Inexact Bundle methods: a computational comparison of different schemes (with applications to energy problems)

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Outline

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1. Bundle Method
2. Bundle Method with Inexact Oracles
3. Preliminary Computational Results
4. Work in Progress

The problem

Minimization problem (P):

$$\text{Minimize } f(x)$$

$$\text{Subject to } x \in X.$$

- $f : \Re^n \to \Re$ is a finite-valued proper convex possibly nondifferentiable function;
- $X \subseteq \Re^n$ is closed convex.

Introduction

Oracle: Given any $x \in X \Rightarrow f(x)$, $s \in \partial f(x)$.

Tentative points: $\{x_i\} \Rightarrow \begin{cases} \text{the } f\text{-value } f_i = f(x_i) \\ \text{any subgradient } s_i \in \partial f(x_i). \end{cases}$

Bundle: At each iteration, a *bundle* of information

$$= \{(x_i, f_i, s_i)\}$$

is maintained to construct a *model* $\check{f}$ of the function f, that is exploited to construct the next tentative point.

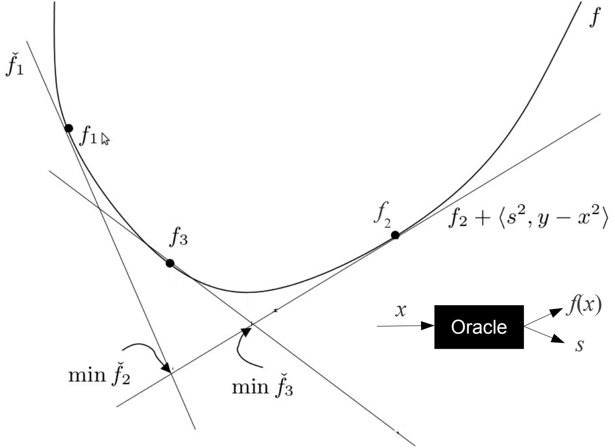
Serious Step / Null Step

One compares $f(\bar{x})$ with $f(x)$, where $x = \bar{x} + d^*$.

Serious Step: If $f(x)$ is "significantly smaller" than $f(\bar{x})$, then $\bar{x}$ is moved to x.

Null Step: Otherwise, $(x, f(x), s \in \partial f(x))$ added to in order to obtain a (hopefully) better direction at the next iteration.

Bundle Method



Bundle methods

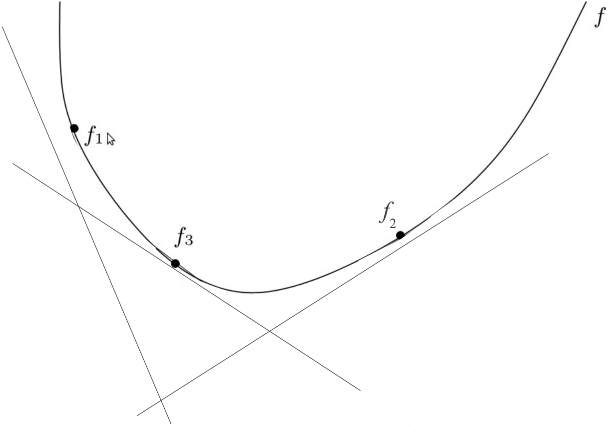
- Applications: [Crainic et al., 2001],[Teo et al., 2010]
- Energy problems: [Belloni et al., 2003], [Sagastizábal, 2012]

- Inexact oracles: subproblems are solved only approximately with an accuracy that varies along iterations under the control of the Bundle solver.

  [Zakeri et al., 2000], [Fábián, 2000], [Kiwiel, 2009], [Richtárik, 2011], [Oliveira and Sagastizábal, 2012].

Outline

Inexact bundle methods



Our goal

- Understanding the impact of different approaches for managing the oracle inexactness in Bundle methods.
- Complex trade off: more exact $\rightarrow$ less iterations but each one more costly.
- Running time is the crucial measure.
- Start with simple "inexact oracle schemes".

Oracles with on-demand accuracy

- Given an evaluation point x and a required accuracy $\epsilon \geq 0$:

  - as function information $f_x = f(x) - \eta$
  - as subgradient information $s_x \in \partial_\eta f(x)$

    where $\eta \leq \epsilon$.

