Acyclic Gambling Games

Rida Laraki
CNRS, Université de Paris Dauphine, et École Polytechnique

With Jérôme Renault, Université de Toulouse Capitole

Introduction

- Bewlew-Kohlberg (resp. Mertens-Neyman) proved existence of the asymptotic (resp. the uniform value) in finite actions and state spaces stochastic games.

Introduction

- Bewlew-Kohlberg (resp. Mertens-Neyman) proved existence of the asymptotic (resp. the uniform value) in finite actions and state spaces stochastic games.
- Recently, two main conjectures on existence without finiteness was proved to be wrong.

- Bewlew-Kohlberg (resp. Mertens-Neyman) proved existence of the asymptotic (resp. the uniform value) in finite actions and state spaces stochastic games.
- Recently, two main conjectures on existence without finiteness was proved to be wrong.
- Vigeral (2013) proved that the limit of discounted values fail to exist in finite state space, compact actions sets and continuous payoffs and transitions.

Introduction

- Bewlew-Kohlberg (resp. Mertens-Neyman) proved existence of the asymptotic (resp. the uniform value) in finite actions and state spaces stochastic games.
- Recently, two main conjectures on existence without finiteness was proved to be wrong.
- Vigeral (2013) proved that the limit of discounted values fail to exist in finite state space, compact actions sets and continuous payoffs and transitions.
- Ziliotto (2013) proved divergence in many cases and in particular compact state space and finite actions sets, and continuous payoffs and transitions

- Bewlew-Kohlberg (resp. Mertens-Neyman) proved existence of the asymptotic (resp. the uniform value) in finite actions and state spaces stochastic games.
- Recently, two main conjectures on existence without finiteness was proved to be wrong.
- Vigeral (2013) proved that the limit of discounted values fail to exist in finite state space, compact actions sets and continuous payoffs and transitions.
- Ziliotto (2013) proved divergence in many cases and in particular compact state space and finite actions sets, and continuous payoffs and transitions
- To avoid oscillations, Botle-Gaubert-Vigeral (2015) proved convergence in finite state space, definable compact action sets and definable transition functions.

Introduction

- Mertens-Zamir, Mertens-Neyman-Rosenberg, Sorin-Vigeral, Laraki, Renault, Li-Venel proved existence of the asymptotic or uniform value in many classes "irreversible" stochastic games, with general compact action and state spaces.

Introduction

- Mertens-Zamir, Mertens-Neyman-Rosenberg, Sorin-Vigeral, Laraki, Renault, Li-Venel proved existence of the asymptotic or uniform value in many classes "irreversible" stochastic games, with general compact action and state spaces.
- Our paper provides a general definition of "irreversibility" in product stochastic games.

Introduction

- Mertens-Zamir, Mertens-Neyman-Rosenberg, Sorin-Vigeral, Laraki, Renault, Li-Venel proved existence of the asymptotic or uniform value in many classes "irreversible" stochastic games, with general compact action and state spaces.
- Our paper provides a general definition of "irreversibility" in product stochastic games.
- We show that "acyclicity" guarantees convergence and any weakening of it implies divergence.

Introduction

- Mertens-Zamir, Mertens-Neyman-Rosenberg, Sorin-Vigeral, Laraki, Renault, Li-Venel proved existence of the asymptotic or uniform value in many classes "irreversible" stochastic games, with general compact action and state spaces.
- Our paper provides a general definition of "irreversibility" in product stochastic games.
- We show that "acyclicity" guarantees convergence and any weakening of it implies divergence.
- We also extend the Mertenz-Zamir characterization of the asymptotic value in repeated games with incomplete information on both sides to product stochastic games.

General Notation

Given a compact metric space S, we denote by :

- $C(S)$: continuous, functions from $S \longrightarrow \mathbf{R}$

General Notation

Given a compact metric space S, we denote by :

- $C(S)$: continuous, functions from $S \longrightarrow \mathbf{R}$
- $\Delta(S)$: Borel probabilities over S.

General Notation

Given a compact metric space S, we denote by :

- $C(S)$: continuous, functions from $S \longrightarrow \mathbf{R}$
- $\Delta(S)$: Borel probabilities over S.
- $\delta_s \in \Delta(S)$: the Dirac measure on $s \in S$.

Given a compact metric space S, we denote by :

- $C(S)$: continuous, functions from $S \to \mathbf{R}$
- $\Delta(S)$: Borel probabilities over S.
- $\delta_s \in \Delta(S)$: the Dirac measure on $s \in S$.
- For $v \in C(S)$, $\tilde{v}$ is the affine extension to $\Delta(S)$:

$$\tilde{v}(p) = \boldsymbol{E}_p(v) \text{ for all } p \text{ in } \Delta(S).$$

General Notation

Given a compact metric space S, we denote by :

- $C(S)$: continuous, functions from $S \to \mathbf{R}$
- $\Delta(S)$: Borel probabilities over S.
- $\delta_s \in \Delta(S)$: the Dirac measure on $s \in S$.
- For $v \in C(S)$, $\tilde{v}$ is the affine extension to $\Delta(S)$:

$$\tilde{v}(p) = \boldsymbol{E}_p(v) \text{ for all } p \text{ in } \Delta(S).$$

- $\Delta(S)$ is endowed with the weak* topology.

General Notation

Given a compact metric space S, we denote by :

- $C(S)$: continuous, functions from $S \to \mathbf{R}$
- $\Delta(S)$: Borel probabilities over S.
- $\delta_s \in \Delta(S)$: the Dirac measure on $s \in S$.
- For $v \in C(S)$, $\tilde{v}$ is the affine extension to $\Delta(S)$:

$$\tilde{v}(p) = \boldsymbol{E}_p(v) \text{ for all } p \text{ in } \Delta(S).$$

- $\Delta(S)$ is endowed with the weak* topology.
- A compatible distance being the Kantorovich-Rubinstein metric : $d_{KR}(p, p') = \sup_{v \in E_1} |\tilde{v}(p) - \tilde{v}(p')|$ where E_1 is the set of 1-Lipschitz functions on S.

