A multi-stage stochastic programming approach for remanufacturing planning under uncertainty

Céline Gicquel [1], Safia Kedad-Sidhoum [2], Quan Vu [1]

[1]Laboratoire de Recherche en Informatique
Université Paris Sud
[2]Laboratoire d'Informatique de Paris 6
Université Pierre et Marie Curie

PGMO Days 2016

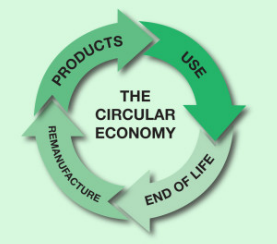
Plan

Plan

Remanufacturing

Context

Transition towards a circular economy

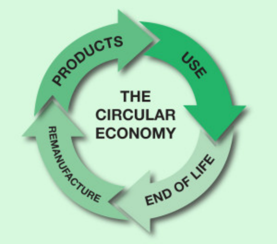


Remanufacturing

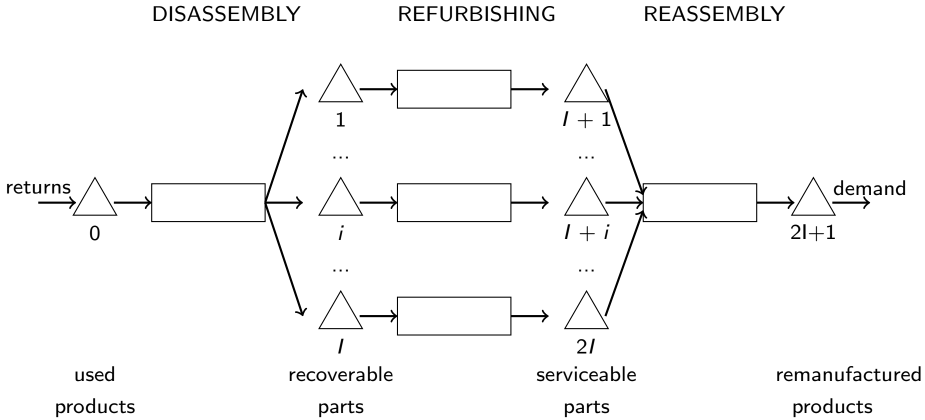
Context
Transition towards a circular economy

Remanufacturing: a way of rehabilitating used products
Obtain new (or like-new) products

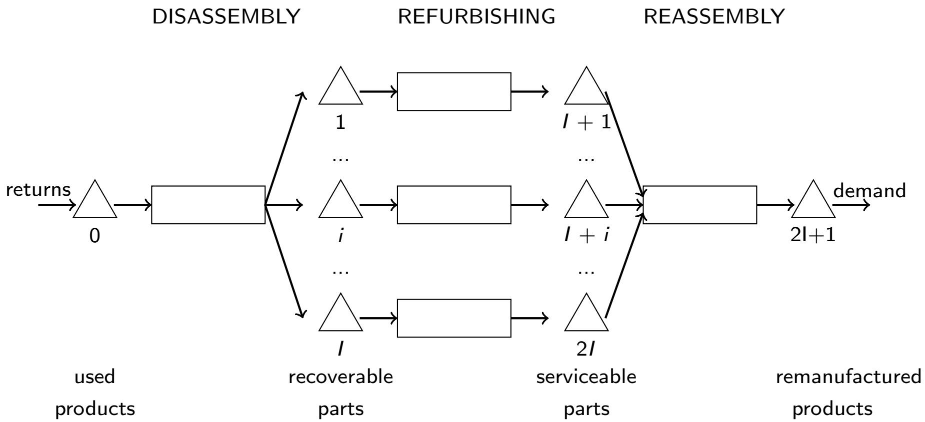
- from end-of-life products returned by consumers
- by replacing or refurbishing damaged components



A three-echelon remanufacturing system



A three-echelon remanufacturing system



Industrial applications: mobile phones, electrical equipment...

[Jayaraman, 2006], [Franke *et al*, 2006],[Han *et al*, 2013], [Ahn *et al*, 2011]

Production planning & lot-sizing

Production planning

Over a multi-period horizon, decide:

- how many used products to disassemble,
- how many parts to refurbish,
- how many remanufactured products to reassemble,

so as to satisfy the demand for remanufactured products.

Production planning & lot-sizing

Production planning

Over a multi-period horizon, decide:

- how many used products to disassemble,
- how many parts to refurbish,
- how many remanufactured products to reassemble,

so as to satisfy the demand for remanufactured products.

