced on the day-ahead market, how can we simultaneously decide a price bidding strategy and an optimal UC strategy taking the market reaction into account?

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Problems with decoupled decisions

- Optimal production from deterministic UC model not sold at the desired price on the market
  $\rightarrow$ possible loss
- Bid for higher quantities on the market to increase profit
  $\rightarrow$ infeasibilities in the UC

Integrated model

(Simplifying) Hypotheses

- Continuous bids
  - welfare maximization problem is an LP
- **Optimistic assumptions**:
  - perfect knowledge of the other players bids
  - price maker: always able to sell at clearing price
- Deterministic data

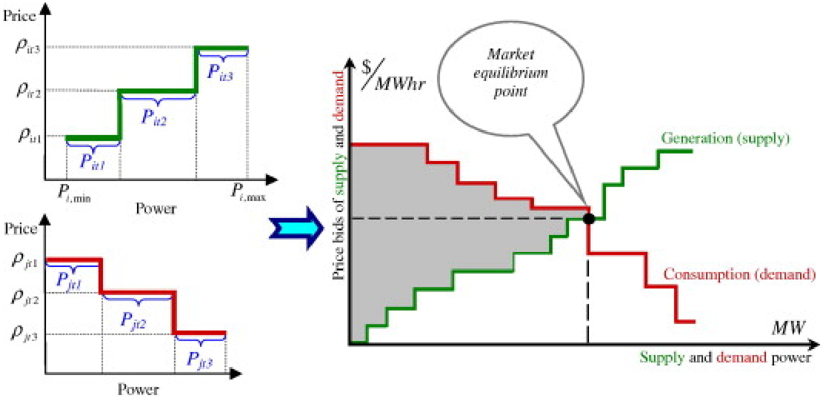
Data

- T: set of time periods
- $c^t(p^t)$: cost of generating p^t units in period t
- P: set of feasible solutions to the unit commitment problem (Carrión et al, IEEE Trans Power Systems 2006)
- S: set of competitors (sellers on the market)
- B: set of buyers on the market
- Q_s^t: quantity offered by seller s in period t
- Q_b^t: quantity offered by buyer b in period t
- π_s^t: price offered by seller s in period t
- π_b^t: price offered by buyer b in period t

Variables

- $p^t \geq 0$: Energy offered in period t
- $\lambda^t \geq 0$: Clearing price in period t
- x_s^t: proportion of quantity Q_s^t cleared in period t
- x_b^t: proportion of quantity Q_b^t cleared in period t
- y_s^t: welfare of seller s in period t
- y_b^t: welfare of buyer b in period t

Market equilibrium



Source: A. Ehsani, A.M. Ranjbar, M. Fotuhi-Firuzabad (2009), A proposed model for co-optimization of energy and reserve in competitive electricity markets, Applied Mathematical Modelling 33(1), 92-109

Welfare maximization problem

Assuming the quantity p^t offered on the market in each period is known, the market clearing LP is:

$$
\begin{aligned}
\max \quad & \sum_{b \in B} \pi_b^t Q_b^t x_b^t - \sum_{s \in S} \pi_s^t Q_s^t x_s^t \\
\text{s.t.} \quad & \sum_{b \in B} Q_b^t x_b^t - \sum_{s \in S} Q_s^t x_s^t = p^t && && (\lambda^t) \\
& 0 \leq x_b^t \leq 1 && b \in B && (y_b^t) \\
& 0 \leq x_s^t \leq 1 && s \in S && (y_s^t)
\end{aligned}
$$

Clearing price computation

The dual problem allows to compute the market clearing price and other players welfare:

$$\begin{aligned}
\min \quad & \lambda^t p^t + \sum_{b \in B} y_b^t + \sum_{s \in S} y_s^t & \\
\text{s.t.} \quad & Q_b^t \lambda^t + y_b^t \geq \pi_b^t Q_b^t & b \in B \\
& -Q_s^t \lambda^t + y_s^t \geq -\pi_s^t Q_s^t & s \in S \\
& y_b^t \geq 0 & b \in B \\
& y_s^t \geq 0 & s \in S
\end{aligned}$$

Combined model

- Objective of the leader: maximize profit
- Profit: difference between revenue (selling production at market clearing price) and production cost
- Constraints: technical constraints from UC problem