We consider a zero-sum product stochastic game where each player controls his gambling house.

- X (resp. Y) is a non empty compact metric set of states controlled by player 1 (resp. player 2).

Ingredient of a gambling game

We consider a zero-sum product stochastic game where each player controls his gambling house.

- X (resp. Y) is a non empty compact metric set of states controlled by player 1 (resp. player 2).
- The transitions of player 1 (resp. player 2) are given by a continuous multifunction $\Gamma : X \rightrightarrows \Delta(X)$ with non empty convex compact values, (resp. $\Lambda : Y \rightrightarrows \Delta(Y)$).

Ingredient of a gambling game

We consider a zero-sum product stochastic game where each player controls his gambling house.

- X (resp. Y) is a non empty compact metric set of states controlled by player 1 (resp. player 2).
- The transitions of player 1 (resp. player 2) are given by a continuous multifunction $\Gamma : X \rightrightarrows \Delta(X)$ with non empty convex compact values, (resp. $\Lambda : Y \rightrightarrows \Delta(Y)$).

Ingredient of a gambling game

We consider a zero-sum product stochastic game where each player controls his gambling house.

- X (resp. Y) is a non empty compact metric set of states controlled by player 1 (resp. player 2).
- The transitions of player 1 (resp. player 2) are given by a continuous multifunction $\Gamma : X \rightrightarrows \Delta(X)$ with non empty convex compact values, (resp. $\Lambda : Y \rightrightarrows \Delta(Y)$).
- The running payoff of P1 is given by a continuous mapping $u : X \times Y \longrightarrow \boldsymbol{R}$, the payoff to P2 is given by $-u$.

How the game is played ?

Given a discount factor $\lambda \in (0, 1]$ and an initial state $(x_1, y_1) \in X \times Y$, the game is played as follows :

- At any stage $t \geq 1$, the payoff of P1 is $u(x_t, y_t)$.

How the game is played ?

Given a discount factor $\lambda \in (0,1]$ and an initial state $(x_1, y_1) \in X \times Y$, the game is played as follows :

- At any stage $t \geq 1$, the payoff of P1 is $u(x_t, y_t)$.
- Independently P1 chooses $p_{t+1} \in \Gamma(x_t)$ and P2 chooses $q_{t+1} \in \Lambda(y_t)$.

How the game is played ?

Given a discount factor $\lambda \in (0,1]$ and an initial state $(x_1, y_1) \in X \times Y$, the game is played as follows :

- At any stage $t \geq 1$, the payoff of P1 is $u(x_t, y_t)$.
- Independently P1 chooses $p_{t+1} \in \Gamma(x_t)$ and P2 chooses $q_{t+1} \in \Lambda(y_t)$.
- New states x_{t+1} and y_{t+1} are selected according to p_{t+1} and q_{t+1}.

How the game is played ?

Given a discount factor $\lambda \in (0, 1]$ and an initial state $(x_1, y_1) \in X \times Y$, the game is played as follows :

- At any stage $t \geq 1$, the payoff of P1 is $u(x_t, y_t)$.
- Independently P1 chooses $p_{t+1} \in \Gamma(x_t)$ and P2 chooses $q_{t+1} \in \Lambda(y_t)$.
- New states x_{t+1} and y_{t+1} are selected according to p_{t+1} and q_{t+1}.
- x_{t+1} and y_{t+1} are publicly announced, and the play goes to stage $t + 1$.

How the game is played ?

Given a discount factor $\lambda \in (0,1]$ and an initial state $(x_1, y_1) \in X \times Y$, the game is played as follows :

- At any stage $t \geq 1$, the payoff of P1 is $u(x_t, y_t)$.
- Independently P1 chooses $p_{t+1} \in \Gamma(x_t)$ and P2 chooses $q_{t+1} \in \Lambda(y_t)$.
- New states x_{t+1} and y_{t+1} are selected according to p_{t+1} and q_{t+1}.
- x_{t+1} and y_{t+1} are publicly announced, and the play goes to stage $t+1$.
- The stream of payoffs is evaluated according to

$$\sum_t \lambda(1-\lambda)^{t-1} u(x_t, y_t).$$

Definition

Let v_λ *be the unique element of* $C(X \times Y)$ *s.t.* $\forall (x,y) \in X \times Y$,

$$\begin{aligned} v_\lambda(x,y) &= \max_{p \in \Gamma(x)} \min_{q \in \Lambda(y)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right) \\ &= \min_{q \in \Lambda(y)} \max_{p \in \Gamma(x)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right). \end{aligned}$$

The Problematic

Definition

Let v_λ *be the unique element of* $C(X \times Y)$ *s.t.* $\forall (x,y) \in X \times Y$,

$$\begin{aligned} v_\lambda(x,y) &= \max_{p \in \Gamma(x)} \min_{q \in \Lambda(y)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right) \\ &= \min_{q \in \Lambda(y)} \max_{p \in \Gamma(x)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right). \end{aligned}$$

- Existence and uniqueness of v_λ follow from standard fixed-point arguments (Sion's theorem).

The Problematic

Definition

Let v_λ *be the unique element of* $C(X \times Y)$ *s.t.* $\forall (x,y) \in X \times Y$,

$$v_\lambda(x,y) = \max_{p \in \Gamma(x)} \min_{q \in \Lambda(y)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right)$$
$$= \min_{q \in \Lambda(y)} \max_{p \in \Gamma(x)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right).$$

- Existence and uniqueness of v_λ follow from standard fixed-point arguments (Sion's theorem).
- $v_\lambda(x,y)$ is the value of the discounted gambling game.