Costs

- Fixed production setup costs
  $\Rightarrow$ A small number of large-size production lots
- Inventory holding costs
  $\Rightarrow$ A large number of small-size production lots

Production planning & lot-sizing

Production planning

Over a multi-period horizon, decide:

- how many used products to disassemble,
- how many parts to refurbish,
- how many remanufactured products to reassemble,

so as to satisfy the demand for remanufactured products.

Costs

- Fixed production setup costs
  $\Rightarrow$ A small number of large-size production lots
- Inventory holding costs
  $\Rightarrow$ A large number of small-size production lots

Lot-sizing

Plan production so as to optimize the trade-off between setup and inventory holding costs

Assumptions

Assumptions

Assumptions

Assumptions

1. a single type of returned/remanufactured product

Assumptions

Assumptions

1. a single type of returned/remanufactured product
2. homogenous quality of the returned products

Assumptions

Assumptions

1. a single type of returned/remanufactured product
2. homogenous quality of the returned products
3. deterministic yield of the disassembly and refurbishing processes

Assumptions

Assumptions

1. a single type of returned/remanufactured product
2. homogenous quality of the returned products
3. deterministic yield of the disassembly and refurbishing processes
4. uncapacitated production processes

Assumptions

Assumptions

1. a single type of returned/remanufactured product
2. homogenous quality of the returned products
3. deterministic yield of the disassembly and refurbishing processes
4. uncapacitated production processes

Deterministic problem: model

Notation

- planning periods: $t = 1...T$
- products: $i = 0...2I + 1$
- production processes: $p = 0...I + 1$

Deterministic problem: model

Notation

- planning periods: $t = 1...T$
- products: $i = 0...2I + 1$
- production processes: $p = 0...I + 1$
- α_i: quantity of product i needed to produce one remanufactured product
  $\alpha_0 = 1$; $\alpha_i = \alpha_{i+1}$, $\forall i = 1...I$; $\alpha_{2I+1} = 1$

Deterministic problem: model

Notation

- planning periods: $t = 1 \ldots T$
- products: $i = 0 \ldots 2I + 1$
- production processes: $p = 0 \ldots I + 1$
- α_i: quantity of product i needed to produce one remanufactured product $\alpha_0 = 1$; $\alpha_i = \alpha_{i+1}$, $\forall i = 1 \ldots I$; $\alpha_{2I+1} = 1$
- r_t: quantity of used products returned in t
- d_t: demand for remanufactured products in t

Deterministic problem: model

Notation

- planning periods: $t = 1 ... T$
- products: $i = 0 ... 2I + 1$
- production processes: $p = 0 ... I + 1$
- α_i: quantity of product i needed to produce one remanufactured product
  $\alpha_0 = 1$; $\alpha_i = \alpha_{i+1}, \forall i = 1 ... I$; $\alpha_{2I+1} = 1$
- r_t: quantity of used products returned in t
- d_t: demand for remanufactured products in t

Decision variables

- $Y_p^t \in \{0, 1\}$: setup variable for process p in t
- X_p^t: production quantity for process p in t
- I_i^t: inventory level for product i at the end of t
- L^t: lost sales of remanufactured product in t

Deterministic problem: formulation

MILP formulation

$$Z^* = min \sum_{t=1}^{T} \Big[\sum_{p=0}^{I+1} s_p Y_p^t + \sum_{i=0}^{2I+1} h_i I_i^t + lL^t \Big]$$

$$I_0^t = I_0^{t-1} + r^t - X_0^t \qquad \forall t$$

$$I_i^t = I_i^{t-1} + \alpha_i X_0^t - X_i^t \qquad \forall t, \forall i = 1...I$$

$$I_i^t = I_i^{t-1} + X_{i-I}^t - \alpha_{i-I} X_{I+1}^t \qquad \forall t, \forall i = I+1...2I$$

$$I_{2I+1}^t = I_{2I+1}^{t-1} + X_{I+1}^t - d^t + L^t \qquad \forall t$$

$$X_p^t \leq M Y_p^t \qquad \forall t, \forall p = 0...I+1$$

$$Y_p^t \in \{0,1\} \qquad \forall t, \forall p = 0...I+1$$

Deterministic problem: formulation

MILP formulation

Setup Inventory Lost sales

$$Z^* = min \sum_{t=1}^{T} \left[\sum_{p=0}^{I+1} s_p Y_p^t + \sum_{i=0}^{2I+1} h_i I_i^t + lL^t \right]$$