Bilevel formulation

$$
\begin{aligned}
\max_{p^t} \quad & \sum_{t \in T} \left(\lambda^t p^t - c^t(p^t) \right) \\
\text{s.t.} \quad & (p^t)_{t \in T} \in P \\
& \min_{\lambda^t, y_b^t, y_s^t} \quad \lambda^t p^t + \sum_{b \in B} y_b^t + \sum_{s \in S} y_s^t && t \in T \\
& \quad \text{s.t.} \quad Q_b^t \lambda^t + y_b^t \geq \pi_b^t Q_b^t && b \in B \\
& \qquad\quad - Q_s^t \lambda^t + y_s^t \geq -\pi_s^t Q_s^t && s \in S \\
& \qquad\quad y_b^t \geq 0 && b \in B \\
& \qquad\quad y_s^t \geq 0 && s \in S
\end{aligned}
$$

MIP reformulation

Model properties

- **Bilevel bilinear/linear model**
- As the second level is linear: use **duality** to transform the problem into a single level one
- Replace the second level objective by
  - dual constraints (i.e. the **welfare maximization problem**)
  - complementarity constraints

Single level reformulation

$$\begin{aligned}
\max \quad & \sum_{t \in T} \left(\lambda^t p^t - c^t(p^t) \right) \\
\text{s.t.} \quad & (p^t)_{t \in T} \in P \\
& \sum_{b \in B} Q_b^t x_b^t - \sum_{s \in S} Q_s^t x_s^t = p^t \qquad t \in T
\end{aligned}$$

$$\begin{aligned}
& Q_b^t \lambda^t + y_b^t \geq \pi_b^t Q_b^t \\
& x_b^t \left(Q_b^t \lambda^t + y_b^t - \pi_b^t Q_b^t \right) = 0 \\
& y_b^t \left(1 - x_b^t \right) = 0 \\
& y_b^t \geq 0 \\
& 0 \leq x_b^t \leq 1 \\
& \qquad b \in B, t \in T
\end{aligned}$$

$$\begin{aligned}
& -Q_s^t \lambda^t + y_s^t \geq -\pi_s^t Q_s^t \\
& x_s^t \left(-Q_s^t \lambda^t + y_s^t + \pi_s^t Q_s^t \right) = 0 \\
& y_s^t \left(1 - x_s^t \right) = 0 \\
& y_s^t \geq 0 \\
& 0 \leq x_s^t \leq 1 \\
& \qquad s \in S, t \in T
\end{aligned}$$

Elimination of y_b^t and y_s^t

Bilinear terms $x_b^t y_b^t$ can be eliminated by using

$$y_b^t \left(1 - x_b^t\right) = 0 \Rightarrow x_b^t y_b^t = y_b^t$$

and therefore

$$x_b^t \left(Q_b^t \lambda^t + y_b^t - \pi_b^t Q_b^t\right) = 0$$

can be rewritten as

$$y_b^t = \pi_b^t Q_b^t x_b^t - Q_b^t \lambda^t x_b^t$$

so y_b^t can be eliminated. A similar transformation allows to eliminate y_s^t.

Discrete choice model

Lemma

There exists an optimal solution such that $\lambda^t \in \{\pi_s^t\}_{s \in S} \cup \{\pi_b^t\}_{b \in B}$, *for all* $t \in T$

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Let:

- $I^t := \{1, \ldots, |\{\pi_s^t\}_{s \in S} \cup \{\pi_b^t\}_{b \in B}|\}$
- $\{\lambda_i^t\}_{i \in I^t} := \{\pi_s^t\}_{s \in S} \cup \{\pi_b^t\}_{b \in B}$.

Discrete choice model

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There exists an optimal solution such that $\lambda^t \in \{\pi_s^t\}_{s \in S} \cup \{\pi_b^t\}_{b \in B}$*, for all* $t \in T$

Let:

- $I^t := \{1, \ldots, |\{\pi_s^t\}_{s \in S} \cup \{\pi_b^t\}_{b \in B}|\}$
- $\{\lambda_i^t\}_{i \in I^t} := \{\pi_s^t\}_{s \in S} \cup \{\pi_b^t\}_{b \in B}$.