- Asymptotically Exact: $\epsilon \to 0$ along the iterative process.

Scheme 1

- Stopping criteria of Bundle method: $(t^*||s||^2 + \sigma) \leq \delta \max(1, |f(x)|)$
- MaxEps: a maximum relative required accuracy

- First iteration:

  $\epsilon =$ MaxEps

- Other iterations:

  OptMeasure $= (t^*||s||^2 + \sigma)/\max(1, |f(x)|)$

  $\epsilon = \min(0.1f(x), 10^{(-log(SS))}) \times RedPar \times OptMeasure$

  If ($\epsilon >$ MaxEps) then

  $\epsilon =$ MaxEps

Scheme 2

- MaxEps: a maximum relative required accuracy
- First iteration:

  $\epsilon =$ MaxEps
- Bundle method stops with an "optimal" solution:

  If $(\epsilon > \delta)$ then

  $\epsilon = 0.1\epsilon$

  Optimization continues...

Scheme 3

- MaxEps: a maximum relative required accuracy
- First iteration:

  $\epsilon = \text{MaxEps}$
- After each *k* iterations:

  $\epsilon = 0.1\epsilon$

Outline

Fixed-Charge Multicommodity Min-Cost Flow problem

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{k\in K} \sum_{(i,j)\in A} c_{ij}^k x_{ij}^k + \sum_{(i,j)\in A} f_{ij} y_{ij} \\
\text{subject to} \quad & \textit{flow conservation constraints}, && (1) \\
& \sum_{k\in K} x_{ij}^k \le u_{ij} y_{ij}, \qquad \forall\,(i,j)\in A, && (2) \\
& x_{ij}^k \le b_{ij}^k y_{ij}, \qquad \forall\,(i,j)\in A, k\in K, && (3) \\
& x_{ij}^k \ge 0, \qquad \forall\,(i,j)\in A, k\in K, && (4) \\
& y_{ij}\in\{0,1\}, \qquad \forall\,(i,j)\in A. && (5)
\end{aligned}
$$

Fixed-Charge Multicommodity Min-Cost Flow problem

$$\text{Maximize } Z(\alpha)$$

$$
\begin{aligned}
Z(\alpha) = \text{Minimize } & \sum_{k\in K} \sum_{(i,j)\in A} (c_{ij}^k + \alpha_{ij}) x_{ij}^k + \sum_{(i,j)\in A} (f_{ij} - u_{ij}\alpha_{ij}) y_{ij} \\
\text{subject to } & \textit{flow conservation constraints,} \\
& x_{ij}^k \leq b_{ij}^k y_{ij}, \qquad \forall\, (i,j) \in A, k \in K, \\
& x_{ij}^k \geq 0, \qquad \forall\, (i,j) \in A, k \in K, \\
& y_{ij} \in \{0,1\}, \qquad \forall\, (i,j) \in A.
\end{aligned}
$$

- Bundle method:

  From [Frangioni, 2002].

  $(t^*||s||^2 + \sigma) \leq \delta \max(1, f(x))$ with $\delta = 10^{-6}$ and $t^* = 10$.

  RedPar $= 0.1$; MaxEps $= 0.1$; $k = 5$.

- Instances: 11 Mulgen problems (by Bernard Gendron).

Number of instances	$\|V\|$	$\|A\|$	$\|K\|$
4	20	289	40
3	20	287	200
3	30	517	100

Exact Oracle

Instance			Time Oracle = 1000s				Time Oracle = 60 s			
$\lvert V\rvert$	$\lvert A\rvert$	$\lvert K\rvert$	Gap	Orac	t	t(Orac)	Gap	Orac	t	t(Orac)
20	289	40	0	6	1.95	1.93	0.0	6	1.87	1.85
20	289	40	0	13	4.54	4.48	0.0	13	4.44	4.4
20	289	40	0	68	21.31	20.97	0.0	68	20.3	19.99
20	289	40	0	39	13.96	13.84	0.0	39	13.84	13.66
20	287	200	0	1	–	–	8.54E-01	4	354.5	354.42
20	287	200	0	1	–	–	6.75E-01	11	–	–
20	287	200	0	1	–	–	3.24E-01	11	–	–
30	517	100	0	19	–	–	7.14E-04	19	–	–
30	517	100	0	1	–	–	8.71E-02	9	–	–
30	517	100	0	65	–	–	0.0	65	–	–

Inexact Oracle

Time limit = 1000s.