Inventory balance:

$$I_0^t = I_0^{t-1} + r^t - X_0^t \quad \forall t$$

$$I_i^t = I_i^{t-1} + \alpha_i X_0^t - X_i^t \quad \forall t, \forall i = 1...I$$

$$I_i^t = I_i^{t-1} + X_{i-I}^t - \alpha_{i-I} X_{I+1}^t \quad \forall t, \forall i = I+1...2I$$

$$I_{2I+1}^t = I_{2I+1}^{t-1} + X_{I+1}^t - d^t + L^t \quad \forall t$$

Setup-production:

$$X_p^t \leq M Y_p^t \quad \forall t, \forall p = 0...I+1$$

$$Y_p^t \in \{0, 1\} \quad \forall t, \forall p = 0...I+1$$

Research objectives

Uncertain input data

Difficulty to forecast the future returns and demand

$\rightarrow r_t$ and d_t = stochastic parameters

Research objectives

Uncertain input data

Difficulty to forecast the future returns and demand

$\rightarrow r_t$ and d_t = stochastic parameters

Assumption

Known probability distribution of the stochastic parameters

Research objectives

Uncertain input data

Difficulty to forecast the future returns and demand

$\rightarrow r_t$ and d_t = stochastic parameters

Assumption

Known probability distribution of the stochastic parameters

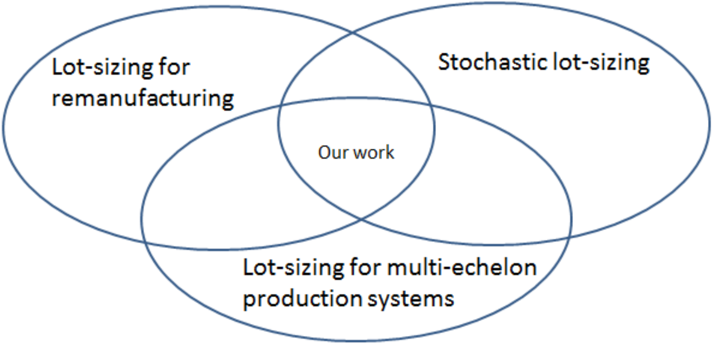
Objective

Develop a stochastic programming approach for the problem

- modeling: a multi-stage approach based on a scenario tree
- solving: polyhedral approach, Cut & Branch algorithm

Plan

Literature review

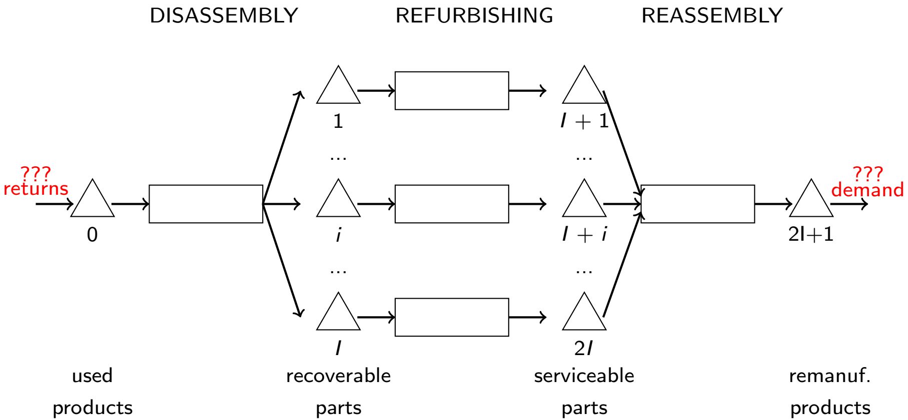


Our contributions

1. **Lot-sizing for remanufacturing**
   Improvement of the problem modeling:
   - Multi-echelon system
   - Stochastic input data
2. **Stochastic lot-sizing**
   Extension of an existing polyhedral solution approach
   - from a single-echelon single-product problem
   - to a multi-echelon multi-product problem

Plan

Uncertain returns and demand



Multi-stage stochastic programming

Assumption

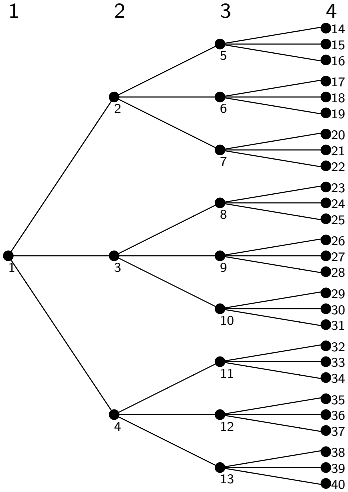
- Not all decisions have to be made before uncertainty realization.
- Some can be postponed to a future point in time.