The Problematic

Definition

Let v_λ *be the unique element of* $C(X \times Y)$ *s.t.* $\forall (x,y) \in X \times Y$,

$$\begin{aligned} v_\lambda(x,y) &= \max_{p \in \Gamma(x)} \min_{q \in \Lambda(y)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right) \\ &= \min_{q \in \Lambda(y)} \max_{p \in \Gamma(x)} \left(\lambda u(x,y) + (1-\lambda)\tilde{v}_\lambda(p,q)\right). \end{aligned}$$

- Existence and uniqueness of v_λ follow from standard fixed-point arguments (Sion's theorem).
- $v_\lambda(x,y)$ is the value of the discounted gambling game.
- Our goal is to establish tight conditions for convergence/divergence of v_λ (as $\lambda \to 0$) and a characterization of the limit.

Example 1 : red-and-black casino

- $X \subset [0, \infty)$ is the space of fortune.

Example 1 : red-and-black casino

- $X \subset [0, \infty)$ is the space of fortune.
- At each fortune $x \geq 0$, the gambler can stake any $s \in [0, x]$.

Example 1 : red-and-black casino

- $X \subset [0, \infty)$ is the space of fortune.
- At each fortune $x \geq 0$, the gambler can stake any $s \in [0, x]$.
- The gambler wins back the stake s and an equal amount more with probability $w \in]0, 1[$, but looses with probability $1 - w$:

$$\Gamma_w(x) = \{w\delta(x+s)+(1-w)\delta(x-s) \;:\; 0 \leq s \leq x\}, \qquad x \in X,$$

Example 1 : red-and-black casino

- $X \subset [0, \infty)$ is the space of fortune.
- At each fortune $x \geq 0$, the gambler can stake any $s \in [0, x]$.
- The gambler wins back the stake s and an equal amount more with probability $w \in]0, 1[$, but looses with probability $1 - w$:

$$\Gamma_w(x) = \{w\delta(x+s)+(1-w)\delta(x-s) \, : \, 0 \leq s \leq x\}, \qquad x \in X,$$

- Dubins & Savage (1965) introduced and solved the one player model.

Example 1 : red-and-black casino

- $X \subset [0, \infty)$ is the space of fortune.
- At each fortune $x \geq 0$, the gambler can stake any $s \in [0, x]$.
- The gambler wins back the stake s and an equal amount more with probability $w \in]0, 1[$, but looses with probability $1 - w$:

$$\Gamma_w(x) = \{w\delta(x+s)+(1-w)\delta(x-s) \,:\, 0 \leq s \leq x\}, \qquad x \in X,$$

- Dubins & Savage (1965) introduced and solved the one player model.
- What if there are two players competing $u(x, y) = x - y$?

Splitting Games

- X and Y are two convex compact metric sets.

Splitting Games

- X and Y are two convex compact metric sets.
- $\Gamma(x)$ is the set of probability distributions on X that are centered at x.

Splitting Games

- X and Y are two convex compact metric sets.
- $\Gamma(x)$ is the set of probability distributions on X that are centered at x.
- $\Gamma(y)$ is the set of probability distributions on Y that are centered at y.

Splitting Games

- X and Y are two convex compact metric sets.
- $\Gamma(x)$ is the set of probability distributions on X that are centered at x.
- $\Gamma(y)$ is the set of probability distributions on Y that are centered at y.
- Laraki (2001), Sorin (2002), Oliu-Barton (2015).

- Recall that $\Gamma : X \rightrightarrows \Delta(X)$.

Extending a Gambling House

- Recall that $\Gamma : X \rightrightarrows \Delta(X)$.
- $\tilde{\Gamma} : \Delta(X) \rightrightarrows \Delta(X)$ extends linearly Γ.

- Recall that $\Gamma : X \rightrightarrows \Delta(X)$.
- $\tilde{\Gamma} : \Delta(X) \rightrightarrows \Delta(X)$ extends linearly Γ.
- The graph of $\tilde{\Gamma}$ is defined as the closure of the convex hull of the graph of Γ, viewed as a subset of $\Delta(X) \times \Delta(X)$

$$\{(\delta_x, p), x \in X, p \in \Gamma(x)\}$$

Define inductively $(\tilde{\Gamma}^n)_n : \Delta(X) \rightrightarrows \Delta(X)$ by :

- $\tilde{\Gamma}^0(p) = \{p\}$ for every state p in $\Delta(X)$,

Iterating the Gambing House

Define inductively $(\tilde{\Gamma}^n)_n : \Delta(X) \rightrightarrows \Delta(X)$ by :

- $\tilde{\Gamma}^0(p) = \{p\}$ for every state p in $\Delta(X)$,
- For each $n \geq 0$, $\tilde{\Gamma}^{n+1} = \tilde{\Gamma}^n \circ \tilde{\Gamma}$.

Iterating the Gambing House

Define inductively $(\tilde{\Gamma}^n)_n : \Delta(X) \rightrightarrows \Delta(X)$ by :

- $\tilde{\Gamma}^0(p) = \{p\}$ for every state p in $\Delta(X)$,
- For each $n \geq 0$, $\tilde{\Gamma}^{n+1} = \tilde{\Gamma}^n \circ \tilde{\Gamma}$.
- $\Gamma^\infty(x)$ is the reachable set of Player 1 from state x in X and is defined as the closure of $\bigcup_{n\geq 0} \tilde{\Gamma}^n(\delta_x)$.

Define inductively $(\tilde{\Gamma}^n)_n : \Delta(X) \rightrightarrows \Delta(X)$ by :

- $\tilde{\Gamma}^0(p) = \{p\}$ for every state p in $\Delta(X)$,
- For each $n \geq 0$, $\tilde{\Gamma}^{n+1} = \tilde{\Gamma}^n \circ \tilde{\Gamma}$.
- $\Gamma^\infty(x)$ is the reachable set of Player 1 from state x in X and is defined as the closure of $\bigcup_{n \geq 0} \tilde{\Gamma}^n(\delta_x)$.
- Similarly we define $\Lambda^\infty(x)$.

