Piecewise linear bounding of energy conversion functions and resulting MILP-based solution methods

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Plan

1. Motivation and State of the art
2. From continuous to (dis)continuous pwl $\delta$-approximators
3. From pwl $\delta$-approximators to pwl $\delta$-bounding
4. Weaknesses and limitations of a $\delta$-bounding/$\delta$-approximation
5. Using $\epsilon$-relative tolerance
6. Computational evaluation
7. Conclusion
Energy in hybrid electric vehicles

Electric propulsion motor powered by:
- onboard generator: e.g. hydrogen fuel cell (FC)
- reversible source: e.g. supercapacitor (SE)

Given the power request of a driver on a predefined road section ...

... and the characteristics of the energy sources: power limitations (kW), efficiency (%), storage capacity (kWs) ...

... Find at each instant the optimal power split between the energy sources to minimize the total fuel consumption.
Mathematical model

\[
\min \sum_{i=1}^{n} f_{FC}(x_i)
\]  

s.t. Power demand satisfaction

\[
x_i + y_i \geq P_i, \quad \forall i \in [1...n]
\]  

Final SE energy level higher or equal to initial energy level

\[
\sum_{i=1}^{n} f_{SE}(y_i) \leq 0
\]  

SE energy level within bounds

\[
E_{SE_0} - E_{SE_{\text{max}}} \leq \sum_{k=1}^{i} f_{SE}(y_k) \leq E_{SE_0} - E_{SE_{\text{min}}}, \quad \forall i \in [1...n]
\]  

Variables domains

\[
x_i \in [0, P_{FC_{\text{max}}}], y_i \in [P_{SE_{\text{min}}}, P_{SE_{\text{max}}}], \quad \forall i \in [1...n]
\]
Water pumping and desalination process

Electrical model

- \( V_m, I_m \): electrical tension, current
- \( T_m \): motor electromag. torque
- \( \Omega \): rotation speed
- \( k_\Phi \): torque equivalent coefficient
- \( r \): stator resistance

Electric motor equations (inertia neglected):

\[
V_m = rI_m + k_\Phi \Omega \tag{6}
\]
\[
T_m = \Phi_m I_m \tag{7}
\]

Electrical power needed: \( P_e = V_m I_m \).

Pressure drop in the pipe

- \( \Delta \text{Pipe}, \rho \): pressure drop, water density
- \( h \): height of water pumping

Static+Dynamic pressure

\[
\Delta \text{Pipe} = kq^2 + \rho gh \tag{8}
\]

Mechanical-Hydraulic conv.

- \( P_p \): output pressure
- \( q \): debit of water
- \( a, b \): non linear girator coefs
- \( c \): hydraulic friction
- \( p_0 \): suction pressure
- \( s_p + s_m \): mechanical losses

Static equations of the motor-pump (mechanical inertia neglected):

\[
P_p = (a\Omega + bq)\Omega - (cq^2 + p_0) \tag{9}
\]
\[
T_m = (a\Omega + bq)q + (s_m + s_p)\Omega \tag{10}
\]
Subsystem pump 2 + Reverse Osmosis module is modeled with equation $P_e = f^2(q, h)$ as follows:

$$P_e = r \cdot K(q_c, h) + ((s_m+s_p) \cdot \Omega(q_c, h) + (q_c + F(q_c)/R_{Me}) \cdot M(q_c, h)) \cdot \Omega(q_c, h)$$

where

$$\begin{align*}
    F(q_c) &= (R_{Mod} + R_{Valve}) \cdot q_c^2 \\
    G(q_c) &= (b \cdot (q_c + F(q_c)/R_{Me})) \\
    M(q_c, h) &= a \cdot \Omega(q_c, h) + G(q_c) \\
    \Omega(q_c, h) &= \frac{-G(q_c) + \sqrt{G(q_c)^2 - 4 \cdot a \cdot (-\rho_0 + \rho g (h - l_{out}) + (k+c) \cdot ((q_c + F(q_c)/R_{Me})^2 + F(q_c))))}}{2 \cdot a} \\
    K(q_c, h) &= (((s_m + s_p) \cdot \Omega(q_c, h) + (q_c + F(q_c)/R_{Me}) \cdot (a \cdot \Omega(q_c, h) + G(q_c))) / k_{\phi})^2
\end{align*}$$
Mathematical model