New variables:

$$z_i^t = \begin{cases} 1 & \text{if } \lambda^t = \lambda_i^t \\ 0 & \text{otherwise} \end{cases}$$

Linearization

- Substitute λ^t by $\sum_{i \in I^t} \lambda_i^t z_i^t$ with $\sum_{i \in I^t} z_i^t = 1$

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- The products of two continuous variables $\lambda^t p^t$, $\lambda^t x_b^t$ and $\lambda^t x_s^t$ are replaced by products of a binary and a continuous variable: $z_i^t p^t$, $z_i^t x_b^t$ and $z_i^t x_s^t$

Linearization

- Substitute λ^t by $\sum_{i \in I^t} \lambda_i^t z_i^t$ with $\sum_{i \in I^t} z_i^t = 1$
- The products of two continuous variables $\lambda^t p^t, \ \lambda^t x_b^t$ and $\lambda^t x_s^t$ are replaced by products of a binary and a continuous variable: $z_i^t p^t, \ z_i^t x_b^t$ and $z_i^t x_s^t$
- Classical linearization technique:
  - $P_i^t = z_i^t p^t$
  - $X_{ib}^t = z_i^t x_b^t$
  - $X_{is}^t = z_i^t x_s^t$

Strenghtening balance constraints

$$\sum_{b \in B} Q_b^t x_b^t - \sum_{s \in S} Q_s^t x_s^t = p^t \qquad t \in T$$

can be replaced by the stronger set of equations

$$\sum_{b \in B} Q_b^t X_{ib}^t - \sum_{s \in S} Q_s^t X_{is}^t = P_i^t \qquad i \in I^t, t \in T$$

$$\sum_{i \in I^t} P_i^t = p^t \qquad t \in T$$

(Strengthened) linearized model

$$
\begin{aligned}
\max \quad & \sum_{t \in T} \left(\sum_{i \in I^t} \lambda_i^t P_i^t \right) - c^t(p^t) \\
\text{s.t.} \quad & (p^t)_{t \in T} \in P \\
& \sum_{b \in B} Q_b^t X_{ib}^t - \sum_{s \in S} Q_s^t X_{is}^t = P_i^t && i \in I^t, t \in T \\
& \sum_{i \in I^t} P_i^t = p^t && t \in T \\
& \sum_{i \in I^t} z_i^t = 1 && t \in T \\
& 0 \leq P_i^t \leq \overline{Q}^t z_i^t && i \in I^t, t \in T \\
& P_i^t \geq p^t - \overline{Q}^t (1 - z_i^t) && i \in I^t, t \in T \\
& z_i^t \in \{0, 1\} && i \in I^t, t \in T
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{i \in I^t} \lambda_i^t (z_i^t - X_{ib}^t) - \pi_b^t (1 - x_b^t) \geq 0 \\
&-\sum_{i \in I^t} \lambda_i^t X_{ib}^t + \pi_b^t x_b^t \geq 0 \\
&\sum_{i \in I^t} X_{ib}^t = x_b^t \\
&0 \leq X_{ib}^t \leq z_i^t && i \in I^t \\
&X_{ib}^t \geq x_b^t + z_i^t - 1 && i \in I^t \\
&0 \leq x_b^t \leq 1 \\
& && b \in B, t \in T
\end{aligned}
$$

$$
\begin{aligned}
&-\sum_{i \in I^t} \lambda_i^t (z_i^t - X_{is}^t) + \pi_s^t (1 - x_s^t) \geq 0 \\
&\sum_{i \in I^t} \lambda_i^t X_{is}^t - \pi_s^t x_s^t \geq 0 \\
&\sum_{i \in I^t} X_{is}^t = x_s^t \\
&0 \leq X_{is}^t \leq z_i^t && i \in I^t \\
&X_{is}^t \geq x_s^t + z_i^t - 1 && i \in I^t \\
&0 \leq x_s^t \leq 1 \\
& && s \in S, t \in T
\end{aligned}
$$

MIP numerical experiments

Instances

- J: number of generators
- S: number of bids
- p: penetration of the GC in the market
- 24 time periods
- Demand following a classical duck curve
- UC data from Carrión
- 5 instances for each $(|J|, |p|, |S|)$ combination

Small instances

Instance			No strenghtening			Strengthened model		
$\lvert J\rvert$	p	$\lvert S\rvert$	LP Gap	Solved	Time	LP Gap	Solved	Time
10	5%	50	137.71	5	7.7	3.83	5	3.2
		100	148.39	5	135.9	3.68	5	6.4
	10%	50	158.93	5	93.6	4.11	5	3.7
		100	165.94	5	313.5	4.79	5	11.1
20	5%	50	137.74	5	71.7	3.83	5	20.4
		100	148.37	5	1153.9	3.68	5	45.6
	10%	50	158.91	5	702.5	5.11	5	15.6
		100	165.97	3	2965.0	4.21	5	22.9

