Using non-parametric statistical tests to compare solutions in evolutionary framework for maintenance schedule optimisation

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1. Maintenance schedule optimisation

2. Proposed Method
   - Using statistical tests in EAs
   - Racing Algorithm

3. Experiments
   - Experimental setup
   - Results

4. Going further
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Simulation-optimisation for maintenance scheduling

The problem:

- Given a set of $c$ components, find the maintenance dates for each component $X = \{x_1, \ldots, x_c\} \in \mathbb{R}^c$ to maximise the Net Present Value

You need two things:

- A realistic model to simulate the life cycle of the system/asset/components
- An good optimisation algorithm
Event Model

- Preventive Spare Supply
- Unplanned Spare Supply
- Spare Delivery
- Make Replacement
- Replenish Old component
- Awaiting Replacements?
- Corrective Replacement Needed
- Preventive Replacement Planned
- Failures

Decision points:
- Spare Not needed or available?
- Yes: Make Replacement
- No: Preventive Spare Supply
- Yes: Preventive Replacement Planned
- Yes: Failures

Symbols:
- Deterministic Event
- Probabilistic Event
Instance definition

- Number of components, $c$.
- Weibull distribution parameters: $\{\lambda, \beta, \gamma\}^c$.
- Stock level.
- Yearly cost of unavailability.
- Failure cost.
- Maintenance cost.
- Spare part cost.
- Storage cost.
- Time to buy a spare part after failure.
- Preventive maintenance time.
Calculating NPV

Given a set of failure times obtained from the failure probabilities of each component

Preventive Maintenance $x_1$ (Rejuvenating) – failure $t_1$ – Reference strategy

Preventive Maintenance $x_2$ (Rejuvenating) – failure $t_2$ – Assessed strategy

Life Span

Reference strategy
Assessed strategy
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The situation

What we have:

• Optimisation algorithm based on the comparison of solutions using deterministic cost function.

• A stochastic cost function.

What can be done:

• Use MonteCarlo simulation and use mean or median to evaluate solutions:
  • Problem: how many replications do we need to have a robust comparison?

• Use surrogate models to approximate the stochastic code
  • Problem: To what extent can we trust our surrogate model?
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• Since the cost function returns a random sample, we can apply statistical tests.
• Sample size can be augmented until significance is found.
• Eliminates the bias of the arbitrary number of replications.
Non-parametric statistical tests

• Let us consider two of them: Friedman and Wilcoxon test
• Comparison on related samples
• In this case let’s consider:
  • a set $n$ failure scenarios $S = \{s_1, ..., s_n\}$ each of which is defined by the failure dates of each component.
  • a set of $k$ maintenance schedule $P = \{p_1, ..., s_k\}$
  • the NPV of each schedule $p_j$ can be evaluated on each failure scenario $s_i$ by $f(p_j, s_i)$.

• Every comparisons of $p_j$ is performed on the same failure scenarios
• Purposes:
  • Remove the bias of comparing solutions on different failure scenarios
  • Use the only number of simulations necessary to decide whether a solution should be maintained in the population or not.
Friedman’s Test

• Answers the question: ”in a set of $k$ sample, do at least two of the samples represent populations with different median values?”

• Given a matrix of results $\{f(p_i, s_j)\}_{n \times k}$

• Replace the results by the ranking of each $p_i$ on each $s_i$, giving a matrix of ranks $\{r_{ij}\}_{n \times k}$

• Apply the Friedman’s test to detect statistical difference

• If statistical differences, apply Holm’s procedure to identify statistical differences on ranks between the highest ranked $p_i$ and the rest of the population.
Wilcoxon test

Procedure

- Also known as signed-rank test.
- Comparison on each pair of instances (failure scenarios) of two related samples.

Given two maintenance schedule \( p_1 \) and \( p_2 \) simulated on \( n \) failure scenarios giving two samples \( \{f(p_1, s_1), ... f(p_1, s_n)\} \) and \( \{f(p_2, s_1), ... f(p_2, s_n)\} \)

1. Calculate each paired difference, \( d_i = f(p_1, s_i) - f(p_2, s_i) \)
2. Rank the \( d_i \) ignoring the signs: assign rank 1 to the smallest \( |d_i| \), rank 2 to the next etc. using fractional ranking.
3. Label each rank with its sign, according to the sign of \( d_i \).
4. Calculate \( R^+ \), the sum of the ranks of the positive \( d_i \) and \( R^- \), the sum of the ranks of the negative \( d_i \).
5. Choose \( R = \min(R^+, R^-) \)
6. Calculate \( p - value \) using normal approximation
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Racing Algorithm

Origins:
- Initially proposed in machine learning to test on data
- Further used in parameter tuning (IRACE)

Idea:
- Perform selection over a population
- Iteratively test solutions on instances until statistical difference is reached
### Racing Example