\[
\min \sum_{i \in \mathbb{N}} \Delta^t r_i \\
\text{s.t. Check if water tanks are full} \quad r_0 = 1 \\
\quad r_i - r_{i-1} \leq 0, \quad \forall i \in \mathbb{N}_1 \\
\quad \sum_{j \in \mathbb{N}_T} h^j_i + (\sum_{j \in \mathbb{N}_T} h^j_{\max}) r_i \geq \sum_{j \in \mathbb{N}_T} h^j_{\max}, \quad \forall i \in \mathbb{N}_1
\]

Do not exceed available power

\[
f^1(q^1_i) + f^2(q^2_i, h^1_i) + f^3(q^3_i, h^2_i) \leq P_i, \quad \forall i \in \mathbb{N}_1
\]

Compute water level in tanks

\[
h^j_0 = h^j_{\text{init}}, \quad \forall j \in \mathbb{N}_T \\
\quad h^1_i - h^1_{i-1} - \frac{\Delta^t}{\Delta b} q^1_i + \frac{\Delta^t}{\Delta b} (q^2_i + f^4(q^2_i)) \leq 0, \quad \forall i \in \mathbb{N}_1 \\
\quad h^2_i - h^2_{i-1} - \frac{\Delta^t}{\Delta b} f^4(q^2_i) + \frac{\Delta^t}{\Delta b} q^3_i \leq 0, \quad \forall i \in \mathbb{N}_1 \\
\quad h^3_i - h^3_{i-1} - \frac{\Delta^t}{\Delta b} q^3_i \leq 0, \quad \forall i \in \mathbb{N}_1
\]

Variables domains

\[
h^k_i \in [h^k_{\min}, h^k_{\max}], q^k_i \in \left[\frac{\Delta b}{\Delta t} h^k_{\min}, \frac{\Delta b}{\Delta t} h^k_{\max}\right], \quad \forall i \in \mathbb{N}_1, k \in \mathbb{N}_P \\
\quad r_i \in [0, 1], \quad \forall i \in \mathbb{N}_1
\]
Introduction

Challenges

Non-linearities come from energy efficiency or energy conversion functions

Context

The resulting optimization problem can be neither convex or concave even if the energy conversion function is convex or concave

Objective

Address the (combinatorial) optimization challenge of integrating energy sources in deterministic (scheduling) models with constraints related to their physical, technological and performance characteristics.

Use of piecewise linear approximation

Focus on the problem of minimising the number of line-segments with a predefined error (no iterative procedure)
State of the art on pwl approximators of univariate functions


- Two exact approaches and two heuristics for the computation of optimal piecewise linear approximators.


- close to RK2012 but different in the following aspects
  - do not target on computing optimal (minimal) breakpoint systems
  - estimate the approximation error (or errors for over- and under-estimating) for the general case of indefinite functions, while RK2012 solve non-convex NLP problems to global optimality leading to the tightest approximators.


- first computation of breakpoints for a given error tolerance
- pwl interpolators using equidistant breakpoints for concave quadratic minimisation problems. By concavity, their interpolators are underestimators.
- condition on the number of breakpoints to achieve a given error tolerance.
State of the art on pwl approximators of univariate functions


Principle

Given

- function \( f \) defined on a closed domain \( D = [X_-, X_+] \)
- a \( \delta \)-approximation \( l \) on \( D \) to compute: \( |l(x) - f(x)| \leq \delta, \forall x \in D \)
- \( B \) : "sufficiently large" set of finite breakpoints
- \( x_b \) : the value of breakpoint \( b \in B \)
- \( s_b \in [-\delta, +\delta] \) : deviation from \( f(x_b) \)
- \( \phi(x_b) = f(x_b) + s_b \)

Then \( l(x) = \sum_{b \in B} \phi(x_b)\lambda_b \) with \( x = \sum_{b \in B} x_b\lambda_b \) and \( \sum_{b \in B} \lambda_b = 1 \).