Standard Conditions in Gambling Houses

- The game is called non-expansive if :

  $\forall x \in X, \forall x' \in X, \forall p \in \Gamma(x), \exists p' \in \Gamma(x'),\ s.t.\ d_{KR}(p, p') \leq d(x, x'),$

  and similarly for Λ

Standard Conditions in Gambling Houses

- The game is called non-expansive if :

  $$\forall x \in X, \forall x' \in X, \forall p \in \Gamma(x), \exists p' \in \Gamma(x'),\ s.t.\ d_{KR}(p, p') \leq d(x, x'),$$

  and similarly for Λ
- The game is called leavable if :

  $$\forall x \in X, \delta_x \in \Gamma(x) \text{ and } \forall y \in Y, \delta_y \in \Lambda(y)$$

- The game is called non-expansive if :

$$\forall x \in X, \forall x' \in X, \forall p \in \Gamma(x), \exists p' \in \Gamma(x'),\ s.t.\ d_{KR}(p,p') \leq d(x,x'),$$

and similarly for Λ

- The game is called leavable if :

$$\forall x \in X, \delta_x \in \Gamma(x) \textit{ and } \forall y \in Y, \delta_y \in \Lambda(y)$$

- Example 1 : When X and Y are finite, the game is non-expansive.

Standard Conditions in Gambling Houses

- The game is called non-expansive if :

$$\forall x \in X, \forall x' \in X, \forall p \in \Gamma(x), \exists p' \in \Gamma(x'),\ s.t.\ d_{KR}(p, p') \leq d(x, x'),$$

  and similarly for Λ
- The game is called leavable if :

$$\forall x \in X, \delta_x \in \Gamma(x) \textit{ and } \forall y \in Y, \delta_y \in \Lambda(y)$$

- Example 1 : When X and Y are finite, the game is non-expansive.
- Example 2 : Splitting games is non-expansive and leavable,

Standard Conditions in Gambling Houses

- The game is called non-expansive if :

$$\forall x \in X, \forall x' \in X, \forall p \in \Gamma(x), \exists p' \in \Gamma(x'),\ s.t.\ d_{KR}(p, p') \leq d(x, x'),$$

  and similarly for Λ
- The game is called leavable if :

$$\forall x \in X, \delta_x \in \Gamma(x) \text{ and } \forall y \in Y, \delta_y \in \Lambda(y)$$

- Example 1 : When X and Y are finite, the game is non-expansive.
- Example 2 : Splitting games is non-expansive and leavable,
- Example 3 : Red-black casinos are leavable. They are non-expansive if and only if $w \leq \frac{1}{2}$.

Excessive, Depressive, Balanced

Definition

1) v is excessive if : $\forall (x,y),\ v(x,y) = \max_{p \in \Gamma(x)} \tilde{v}(p,y)$.

Excessive, Depressive, Balanced

Definition

1) v is excessive if : $\forall(x, y),\ v(x, y) = \max_{p \in \Gamma(x)} \tilde{v}(p, y)$.
2) v is depressive if : $\forall(x, y),\ v(x, y) = \min_{q \in \Gamma(y)} \tilde{v}(x, q)$.

Excessive, Depressive, Balanced

Definition

1) v is excessive if : $\forall (x,y),\ v(x,y) = \max_{p\in\Gamma(x)} \tilde{v}(p,y)$.
2) v is depressive if : $\forall (x,y),\ v(x,y) = \min_{q\in\Gamma(y)} \tilde{v}(x,q)$.
3) v is balanced if $\forall (x,y)$,
$$v(x,y) = \max_{p\in\Gamma(x)} \min_{q\in\Gamma(y)} \tilde{v}(p,q) = \min_{q\in\Gamma(y)} \max_{p\in\Gamma(x)} \tilde{v}(p,q).$$

Excessive, Depressive, Balanced

Definition

1) v is excessive if : $\forall (x,y)$, $v(x,y) = \max_{p \in \Gamma(x)} \tilde{v}(p,y)$.
2) v is depressive if : $\forall (x,y)$, $v(x,y) = \min_{q \in \Gamma(y)} \tilde{v}(x,q)$.
3) v is balanced if $\forall (x,y)$,
$v(x,y) = \max_{p \in \Gamma(x)} \min_{q \in \Gamma(y)} \tilde{v}(p,q) = \min_{q \in \Gamma(y)} \max_{p \in \Gamma(x)} \tilde{v}(p,q)$.

Observe that under non-expansivity and compactness, a uniform limit v of (v_λ) is necessarily continuous, and balanced.

Main result for 1-player games

Theorem

Consider a one player gambling game non-expansive and leavable. Then (v_λ) uniformly converges to v (called the "réduite") v is the unique excessive function in $C(X)$ satisfying :

$$\forall x \in X, \exists p \in \Gamma^{\infty}(x),\ v(x) \leq \tilde{v}(p) \leq \tilde{u}(p).$$

$$\forall x \in X, v(x) \geq u(x)$$

Main result for 1-player games

Theorem

Consider a one player gambling game non-expansive and leavable.
Then (v_λ) uniformly converges to v (called the "réduite")
v is the unique excessive function in $C(X)$ satisfying :

$$\forall x \in X, \exists p \in \Gamma^\infty(x),\ v(x) \leq \tilde{v}(p) \leq \tilde{u}(p).$$

$$\forall x \in X, v(x) \geq u(x)$$

This is the "fundamental theorem" of gambling (Dubins-Savage).