Dynamic multi-stage decision process

Observation of random parameters for stage 1 → Production planning decisions for stage 1 → Observation of random parameters for stage 2 → Production planning decisions for stage 2 → ... → Observation of random parameters for stage S → Production planning decisions for stage S

Uncertainty representation

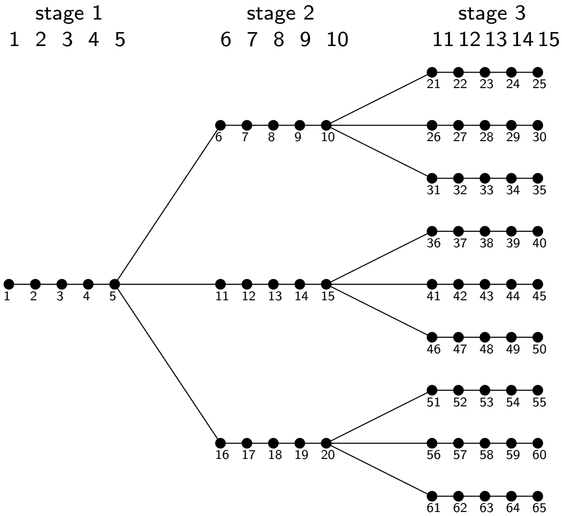
Scenario tree: 1 stage = 1 period



- A node n = a realization of the stochastic returns and demand (r_n, d_n) in the corresponding period
- New information on uncertainty available in each period
  → Every node has several children.

Uncertainty representation

Scenario tree: 1 stage = π periods



- A node n = a realization of the stochastic returns and demand (r_n, d_n) in the corresponding period
- New information on uncertainty available every π periods
  $\rightarrow$ Only a subset of nodes have several children.

[Zanjani *et al*, 2010]

Problem formulation

Notation

- $n = 1...N$: set of nodes
- π_n: probability of node n
- $a(n)$: predecessor of node n in the scenario tree
- r^n: quantity of used products returned in n
- d^n: demand for remanufactured products in n

Problem formulation

Notation

- $n = 1...N$: set of nodes
- π_n: probability of node n
- $a(n)$: predecessor of node n in the scenario tree
- r^n: quantity of used products returned in n
- d^n: demand for remanufactured products in n

Decision variables

- $Y_p^n \in \{0, 1\}$: setup variable for process p in n
- X_p^n: production quantity for process p in n
- I_i^n: inventory level for product i at the end of n
- L^n: lost sales of remanufactured product in n

Problem formulation

MILP formulation

Expected setup/inventory/lost sales costs

$$Z^* = min \sum_{n=1}^{N} \pi_n \Big[\sum_{p=0}^{I+1} s_p Y_p^n + \sum_{i=0}^{2I+1} h_i I_i^n + lL^n \Big]$$

$$I_0^n = I_0^{a(n)} + r^n - X_0^n \qquad \forall n$$

Inventory balance

$$I_i^n = I_i^{a(n)} + \alpha_i X_0^n - X_i^n \qquad \forall n, \forall i = 1 ... I$$

$$I_i^n = I_i^{a(n)} + X_{i-I}^n - \alpha_{i-I} X_{I+1}^n \qquad \forall n, \forall i = I+1 ... 2I$$

$$I_{2I+1}^n = I_{2I+1}^{a(n)} + X_{I+1}^n - d^n + L^n \qquad \forall n$$

Setup-production

$$X_p^n \leq M Y_p^n \qquad \forall n, \forall p = 0 ... I+1$$

$$Y_p^n \in \{0, 1\} \qquad \forall n, \forall p = 0 ... I+1$$

Plan

Problem formulation

MILP formulation

$$Z^* = min \sum_{n=1}^{N} \pi_n \Big[\sum_{p=0}^{I+1} s_p Y_p^n + \sum_{i=0}^{2I+1} h_i I_i^n + lL^n \Big]$$

$$I_0^n = I_0^{a(n)} + r^n - X_0^n \qquad \forall n$$

$$I_i^n = I_i^{a(n)} + \alpha_i X_0^n - X_i^n \qquad \forall n, \forall i = 1...I$$

$$I_i^n = I_i^{a(n)} + X_{i-I}^n - \alpha_{i-I} X_{I+1}^n \qquad \forall n, \forall i = I+1...2I$$

$$I_{2I+1}^n = I_{2I+1}^{a(n)} + X_{I+1}^n - d^n + L^n \qquad \forall n$$

$$X_p^n \leq M Y_p^n \qquad \forall n, \forall p = 0...I+1$$

$$Y_p^n \in \{0,1\} \qquad \forall n, \forall p = 0...I+1$$

Dependent demand

Solution

Echelon stock = Total inventory of an item in the system, as such or as a component of other items