Decision variables

- \( x_b \in [X_-, X_+] \) : breakpoint value
- \( s_b \in [-\delta, +\delta] \) : deviation on bpt \( b \)
- \( \chi_b \in [0, 1] = 1 \) iff bpt \( b \) is used
- \( y_b \geq \frac{1}{M} : = x_b - x_{b-1} \) if \( x_b - x_{b-1} > 0 \) and \( = |X_-, X_+| \) otherwise
- \( \xi_{bx}^{x} \in [0, 1] : = 1 \) iff \( x \in [x_{b-1}, x_b] \)
- \( l_b(x) \in \mathbb{R} : = l(x) \) if \( x \in [x_{b-1}, x_b] \)
State of the art on pwI approximators of univariate functions

OBSC : Objective = minimize number of active breakpoints s.t. Order breakpoints:

\[ x_{b-1} \leq x_b, \forall b \in B \] (22)

Link \( x_b \) and \( \chi_b \):

\[ x_b \geq X_+ + (X_+ - X_-)(1 - \chi_b), \quad \forall b \in B \] (23)
\[ x_b - x_{b-1} \geq \frac{1}{M} \chi_b, \quad \forall b \in B \] (24)
\[ x_b - x_{b-1} \leq (X_+ - X_-)\chi_b, \quad \forall b \in B \] (25)

Verify the \( \delta \)-approximation

\[ |l(x) - f(x)| \leq \delta, \quad \forall x \in D \] (26)

Compute \( y_b \)

\[ y_b = x_b - x_{b-1} + (X_+ - X_-)(1 - \chi_b), \quad \forall b \in B \] (27)

Compute \( \xi_{bx}^x \)

\[ \sum_{b \in B} \xi_{bx}^x = 1, \quad \forall x \in D \] (28)
\[ x_{b-1} - (X_+ - X_-)(1 - \xi_{bx}^x) \leq x \leq x_b + (X_+ - X_-)(1 - \xi_{bx}^x), \quad \forall b \in B, \forall x \in D \] (29)
State of the art on pwil approximators of univariate functions

Compute $l_b(x)$

$$l_b(x) = \phi(x_{b-1}) + \frac{\phi(x_b) - \phi(x_{b-1})}{y_b} (x - x_{b-1}), \quad \forall b \in B, \forall x \in D \quad (30)$$

Compute $l(x)$

$$l(x) = \sum_{b \in B} l_b(x) \xi^x_{bx}, \quad \forall x \in D \quad (31)$$

with $x_0 = X_-$

The problem OBSC is SIP.
Number of breakpoints = $z^* + 1$

Iterative solution procedure:

**Step 1** check the constraint on discrete points $|I|$  
- discretize each interval between two potential breakpoints  
- discretize the entire $[X_-, X_+]$ interval

**Step 2** compute the real maximum error
- solve the NLP problem $z^*_l = \max_{x \in [X_-, X_+]} |l(x) - f(x)|$
- if $z^*_l \leq \delta$, then $l$ is a $\delta$-approximator of $f$, stop. If not increase $|I|$ then go to Step 1.
State of the art on pwll approximators of univariate functions

Focus on Step 1: check the constraint on discrete points

No need for $x_{bx}$ and $l(x)$, replace with variables:

- $x_{bi} \in [X_-, X_+], \forall b \in B$
- $l_{bi} \in \mathbb{R}$

Resulting Model OBSD

Minimize number of active breakpoints: (??)