Main result for 1-player games

Theorem

Consider a one player gambling game non-expansive and leavable. Then (v_λ) uniformly converges to v (called the "réduite") v is the unique excessive function in $C(X)$ satisfying :

$$\forall x \in X, \exists p \in \Gamma^\infty(x),\ v(x) \le \tilde{v}(p) \le \tilde{u}(p).$$

$$\forall x \in X, v(x) \ge u(x)$$

This is the "fundamental theorem" of gambling (Dubins-Savage).
Our theorem is a new characterization.

- Γ is weakly acyclic if there exists $\varphi \in C(X)$ such that :

$$\forall x \in X, \operatorname{Argmax}_{p \in \Gamma(x)} \tilde{\varphi}(p) = \{\delta_x\}$$

Defining Irreversibility in Gambling Games

- Γ is weakly acyclic if there exists $\varphi \in C(X)$ such that :

$$\forall x \in X, \mathrm{Argmax}_{p \in \Gamma(x)} \tilde{\varphi}(p) = \{\delta_x\}$$

- Γ is strongly acyclic if there exists $\varphi \in C(X)$ such that :

$$\forall x \in X, \mathrm{Argmax}_{p \in \Gamma^\infty(x)} \tilde{\varphi}(p) = \{\delta_x\}$$

- Γ is weakly acyclic if there exists $\varphi \in C(X)$ such that :

$$\forall x \in X, \mathrm{Argmax}_{p \in \Gamma(x)} \tilde{\varphi}(p) = \{\delta_x\}$$

- Γ is strongly acyclic if there exists $\varphi \in C(X)$ such that :

$$\forall x \in X, \mathrm{Argmax}_{p \in \Gamma^\infty(x)} \tilde{\varphi}(p) = \{\delta_x\}$$

Observe that our definition of acyclicity implies leavable.

Theorem

Assume the gambling game is non-expansive and strongly acyclic. Then (v_λ) *uniformly converges to the unique continuous function* $v : X \times Y \longrightarrow R$ *satisfying :*

$$v \text{ is excessive and depressive},$$

Theorem

Assume the gambling game is non-expansive and strongly acyclic. Then (v_λ) uniformly converges to the unique continuous function $v : X \times Y \longrightarrow R$ satisfying :

$$v \text{ is excessive and depressive},$$
$$P1 : \forall (x,y) \in X \times Y, \exists p \in \Gamma^\infty(x), v(x,y) \le \tilde{v}(p,y) \le \tilde{u}(p,y),$$

Main results for 2-player games

Theorem

Assume the gambling game is non-expansive and strongly acyclic. Then (v_λ) uniformly converges to the unique continuous function $v : X \times Y \longrightarrow R$ satisfying :

$$v \text{ is excessive and depressive},$$
$$P1 : \forall (x,y) \in X \times Y, \exists p \in \Gamma^{\infty}(x), v(x,y) \leq \tilde{v}(p,y) \leq \tilde{u}(p,y),$$
$$P2 : \forall (x,y) \in X \times Y, \exists q \in \Lambda^{\infty}(y), v(x,y) \geq \tilde{v}(x,q) \geq \tilde{u}(x,q).$$

Main results for 2-player games

Theorem

Assume the gambling game is non-expansive and strongly acyclic. Then (v_λ) *uniformly converges to the unique continuous function* $v : X \times Y \longrightarrow R$ *satisfying :*

$$v \text{ is excessive and depressive},$$
$$P1 : \forall (x,y) \in X \times Y, \exists p \in \Gamma^\infty(x), v(x,y) \leq \tilde{v}(p,y) \leq \tilde{u}(p,y),$$
$$P2 : \forall (x,y) \in X \times Y, \exists q \in \Lambda^\infty(y), v(x,y) \geq \tilde{v}(x,q) \geq \tilde{u}(x,q).$$

It applies to red-black casino when $w \leq \frac{1}{2}$ and to splitting games (with a new characterization of the so called Mertens Zamir system).

Proposition (A)

Assume the transitions are non-expansive. The family $(v_\lambda)_{\lambda \in (0,1]}$ is equicontinuous.

Proposition (A)

Assume the transitions are non-expansive. The family $(v_\lambda)_{\lambda \in (0,1]}$ is equicontinuous.

Proposition (B)

Assume the game is non-expansive and leavable. Let v be a limit point of $(v_\lambda)_{\lambda \in (0,1]}$ for the uniform convergence.
Then v is balanced and satisfies $P1$ and $P2$.

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \rightarrow_{n\rightarrow\infty} 0$.
Assume $v_{\lambda_n}(x,y) > u(x,y) + \lambda_n$ for n large.

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.
Assume $v_{\lambda_n}(x, y) > u(x, y) + \lambda_n$ for n large.
Define inductively $(p_t^n)_{t=0,\ldots,T_n}$ in $\Delta(X)$ by :

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.
Assume $v_{\lambda_n}(x,y) > u(x,y) + \lambda_n$ for n large.
Define inductively $(p_t^n)_{t=0,\dots,T_n}$ in $\Delta(X)$ by :

- $p_0^n = \delta_x$,

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.
Assume $v_{\lambda_n}(x,y) > u(x,y) + \lambda_n$ for n large.
Define inductively $(p_t^n)_{t=0,\ldots,T_n}$ in $\Delta(X)$ by :

- $p_0^n = \delta_x$,
- If (*) : $\tilde{v}_{\lambda_n}(p_t^n, y) > \tilde{u}(p_t^n, y) + \lambda_n$, let p_{t+1}^n be s.t.

$$\lambda_n \tilde{u}(p_t^n, y) + (1-\lambda_n)\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y).$$

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.
Assume $v_{\lambda_n}(x, y) > u(x, y) + \lambda_n$ for n large.
Define inductively $(p_t^n)_{t=0,\ldots,T_n}$ in $\Delta(X)$ by :