Problem formulation

MILP 'echelon stock' formulation

$$Z^* = min \sum_{n=1}^{N} \pi_n \Big[\sum_{p=0}^{I+1} s_p Y_p^n + \sum_{i=0}^{2I+1} eh_i EI_i^n + lL_n \Big]$$

$$EI_0^n = EI_0^{a(n)} + r^n - \alpha_0 (d^n - L^n) \qquad \forall n$$

$$EI_i^n = EI_i^{a(n)} + \alpha_i X_0^n - \alpha_i (d^n - L^n) \qquad \forall n, \forall i = 1 \dots I - 1$$

$$EI_{p+I}^n = EI_{p+I}^{a(n)} + \alpha_{p+I} X_p^n - \alpha_{p+I} (d^n - L^n) \qquad \forall n, \forall p = 0 \dots I + 1$$

$$X_p^n \leq M Y_p^n \qquad \forall n, \forall i = 0..I + 1$$

$$EI_0^n - \frac{EI_1^n}{\alpha_1} \geq 0 \qquad \forall n$$

$$EI_i^n - EI_{i+I}^n \geq 0 \qquad \forall n, \forall i = 1..I$$

$$EI_i^n - EI_{2I+1}^n \geq 0 \qquad \forall n, \forall i = I + 1 \dots 2I$$

$$Y_p^n \in \{0, 1\} \qquad \forall n, \forall p = 0 \dots I + 1,$$

Problem formulation

MILP 'echelon stock' formulation

$$Z^* = min \sum_{n=1}^{N} \pi_n \Big[\sum_{p=0}^{I+1} s_p Y_p^n + \sum_{i=0}^{2I+1} eh_i EI_i^n + lL_n \Big]$$

$$EI_0^n = EI_0^{a(n)} + r^n - \alpha_0(d^n - L^n) \qquad \forall n$$

$$EI_i^n = EI_i^{a(n)} + \alpha_i X_0^n - \alpha_i(d^n - L^n) \qquad \forall n, \forall i = 1...I-1$$

$$EI_{p+I}^n = EI_{p+I}^{a(n)} + \alpha_{p+I} X_p^n - \alpha_{p+I}(d^n - L^n) \qquad \forall n, \forall p = 0...I+1$$

$$X_p^n \leq MY_p^n \qquad \forall n, \forall i = 0..I+1$$

$I + 1$ independent subproblems

$$EI_0^n - \frac{EI_1^n}{\alpha_1} \geq 0 \qquad \forall n$$

$$EI_i^n - EI_{i+I}^n \geq 0 \qquad \forall n, \forall i = 1..I$$

$$EI_i^n - EI_{2I+1}^n \geq 0 \qquad \forall n, \forall i = I+1...2I$$

$$Y_p^n \in \{0, 1\} \qquad \forall n, \forall p = 0...I+1,$$

Subproblems

Subproblem for production process p

$$Z^* = min \sum_{n=1}^{N} \pi_n \left[s_p Y_p^n + eh_{p+l} EI_i^n + lL_n \right]$$

$$EI_{p+l}^n = EI_{p+l}^{a(n)} + \alpha_{p+l} X_p^n - \alpha_{p+l}(d^n - L^n) \qquad \forall n$$

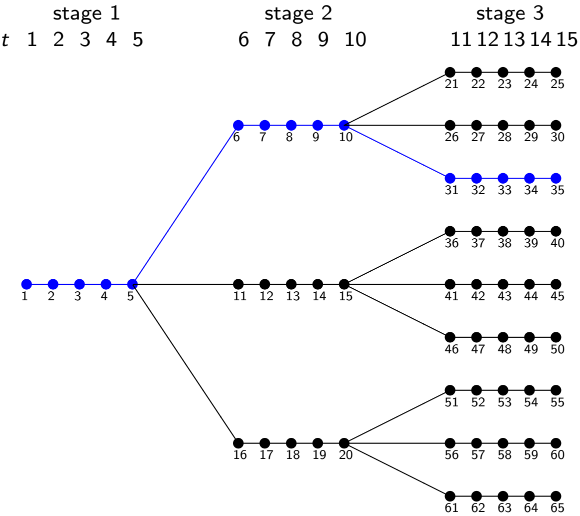
$$X_p^n \leq MY_p^n \qquad \forall n$$

$$Y_p^n \in \{0, 1\} \qquad \forall n,$$

→ Single-item single-echelon single-resource lot-sizing with lost sales
→ But on a tree !