s.t.
Order breakpoints: (22)
Link $x_b$ and $x_b$: (23)-(25)
Compute $y_b$: (27)
Compute $x_{bi}$:

$$x_{bi} = x_{b-1} + \frac{i}{l + 1}(x_b - x_{b-1}), \forall b \in B, \forall i \in l$$  \hspace{1cm} (32)

$$l_{bi} = \phi(x_{b-1}) + \frac{\phi(x_b) - \phi(x_{b-1})}{y_b}(x_{bi} - x_{b-1}), \forall b \in B, \forall i \in l$$ \hspace{1cm} (33)

$$|l_{bi} - f(x_{bi})| \leq \delta, \forall b \in B, \forall i \in l$$ \hspace{1cm} (34)

with $x_B = X_+$
State of the art on pwI approximators of univariate functions

OBSC and OBSD are in general too large and difficult to solve to optimality (for small numbers of breakpoints and discretization points).

Solution 1 : Fix the number of breakpoint ⇒ no more binary variables

- OBSD becomes an NLP problem
- iterative procedure to find $B$ (on top of the procedure to find $I$)

Solution 2 : Greedy heuristics computing $x_b$ given $x_{b-1}, s_{b-1}$ and $\delta$

- Max length of interval $[x_{b-1}, x_b]$(projection on x-axis) : not optimal

- solve the NLP problem of max $x_b$ or try discrete values of $x_b$ and $s_b$ from the discretization of $[X_-, X_+]$ and $[-\delta, \delta]$
1. Motivation and State of the art

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6. Computational evaluation

7. Conclusion
(Dis)continuous pwl approximation

A pwl function $g$ of $n_g$ pieces is denoted $\bigcup_{i=1}^{n_g}([a_i, b_i], [x_i^{\text{min}}, x_i^{\text{max}}])$:
i.e. if $x \in [x_i^{\text{min}}, x_i^{\text{max}}]$, then $g(x) = a_i x + b$.

$x_i^{\text{min}} = x_i^{\text{max}}, \forall i \in \{2...n\}$.

If pwl function $g$ is continuous, then it also verifies:

$a_i x_i^{\text{max}} + b_i = a_{i+1} x_{i+1}^{\text{min}} + b_{i+1}, \forall i \in [1, n-1]$.

Let $l$ be an optimal continuous $\delta-$ pwl approximator of a continuous fonction $f$ over an interval $D \subset \mathbb{R}$.

Let $y$ be the solution value of problem

$max_{|a_1 x + b_1 - f(x)| \leq \delta, \forall x \in [x_1^{\text{min}}, y]} Y$.

Let $q$ that verifies $x_q^{\text{min}} \leq y \leq x_q^{\text{max}}$.

Remark

Either $y = x_1^{\text{max}}$ or $y > x_1^{\text{max}}$. If $y > x_1^{\text{max}}$ then a discontinuous pwl $\delta-$approximator $\tilde{l}$ of $f$ can be obtained as

$\left(\bigcup_{i=q+1}^{n_1}([a_i, b_i], [x_i^{\text{min}}, x_i^{\text{max}}])\right) \cup ([a_1, b_1], [x_1^{\text{min}}, y]) \cup ([a_q, b_q], [y, x_q^{\text{max}}])$.
(Dis)continuous pwL approximation
Non necessarily continuous pwil approximation

Proposition

Any optimal pwil continuous $\delta -$approximator with $n^*$ pieces can be converted into a pwil non-necessarily continuous $\delta -$approximator with $n \leq n^*$ pieces where the projection of the first piece on interval $D$ is of maximal length.
Non necessarily continuous pwls approximation

Theorem

For any continuous fonction $f : \mathbb{D}(= [X_-, X_+]) \rightarrow \mathbb{R}$ on a compactum $D \subset \mathbb{R}$ and any constant $\delta$, there exists an optimal non-necessarily continuous piecewise linear $\delta$-approximator such that each line-segment $i$ has a maximal length projection on the interval $[x_i^{\min}, X_+]$.