- $p_0^n = \delta_x$,
- If (*) : $\tilde{v}_{\lambda_n}(p_t^n, y) > \tilde{u}(p_t^n, y) + \lambda_n$, let p_{t+1}^n be s.t.

$$\lambda_n \tilde{u}(p_t^n, y) + (1 - \lambda_n)\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y).$$

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.
Assume $v_{\lambda_n}(x,y) > u(x,y) + \lambda_n$ for n large.
Define inductively $(p_t^n)_{t=0,\ldots,T_n}$ in $\Delta(X)$ by :

- $p_0^n = \delta_x$,
- If (*) : $\tilde{v}_{\lambda_n}(p_t^n, y) > \tilde{u}(p_t^n, y) + \lambda_n$, let p_{t+1}^n be s.t.

$$\lambda_n \tilde{u}(p_t^n, y) + (1-\lambda_n)\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y).$$

- If $(*)$ then :

$$\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y) + \frac{\lambda_n^2}{1-\lambda_n} > \tilde{v}_{\lambda_n}(p_t^n, y).$$

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.
Assume $v_{\lambda_n}(x,y) > u(x,y) + \lambda_n$ for n large.
Define inductively $(p_t^n)_{t=0,\dots,T_n}$ in $\Delta(X)$ by :

- $p_0^n = \delta_x$,
- If (*) : $\tilde{v}_{\lambda_n}(p_t^n, y) > \tilde{u}(p_t^n, y) + \lambda_n$, let p_{t+1}^n be s.t.

$$\lambda_n \tilde{u}(p_t^n, y) + (1-\lambda_n)\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y).$$

- If $(*)$ then :

$$\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y) + \frac{\lambda_n^2}{1-\lambda_n} > \tilde{v}_{\lambda_n}(p_t^n, y).$$

- Consequently, $\exists\, T_n$ such that $\tilde{v}_{\lambda_n}(p_{T_n}^n, y) \leq \tilde{u}(p_{T_n}^n, y) + \lambda_n$.

Proof proposition B

Let $(\lambda_n)_n$ such that $\|v_{\lambda_n} - v\| \to_{n\to\infty} 0$.
Assume $v_{\lambda_n}(x,y) > u(x,y) + \lambda_n$ for n large.
Define inductively $(p_t^n)_{t=0,\dots,T_n}$ in $\Delta(X)$ by :

- $p_0^n = \delta_x$,
- If (*) : $\tilde{v}_{\lambda_n}(p_t^n, y) > \tilde{u}(p_t^n, y) + \lambda_n$, let p_{t+1}^n be s.t.

$$\lambda_n \tilde{u}(p_t^n, y) + (1-\lambda_n)\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y).$$

- If $(*)$ then :

$$\tilde{v}_{\lambda_n}(p_{t+1}^n, y) \geq \tilde{v}_{\lambda_n}(p_t^n, y) + \frac{\lambda_n^2}{1-\lambda_n} > \tilde{v}_{\lambda_n}(p_t^n, y).$$

- Consequently, $\exists\, T_n$ such that $\tilde{v}_{\lambda_n}(p_{T_n}^n, y) \leq \tilde{u}(p_{T_n}^n, y) + \lambda_n$.
- Define $p^n = p_{T_n}^n$ and consider a limit point $p^* \in \Gamma^\infty(x)$. Then :

$$v(x,y) \leq \tilde{v}(p^*, y) \leq \tilde{u}(p^*, y).$$

Steps of the Proof

Proposition (C)

Assume the gambling game is weakly acyclic and non-expansive. If v in $C(X \times Y)$ is balanced, then v is excessive-depressive.

Proposition (C)

Assume the gambling game is weakly acyclic and non-expansive. If v in $C(X \times Y)$ is balanced, then v is excessive-depressive.

False without weak acyclicity.

Proposition (C)

Assume the gambling game is weakly acyclic and non-expansive. If v in $C(X \times Y)$ is balanced, then v is excessive-depressive.

False without weak acyclicity.

Proposition (D)

Assume the gambling game is strongly-acyclic. There is at most one excessive-depressive function v in $C(X \times Y)$ satisfying $P1$ and $P2$.

Proposition (C)

Assume the gambling game is weakly acyclic and non-expansive. If v in $C(X \times Y)$ is balanced, then v is excessive-depressive.

False without weak acyclicity.

Proposition (D)

Assume the gambling game is strongly-acyclic. There is at most one excessive-depressive function v in $C(X \times Y)$ satisfying $P1$ and $P2$.

Uniqueness is false with only weak-acyclicity.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y}\ v_1(x,y) - v_2(x,y).$$

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y}\ v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y)\in Z} \varphi(x) - \psi(y)$.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y}\ v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y)\in Z}\varphi(x) - \psi(y)$.
- By $P1$ and excessivity in x, there is p in $\Gamma^{\infty}(x_0)$ such that :

$$v_1(x_0, y_0) = \tilde{v}_1(p, y_0) \leq \tilde{u}(p, y_0)$$

.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y}\ v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y)\in Z}\varphi(x) - \psi(y)$.
- By $P1$ and excessivity in x, there is p in $\Gamma^\infty(x_0)$ such that :

$$v_1(x_0, y_0) = \tilde{v}_1(p, y_0) \leq \tilde{u}(p, y_0)$$

.

- Because v_2 is excessive in x, $v_2(p, y_0) \leq v_2(x_0, y_0)$.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y}\ v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y)\in Z}\varphi(x) - \psi(y)$.
- By $P1$ and excessivity in x, there is p in $\Gamma^\infty(x_0)$ such that :

$$v_1(x_0,y_0) = \tilde{v}_1(p,y_0) \leq \tilde{u}(p,y_0)$$

.