Exact			Scheme 1			Scheme 2			Scheme 3		
Gap	Orac	t	Gap	Orac	t	Gap	Orac	t	Gap	Orac	t
0.0	6	1.93	0.0	6	1.82	0.0	12	3.66	0.0	6	1.82
0.0	13	4.48	0.0	13	4.33	0.0	19	6.26	0.0	13	4.37
0.0	68	20.97	0.0	68	20.23	0.0	94	27.41	0.0	68	19.82
0.0	39	13.84	0.0	39	13.73	0.0	61	25.26	0.0	39	13.7
0.0	1	–	0.0	1	–	0.0	1	–	0.0	1	–
0.0	1	–	0.0	1	–	0.0	1	–	0.0	1	–
0.0	1	–	0.0	1	–	0.0	1	–	0.0	1	–
1.43E-03	19	–	0.0	21	–	1.50E-03	41	–	1.65E-03	25	–
0.0	1	–	0.0	1	–	0.0	1	–	0.0	1	–
6.55E-02	65	–	6.55E-02	71	–	0.0	31	116.99	6.47E-02	79	–

Time limit = 60s.

"Exact"			Scheme 1			Scheme 2			Scheme 3		
Gap	Orac	t	Gap	Orac	t	Gap	Orac	t	Gap	Orac	t
0.0	6	1.85	0.0	6	1.82	0.0	12	3.58	0.0	6	1.81
0.0	13	4.4	0.0	13	4.28	0.0	19	6.42	0.0	13	4.34
0.0	68	19.99	0.0	68	20.24	0.0	94	27.31	0.0	68	20.24
0.0	39	13.66	0.0	39	13.7	0.0	61	26.18	0.0	39	13.48
0.0	4	354.42	0.0	4	352.39	0.0	10	959.92	0.0	4	352.72
0.0	11	–	0.0	11	–	0.0	11	–	0.0	11	–
0.0	11	–	0.0	11	–	0.0	11	–	0.0	11	–
2.15E-03	19	–	0.0	21	–	1.50E-03	41	–	1.85E-03	25	–
0.0	9	–	0.0	9	–	0.0	9	–	0.0	9	–
6.55E-02	65	–	6.55E-02	72	–	0.0	31	–	6.47E-02	79	–

Outline

Future research

- Develop different schemes.
- Continues this investigation with the Fixed-Charge Multicommodity Min-Cost Flow problem.
- Apply the most successful ones on the solution of the Unit Commitment Problem.

Thank you for your attention

Belloni, A., Lima, A. D. S., neiro Maceira, M. P., and Sagastizábal, C. (2003).
Bundle relaxation and primal recovery in unit commitment problems: The brazilian case.
Annals of Operations Research, pages 21–44.

Crainic, T. G., Frangioni, A., and Gendron, B. (2001).
Bundle-based relaxation methods for multicommodity capacitated fixed charge network design.
Discrete Applied Mathematics, pages 73–99.

Fábián, C. (2000).
Bundle-type methods for inexact data.
Central European Journal of Operations Research, pages 35–55.

Frangioni, A. (2002).
Generalized bundle methods.
SIAM Journal on Optimization, 13:117–156.

Kiwiel, K. (2009).

Bundle methods for convex minimization with partially inexact oracles.

Technical report, Tech. report, Optimization Online.

Oliveira, W. and Sagastizábal, C. (2012).
Level bundle methods for oracles with on-demand accuracy.
Otimization On-line.

Richtárik, P. (2011).
Approximate level method for nonsmooth convex minimization.
Journal of Optimization Theory and Applications, pages 1–17.

Sagastizábal, C. (2012).
Divide to conquer: decomposition methods for energy optimization.
Math. Program, pages 187–222.

Teo, C., Vishwanathan, S., Smola, A., and Le, Q. (2010).
Bundle methods for regularized risk minimization.
Journal of Machine Learning Research, pages 311–365.

Zakeri, G., Philpott, A., and Ryan, D. (2000).
Inexact cuts in benders decomposition.

SIAM J. Optim, pages 643–657.