proof outline

- There exists a continuous $\delta$-approximator function for any continuous fonction $f$ on a compactum $\mathbb{D}$ and any constant $\delta$ (Duistermaat and Kol, 2004).
- Proposition 2.1 can be applied iteratively on an optimal continuous approximator to obtain a optimal non-necessarily continuous pwls $\delta$—approximator with intervals of maximal length on $\mathbb{D}$. 
Non necessarily continuous pwl approximation

All mathematical models and algorithms from Rebennack and Kallrath (2015) can be adapted by considering two shifts per breakpoint instead of one: $s_b^-$ and $s_B^+$. 

- $s_b^-$ is applied on the piece that ends at breakpoint $b$
- $s_B^+$ is applied on the piece that starts at the breakpoint $b$

Adapted Heuristic 1 based on the maximisation of the (projection) interval length is now an exact method!

New heuristics proposed for convex and concave functions based on tangents.
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Pwl bounding instead of pwl approximation

Pwl bounding of the nonlinear univariate energy transfer functions (Ngueveu et al, 2013)

(a) Linear approximation
(b) Piecewise bounding

Rebennack : 2015

Convert the $\delta$-approximator into a $\frac{\delta}{2}$-underestimator (-overestimator) by shifting the approximator by $\frac{\delta}{2}$
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Weaknesses/Limitations of pwl $\delta -$ approx

Limitations of the pwl $\delta -$ approx

- dependent on the instance: e.g. dependent on time horizon
- dependent on the solution: e.g. solution cost
- difficult to choose a relevant value of $\delta$ for a new instance or worse a new problem

Energy optimization in hybrid electric vehicle

\[ \min \sum_{i=1}^{n} f_{FC}(x_i) \]  
(35)

s.t. Power demand satisfaction
Final SE energy level higher or equal to initial energy level
SE energy level within bounds
Variables domains
Weaknesses/Limitations of pwl $\delta$—approx

Minimize the total energy cost

$$(\text{CF}) \min \quad F = \sum_{t \in T} f \left( \sum_{i \in A} b_i x_{it} \right)$$

s.t.
Satisfaction of the demand for each activity

$$\sum_{t \in T} a_{it} x_{it} \geq p_i, \quad \forall i \in A$$

Validity domain

$$x_{it} \in \{0, 1\}, \quad \forall i \in A, t \in T$$
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Piecewise linear bounding with relative $\epsilon$-tolerance

**Principle**

Piecewise linear bounding a function $f$ on a compactum $\mathbb{D}$ with a tolerance value $\epsilon$ consists in identifying two piecewise linear functions $(\overline{f}^\epsilon, \underline{f}^\epsilon)$ that verify:

\[
\underline{f}^\epsilon(x) \leq f(x) \leq \overline{f}^\epsilon(x), \quad \forall x \in \mathbb{D} \tag{39}
\]

\[
|f(x) - \underline{f}^\epsilon(x)| \leq \epsilon |f(x)|, \quad \forall x \in \mathbb{D} \tag{40}
\]

\[
|\overline{f}^\epsilon(x) - f(x)| \leq \epsilon |f(x)|, \quad \forall x \in \mathbb{D} \tag{41}
\]

**Purpose**

- The bounding is applied directly on the energy conversion function BEFORE insertion into the model
  - univariate function that can be convex or concave, easier to approximate/bound
- Two MILP ($\text{MILP}$ and $\overline{\text{MILP}}$) are obtained
- Guarantees on the quality of the resulting lower and upper bounds
Piecewise linear bounding with relative $\epsilon$-tolerance

The upper and lower bounding pwl functions $\overline{f}_{\epsilon}(x)$ and $\underline{f}_{\epsilon}(x)$ may not share the same breakpoints and number of pieces (no shift).