- Because v_2 is excessive in x, $v_2(p,y_0) \leq v_2(x_0,y_0)$.
- Thus, $\tilde{v}_1(p,y_0) - \tilde{v}_2(p,y_0) \geq v_1(x_0,y_0) - v_2(x_0,y_0)$.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y) \in X \times Y} \; v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y) \in Z} \varphi(x) - \psi(y)$.
- By $P1$ and excessivity in x, there is p in $\Gamma^\infty(x_0)$ such that :

$$v_1(x_0, y_0) = \tilde{v}_1(p, y_0) \leq \tilde{u}(p, y_0)$$

.

- Because v_2 is excessive in x, $v_2(p, y_0) \leq v_2(x_0, y_0)$.
- Thus, $\tilde{v}_1(p, y_0) - \tilde{v}_2(p, y_0) \geq v_1(x_0, y_0) - v_2(x_0, y_0)$.
- Since $(x_0, y_0) \in Z$, $\mathrm{Supp}(p) \times \{y_0\} \subset Z$.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y} \ v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y)\in Z} \varphi(x) - \psi(y)$.
- By $P1$ and excessivity in x, there is p in $\Gamma^\infty(x_0)$ such that :

$$v_1(x_0, y_0) = \tilde{v}_1(p, y_0) \leq \tilde{u}(p, y_0)$$

.

- Because v_2 is excessive in x, $v_2(p, y_0) \leq v_2(x_0, y_0)$.
- Thus, $\tilde{v}_1(p, y_0) - \tilde{v}_2(p, y_0) \geq v_1(x_0, y_0) - v_2(x_0, y_0)$.
- Since $(x_0, y_0) \in Z$, $\mathrm{Supp}(p) \times \{y_0\} \subset Z$.
- By definition of (x_0, y_0) : $\tilde{\varphi}(p) - \psi(y_0) \geq \varphi(x_0) - \psi(y_0)$.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y}\ v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y)\in Z}\varphi(x) - \psi(y)$.
- By $P1$ and excessivity in x, there is p in $\Gamma^\infty(x_0)$ such that :

$$v_1(x_0, y_0) = \tilde{v}_1(p, y_0) \leq \tilde{u}(p, y_0)$$

.

- Because v_2 is excessive in x, $v_2(p, y_0) \leq v_2(x_0, y_0)$.
- Thus, $\tilde{v}_1(p, y_0) - \tilde{v}_2(p, y_0) \geq v_1(x_0, y_0) - v_2(x_0, y_0)$.
- Since $(x_0, y_0) \in Z$, $\mathrm{Supp}(p) \times \{y_0\} \subset Z$.
- By definition of (x_0, y_0) : $\tilde{\varphi}(p) - \psi(y_0) \geq \varphi(x_0) - \psi(y_0)$.
- Acyclicity implies that $p = \delta_{x_0}$.
- Thus, $v_1(x_0, y_0) \leq u(x_0, y_0)$. Similarly, $v_2(x_0, y_0) \geq u(x_0, y_0)$.

Proof proposition D : a maximum principle

Assume v_1 and v_2 two excessive-depressive continuous functions satisfying $P1$ and $P2$ respectively. Show that $v_1 \leq v_2$.

- $v_1 - v_2$ being continuous on $X \times Y$, define the compact set :

$$Z = \mathrm{Argmax}_{(x,y)\in X\times Y}\ v_1(x,y) - v_2(x,y).$$

- Let $(x_0, y_0) \in \mathrm{Argmin}_{(x,y)\in Z}\varphi(x) - \psi(y)$.
- By $P1$ and excessivity in x, there is p in $\Gamma^\infty(x_0)$ such that :

$$v_1(x_0, y_0) = \tilde{v}_1(p, y_0) \leq \tilde{u}(p, y_0)$$

.

- Because v_2 is excessive in x, $v_2(p, y_0) \leq v_2(x_0, y_0)$.
- Thus, $\tilde{v}_1(p, y_0) - \tilde{v}_2(p, y_0) \geq v_1(x_0, y_0) - v_2(x_0, y_0)$.
- Since $(x_0, y_0) \in Z$, $\mathrm{Supp}(p) \times \{y_0\} \subset Z$.
- By definition of (x_0, y_0) : $\tilde{\varphi}(p) - \psi(y_0) \geq \varphi(x_0) - \psi(y_0)$.
- Acyclicity implies that $p = \delta_{x_0}$.
- Thus, $v_1(x_0, y_0) \leq u(x_0, y_0)$. Similarly, $v_2(x_0, y_0) \geq u(x_0, y_0)$.
- Consequently, $v_1(x_0, y_0) - v_2(x_0, y_0) \leq 0$.

Theorem

With only weak acyclicity in the main theorem and everything the same : the limit of v_λ may not exist.

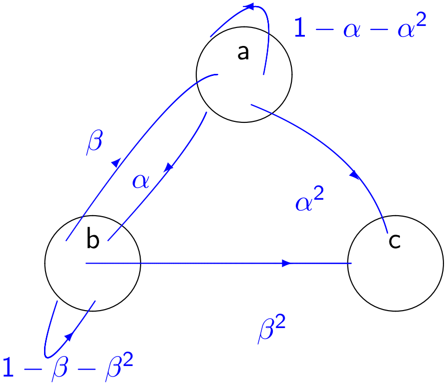
Theorem

With only weak acyclicity in the main theorem and everything the same : the limit of v_λ may not exist.

For a counter example, we adapt a counter example from Ziliotto.

A weakly acyclic game without convergence

We have $X = \{a, b, c\}$, c is absorbing, α and β belong to a compact set $I \subset [0, 1/4]$ such that 0 is in the closure of $I \backslash \{0\}$.



The gambling house of player 2 is a copy, with $Y = \{a', b', c'\}$ and a compact set of choices $I' \subset [0, 1/4]$ but I and I' may be different.

Payoff

The utility function u is supposed to be :

	a'	b'	c'
a	0	1	1
b	1	0	1
c	1	1	0

Player 2 wants to be at the same location as player 1, and player 1 the opposite.