Subproblem formulation strengthening

Path inequalities



Subproblem formulation strengthening

Valid inequalities

Let $k \in \{0, ...N\}$ be a non-leaf node and $l \in \mathcal{L}(k)$ be a leaf.
Let $U \in \mathcal{P}(k, l)$.

$$EI^k_{p+l} \geq \sum_{n \in U} \left[d^n (1 - \sum_{\nu \in \mathcal{P}(k,n)} Y^\nu_p - L^n)\right]$$

Subproblem formulation strengthening

Valid inequalities

Let $k \in \{0, \ldots N\}$ be a non-leaf node and $l \in \mathcal{L}(k)$ be a leaf.
Let $U \in \mathcal{P}(k, l)$.

$$EI^k_{p+l} \geq \sum_{n \in U} \left[d^n (1 - \sum_{\nu \in \mathcal{P}(k,n)} Y^\nu_p - L^n) \right]$$

where:

- $\mathcal{L}(n)$: set of leaves in the subtree originated from n
- $\mathcal{P}(n, m)$: path between node n and m in the scenario tree

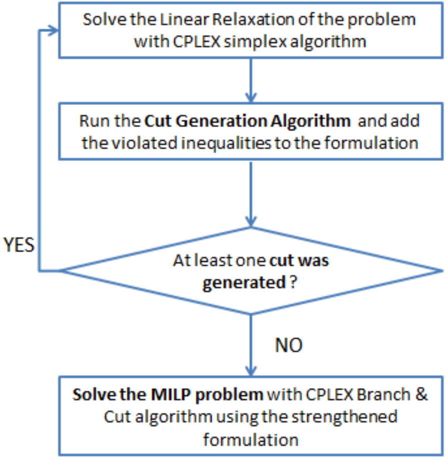
Separation

Polynomial in $O(N^2)$

Key point in practice

Careful selection of the valid inequalities to be added to the formulation

Cut & Branch algorithm



Preliminary computational results

Instances
Random generation based on the numerical values used by [Ahn *et al*, 2011]

Preliminary computational results

Instances

Random generation based on the numerical values used by [Ahn *et al*, 2011]

Numerical results

		CPLEX alone			Cut & Branch algorithm			
N	I	LP Gap	Opt Gap	Time	Cuts	LP Gap	Opt Gap	Time
105	5	4.7%	0.0%	2.4s	965	1.1%	0.0%	2.7s
200	5	7.2%	0.0%	9.6s	2241	1.2%	0.0%	7.5s
425	5	13.1%	0.0%	91s	4057	1.7%	0.0%	14s
605	5	14.6%	0.0%	522s	6438	1.5%	0.0%	33s
728	5	5.4%	0.0%	588s	4181	1.5%	0.0%	212s
1092	5	8.9%	0.0%	746s	8994	1.4%	0.0%	197s
1457	5	10.1%	0.1%	900s	14310	1.3%	0.0%	650s
1705	5	15.9%	0.2%	900s	17191	1.6%	0.1%	900s

- Average values for 5 randomly generated instances
- Resolution with CPLEX 12.6 on a PC running under Windows 10, Intel Core i7, 8 GB of RAM
- Time limit : 900s

Plan

Conclusion and perspectives

Conclusion

- Lot sizing for a three-echelon remanufacturing system
- Uncertainty on returns and demand
- Multi-stage stochastic programming approach
- Polyhedral approach based on 'path inequalities'
- Significant reduction of the computation time

Conclusion and perspectives

Conclusion

- Lot sizing for a three-echelon remanufacturing system
- Uncertainty on returns and demand
- Multi-stage stochastic programming approach
- Polyhedral approach based on 'path inequalities'
- Significant reduction of the computation time

Perspectives

- Solving
  - More efficient cut generation for the 'path inequalities'
  - Extension to 'tree inequalities'
- Modeling
  - Heterogenous quality of the returns
  - Uncertainty on the disassembly yield

Thank you for your attention !