Adapted algorithms for convex, concave and piecewise convex and concave energy efficiency functions have been proposed in (Ngueveu et al, 2013).
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4. Weaknesses and limitations of a $\delta$-bounding/$\delta$-approximation

5. Using $\epsilon$-relative tolerance

6. Computational evaluation

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Water pumping and desalination process

<table>
<thead>
<tr>
<th></th>
<th>Pump 1</th>
<th>(Pump 2+RO)</th>
<th>Pump 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>n_1</td>
<td>n_2</td>
<td>n_3</td>
<td></td>
</tr>
<tr>
<td>ε</td>
<td>n_1</td>
<td>n_2</td>
<td>n_3</td>
</tr>
<tr>
<td>5%</td>
<td>2</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>1%</td>
<td>5</td>
<td>21</td>
<td>8</td>
</tr>
<tr>
<td>0.5%</td>
<td>8</td>
<td>35</td>
<td>13</td>
</tr>
<tr>
<td>0.3%</td>
<td>10</td>
<td>43</td>
<td>17</td>
</tr>
</tbody>
</table>

**Table – Number of sectors per tolerance value**

<table>
<thead>
<tr>
<th></th>
<th>MILP UB</th>
<th>MILP LB</th>
<th>MILP UB*</th>
<th>Gap</th>
<th>opt</th>
</tr>
</thead>
<tbody>
<tr>
<td>n_1</td>
<td>n_2</td>
<td>n_3</td>
<td>n_4</td>
<td>n_5</td>
<td>n_6</td>
</tr>
<tr>
<td>ε</td>
<td>UB</td>
<td>s</td>
<td>LB</td>
<td>s</td>
<td>UB*</td>
</tr>
<tr>
<td>5%</td>
<td>20580</td>
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<td>19740</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>1%</td>
<td>20100</td>
<td>15</td>
<td>19920</td>
<td>140</td>
<td>-</td>
</tr>
<tr>
<td>0.5%</td>
<td>20040</td>
<td>178</td>
<td>19980</td>
<td>117</td>
<td>-</td>
</tr>
<tr>
<td>0.3%</td>
<td>20040</td>
<td>64</td>
<td>19980</td>
<td>321</td>
<td>19980</td>
</tr>
</tbody>
</table>

**Table – Upper and Lower bounds values obtained**
### Scheduling under non-reversible energy source

<table>
<thead>
<tr>
<th># instances</th>
<th>$\epsilon=5%$</th>
<th>$\epsilon=1%$</th>
<th>$\epsilon=0.5%$</th>
<th>$\epsilon=0.1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best UB</td>
<td>MILP</td>
<td>232</td>
<td>265</td>
<td>264</td>
</tr>
<tr>
<td></td>
<td>MILP $\times (1+\epsilon)%$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Best LB</td>
<td>MILP</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>MILP $\times (1-\epsilon)%$</td>
<td>233</td>
<td>265</td>
<td>264</td>
</tr>
<tr>
<td>Gap</td>
<td>min</td>
<td>2.64%</td>
<td>0.53%</td>
<td>0.27%</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>5.00%</td>
<td>0.83%</td>
<td>0.40%</td>
</tr>
<tr>
<td></td>
<td>avg</td>
<td>3.54%</td>
<td>0.63%</td>
<td>0.31%</td>
</tr>
</tbody>
</table>


- Impact of non reversible sources functions: aggregability, ...
- Complexity analysis, Extended formulation, Dantzig-Wolfe Decomposition, Branch-and-price
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Conclusion

Done

- Energy sources characteristics in (combinatorial) optimization pbs.
- Resolution scheme: piecewise bounding and integer programming
- Non-necessarily continuous piecewise linear functions
- Relative $\epsilon$—tolerance
- 2 similar MILPs to solve
New PGMO Project OPAL

Optimizing energy allocation using lot sizing models

Participants:
- LAAS: Christian Artigues, Pierre Lopez and Sandra U. Ngueveu
- LIP6: Safia Kedad-Sidhoun
- LIMOS: Nabil Absi
Bibliography