<table>
<thead>
<tr>
<th>$S$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$p_5$</th>
<th>$p_6$</th>
<th>$p_7$</th>
<th>$p_8$</th>
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<td>$s_{13}$</td>
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<tr>
<td>$s_{14}$</td>
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<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
<td>=</td>
</tr>
</tbody>
</table>
Racing pseudo-code

1: Generate initial population at random $P = \{p_1, \ldots, p_k\}$
2: Generate set of failure scenarios $S = \{s_1, \ldots, s_n\}$
3: $\lambda = \mu k$
4: \textbf{while} !termination criterion not reached \textbf{do}
5: \hspace{1em} \textbf{while} $k > t$ \textbf{do}
6: \hspace{2em} $i = i + 1$
7: \hspace{1em} \textbf{for} $j$ in 1 to $k$ \textbf{do}
8: \hspace{2em} Evaluate $x_{ij} = f(p_j, s_i)$
9: \hspace{1em} \textbf{end for}
10: \hspace{1em} Perform Holm’s procedure on $\{x_{ij}\}_{i,k}$
11: \hspace{1em} Remove from $P$ all individuals $p_j$ if significantly worse than the ”best” individual in $P$
12: \hspace{1em} $k = |P|$
13: \hspace{1em} \textbf{end while}
14: \hspace{1em} Generate new population from $P$
15: \textbf{end while}
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Instance generation

- Number of components $c = \{4, 8, 12, 16, 20, 40\}$
- Weibull distribution parameters:
  - $\Lambda = \{\lambda_1, ..., \lambda_c\} \in [0.01, 0.1]^c$
  - $B = \{\beta_1, ..., \beta_c\} \in [1, 4]^c$
  - $\Gamma = \{\gamma_1, ..., \gamma_c\}$ always equal to 0
- Components age at $t = 0 : 0$
- stock level = 0.25$c$
- Yearly cost of unavailable = 10000
- Failure cost $C_{\text{failure}} = 1000$
- Maintenance cost $C_{\text{maintenance}} = 100$
- Spare part cost $C_{\text{spare}} = 10$
- Storage cost $C_{\text{storage}} = 1$
- Time to buy a spare part after failure $t_{\text{failure}} = 1$
- Preventive maintenance time $t_{\text{maintenance}} = 1$
Experiments

Benchmark:
- 25 instances in each dimensions, generating random failure probabilities.
- Each instance is ran 25 times with different seeds (using the same failure scenarios in each run)
- A budget of 500K simulations

Racing algorithm tested:
- Univariate EDA:
  - Generate new solutions from the $x_i = \mathcal{N}(\mu_i, \sigma_i^2)$
- Statistical tests:
  - Friedman and Wilcoxon
  - Significance level: $\alpha = \{0.05, 0.1\}$
- Population size : 50
- Elitism : $\lambda = 0.5$
- Each instance is ran 25 times with different seeds (using the same failure scenarios in each run)
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Comparison of the statistical tests

Table: Mean ranking of each statistical tests over Friedman’s $p - value = 1.48E-10$

<table>
<thead>
<tr>
<th>Test</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>wilcoxon-0.05</td>
<td>2.66</td>
</tr>
<tr>
<td>wilcoxon-0.1</td>
<td>3.89</td>
</tr>
<tr>
<td>friedman-0.05</td>
<td>1.71</td>
</tr>
<tr>
<td>friedman-0.1</td>
<td>1.72</td>
</tr>
</tbody>
</table>

Table: Holm / Hochberg Table for $\alpha = 0.05$

<table>
<thead>
<tr>
<th>i</th>
<th>algorithm</th>
<th>$z = (R_0 - R_i)/SE$</th>
<th>$p$</th>
<th>Holm/Hochberg/Hommel</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>wilcoxon-0.1</td>
<td>14.62</td>
<td>1.98E-48</td>
<td>0.016</td>
</tr>
<tr>
<td>2</td>
<td>wilcoxon-0.05</td>
<td>6.39</td>
<td>1.60E-10</td>
<td>0.025</td>
</tr>
<tr>
<td>1</td>
<td>friedman-0.1</td>
<td>0.089</td>
<td>0.93</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Racing effect

(a) $\lambda = 0.1$, gain = 0.325

(b) $\lambda = 0.25$, gain = 0.555

(c) $\lambda = 0.5$, gain = 0.755

(d) $\lambda = 0.75$, gain = 0.891
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Statistical tests in Steady-State EA

• One on one comparison: using Wilcoxon test
• Risk of comparing two similar solutions with no statistical different
  • Waste of simulations to compare equivalent solutions
  • When to stop in the case of equivalent solution? (back to the original problem)
• Which replacement strategy do you use?
Thanks!
Questions?
Code?
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