Uniqueness in proposition D fails

The gambling game is weakly-acyclic

thus excessive-depessive = balanced.

Uniqueness in proposition D fails

The gambling game is weakly-acyclic

thus excessive-depessive = balanced.

However we can show that :

A function $v : X \times Y \longrightarrow \boldsymbol{R}$ is excessive-depressive and satisfies $P1$ and $P2$ if and only if there exists a number $x \in [0,1]$ s. t. :

$v =$

	a'	b'	c'
a	x	x	1
b	x	x	1
c	0	0	0

Uniqueness in proposition D fails

The gambling game is weakly-acyclic

thus excessive-depessive = balanced.

However we can show that :

A function $v : X \times Y \longrightarrow \boldsymbol{R}$ is excessive-depressive and satisfies $P1$ and $P2$ if and only if there exists a number $x \in [0,1]$ s. t. :

$v =$

	a'	b'	c'
a	x	x	1
b	x	x	1
c	0	0	0

There are infinitely many such functions.

Uniqueness in proposition D fails

The gambling game is weakly-acyclic

thus excessive-depessive = balanced.

However we can show that :

A function $v : X \times Y \longrightarrow \boldsymbol{R}$ is excessive-depressive and satisfies $P1$ and $P2$ if and only if there exists a number $x \in [0, 1]$ s. t. :

$v =$

	a'	b'	c'
a	x	x	1
b	x	x	1
c	0	0	0

There are infinitely many such functions.

Consequently : $P1$ and $P2$ + excessive-depressive are not sufficient to characterize a unique limiting value (if such a limit exists ! ! !).

Existence of the limit ?

Proposition

Assume $I' = [0, 1/4]$
A) If $I = [0, 1/4]$*, the limit value exists with* $x = \frac{1}{2}$*.*

Existence of the limit ?

Proposition

Assume $I' = [0, 1/4]$
A) If $I = [0, 1/4]$, *the limit value exists with* $x = \frac{1}{2}$.

B) If $I = \{0, \frac{1}{4}\}$, *the limit value exists with* $x = 0$.

Existence of the limit ?

Proposition

Assume $I' = [0, 1/4]$
A) If $I = [0, 1/4]$, *the limit value exists with* $x = \frac{1}{2}$.

B) If $I = \{0, \frac{1}{4}\}$, *the limit value exists with* $x = 0$.

C) If $I = \{\frac{1}{2^{2n}}, n \in \mathbb{N}^*\} \cup \{0\}$, (v_λ) *may diverge.*

Remarks

- Convergence in A) and B) is implied by Bolte-Gaubert-Vigeral 2015.

Remarks

- Convergence in A) and B) is implied by Bolte-Gaubert-Vigeral 2015.
- They show that any stochastic game with finitely many states, separable-definable transitions and definable compact action sets has a uniform value.

Remarks

- Convergence in A) and B) is implied by Bolte-Gaubert-Vigeral 2015.
- They show that any stochastic game with finitely many states, separable-definable transitions and definable compact action sets has a uniform value.
- Example in C is weakly acyclic, has finitely many states, separable-definable transitions but the compact action set of player 1 is not definable.

Remarks

- Convergence in A) and B) is implied by Bolte-Gaubert-Vigeral 2015.
- They show that any stochastic game with finitely many states, separable-definable transitions and definable compact action sets has a uniform value.
- Example in C is weakly acyclic, has finitely many states, separable-definable transitions but the compact action set of player 1 is not definable.
- Example C) may be related to a counter-example of Ziliotto 2013.

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.
- Moreover we offer a characterization of the asymptotic value that extends the Mertens-Zamir system of equations.

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.
- Moreover we offer a characterization of the asymptotic value that extends the Mertens-Zamir system of equations.
- We can prove existence of the uniform value under a bounded variation assumption (splitting games by Oliu-Barton).

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.
- Moreover we offer a characterization of the asymptotic value that extends the Mertens-Zamir system of equations.
- We can prove existence of the uniform value under a bounded variation assumption (splitting games by Oliu-Barton).
- Under progress :

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.
- Moreover we offer a characterization of the asymptotic value that extends the Mertens-Zamir system of equations.
- We can prove existence of the uniform value under a bounded variation assumption (splitting games by Oliu-Barton).
- Under progress :
  - Non existence of the uniform value without bounded variation ;

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.
- Moreover we offer a characterization of the asymptotic value that extends the Mertens-Zamir system of equations.
- We can prove existence of the uniform value under a bounded variation assumption (splitting games by Oliu-Barton).
- Under progress :
  - Non existence of the uniform value without bounded variation ;
  - Convergence and characterization without leavablity ;

Conclusion, Extensions

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.
- Moreover we offer a characterization of the asymptotic value that extends the Mertens-Zamir system of equations.
- We can prove existence of the uniform value under a bounded variation assumption (splitting games by Oliu-Barton).
- Under progress :
  - Non existence of the uniform value without bounded variation ;
  - Convergence and characterization without leavablity ;
  - Convergence when only one house is strongly acyclic ;

- The recent counter examples rise the question : under which conditions does v_λ converges for compact state spaces ?.
- It was in the "air" that a sufficient (and perhaps necessary) condition is "irreversiblity".
- Our paper made this statement precise in the class of product stochastic games : if the game is strongly acyclique v_λ converges and if it is weakly acyclique, it may diverge.
- Moreover we offer a characterization of the asymptotic value that extends the Mertens-Zamir system of equations.
- We can prove existence of the uniform value under a bounded variation assumption (splitting games by Oliu-Barton).
- Under progress :
  - Non existence of the uniform value without bounded variation ;
  - Convergence and characterization without leavablity ;
  - Convergence when only one house is strongly acyclic ;
  - Relaxing separability (the dependent case).