Big instances

$\|J\|$	p	$\|S\|$	LP gap	Root gap(%)	Solved	Final gap	Time(s)	Nodes
20	10%	100	4.21	0.1	5	-	22.9	2136
		150	4.16	0.09	5	-	48.8	2302
		200	4.54	0.12	5	-	170.8	4457
	15%	100	5.72	0.13	5	-	39.7	1457
		150	5.66	0.17	5	-	319.6	5889
		200	5.94	0.15	4	0.08	1602.1	7433
	20%	100	7.78	0.19	5	-	477.9	5228
		150	7.42	0.2	5	-	1588.5	6138
		200	7.49	0.22	0	0.08	3602.4	6483
30	10%	100	4.2	0.11	5	-	106.4	5096
		150	4.15	0.11	5	-	252.6	5939
		200	4.53	0.14	5	-	963.1	7938
	15%	100	5.71	0.12	5	-	233.7	2643
		150	5.66	0.19	4	0.08	1721.4	9740
		200	5.94	0.18	3	0.09	2143.6	6058
	20%	100	7.77	0.24	5	-	1285.7	7037
		150	7.5	0.31	0	0.14	3601.7	6468
		200	7.83	0.57	0	0.48	3602.3	3883
40	10%	100	4.2	0.11	5	-	196.0	10533
		150	4.15	0.12	5	-	457.3	6991
		200	4.53	0.14	4	0.03	2267.0	11783
	15%	100	5.71	0.13	5	-	301.7	3804
		150	5.66	0.19	3	0.09	2936.9	10879
		200	5.94	0.17	3	0.08	2968.9	7698
	20%	100	7.78	0.26	4	0.07	2288.2	9072
		150	7.61	0.43	1	0.34	3564.2	6051
		200	7.79	0.54	0	0.46	3602.8	3492

Rolling Horizon Heuristic

Rolling Horizon Heuristic

- Classical approach for multi-period problems
- Static roll s, dynamic roll d
- Iteration t:
  - Binary variables for periods $0, \ldots, ts$ fixed from the solution of iteration $t-1$
  - Solve the problem with variables for $ts+1, \ldots, ts+s+d$ considered as binary, variables for periods $> ts+s+d$ continuous
- Iterate until $ts \geq T$

Numerical results

Instance			MIP	RH $s = 6$ $d = 1$			
$\|J\|$	p	$\|S\|$	Time(s)	Time(s)	Root gap	Min gap	Best gap
20	15%	100	39.7	29.9	0.16	0.04	0.04
		150	319.6	72.4	0.23	0.06	0.06
		200	1602.1	142.3	0.22	0.08	0.06
	20%	100	477.9	58.8	0.3	0.11	0.11
		150	1588.5	120.2	0.28	0.07	0.07
		200	3602.4	315.5	0.28	0.14	0.06
30	15%	100	233.7	32.3	0.15	0.03	0.03
		150	1721.4	79.0	0.23	0.06	0.05
		200	2143.6	163.9	0.2	0.05	0.02
	20%	100	1285.7	77.4	0.28	0.04	0.04
		150	3601.7	148.7	0.29	0.12	-0.02
		200	3602.3	430.4	0.26	0.16	-0.31
40	15%	100	301.7	43.0	0.19	0.06	0.06
		150	2936.9	103.5	0.24	0.08	0.05
		200	2968.9	192.9	0.2	0.06	0.03
	20%	100	2288.2	83.6	0.29	0.05	0.03
		150	3564.2	174.5	0.3	0.14	-0.13
		200	3602.8	452.3	0.27	0.19	-0.26

Concluding remarks

Summary

- Bilevel bilinear/linear model for integration of UC and market clearing strategies
- Reformulation and strenghtening as a tractable MIP
- Efficient rolling horizon heuristic for large instances

Future Research

- Strengthen the model (valid inequalities)
- Robust version of the model:
  - Uncertain demand curves in the day-ahead market model
  - Uncertainty in renewable energy in the UC model
- More realistic market mechanisms (non-convex bids)

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