Approximate Quantity Decomposition for Long-term Electricity Market Problems

Theoretical aspects

Nicolas Grebille

EDF R&D – École polytechnique

Short acknowledgment

Approximate Quantity Decomposition
for Long-term Electricity Market Problems

Joint work with

Dr Anes Dallagi	EDF R&D
Prof. Frédéric Bonnans	École polytechnique, INRIA
Prof. Daniel Ralph	Cambridge university

Based on a study by

Dr Anes Dallagi	EDF R&D
Dr Arnaud Lenoir	EDF R&D

Plan

Plan

Overview

Long-term model

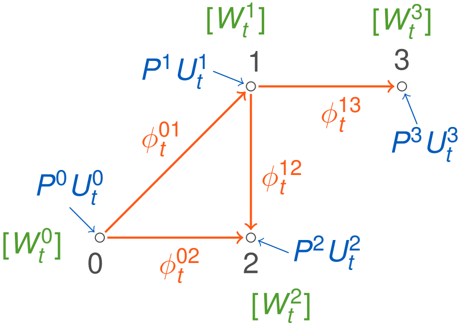
- 20 to 30 years from now;
- discrete time step $t \in \{0, \ldots, T\}$;
- example: step 1 day, total period 1 years.

Network with interconnections

- A set $\mathcal{A}$ of areas ($\equiv$ a country) with production units
- A set $\mathcal{E}$ of links (commercial connection)

Example (four areas)

At each time step t:



- Control U_t^a: the production in the area a is $P^a U_t^a$;
- Control ϕ_t^e: algebraic flow in the edge e;
- A noise W_t^a is observed (demand, inflows in the hydraulic storage).

The model

$$\min_{U,\phi,X} \mathbf{E}\left[\sum_{t=0}^{T}\left(\sum_{a\in\mathcal{A}} c_t^a(U_t^a) + \sum_{e\in\mathcal{E}} \ell_t^e(\phi_t^e)\right)\right]$$

subject to $(\forall a)$:

$$D_t^a = P^a U_t^a + \sum_{e\in\mathcal{E}} \delta_{ae}\phi_t^e$$

$$\begin{cases} X_{t+1}^a = f_t^a(X_t^a, U_t^a, W_t^a) \\ X_0^a = x_0^a \end{cases}$$

$$\begin{cases} 0 \leq U_t^a \leq u_M^a \\ 0 \leq X_t^a \leq x_M^a \\ -\phi_M^e \leq \phi_t^e \leq \phi_M^e \end{cases}$$

$$\begin{cases} U_t^a \preceq \mathcal{F}_t = \sigma(W_0, \dots, W_t) \\ \phi_t^e \preceq \mathcal{F}_t = \sigma(W_0, \dots, W_t) \end{cases}$$

Remarks

About this model

- Reference model for other studies;
- Hard to solve, but approximate methods are known to work (so we can make numerical comparisons)
- Advantage: interpretation is simple (one optimizer)

Decomposition methods

- Idea: Separate the model by area;
- Other methods exists, such as the price decomposition

Plan

The model

$$\min_{U,\phi,X} \ \mathbf{E}\left[\sum_{t=0}^{T}\left(\sum_{a\in\mathcal{A}} c_t^a(U_t^a) + \sum_{e\in\mathcal{E}} \ell_t^e(\phi_t^e)\right)\right]$$

subject to ($\forall a$):

$$D_t^a = P^a U_t^a + \sum_{e\in\mathcal{E}} \delta_{ae}\phi_t^e$$

$$\begin{cases} X_{t+1}^a = f_t^a(X_t^a, U_t^a, W_t^a) \\ X_0^a = x_0^a \end{cases}$$

$$\begin{cases} 0 \leq U_t^a \leq u_M^a \\ 0 \leq X_t^a \leq x_M^a \\ -\phi_M^e \leq \phi_t^e \leq \phi_M^e \end{cases}$$

$$\begin{cases} U_t^a \preceq \mathcal{F}_t = \sigma(W_0, \ldots, W_t) \\ \phi_t^e \preceq \mathcal{F}_t = \sigma(W_0, \ldots, W_t) \end{cases}$$

Quantity process

Coupling constraints:

$$D_t^a = P^a U_t^a + \sum_{e \in \mathcal{E}} \delta_{ae} \phi_t^e$$

We introduce the *quantity* process:

$$Q_t^a = \sum_{e \in \mathcal{E}} \delta_{ae} \phi_t^e$$

(sum of imports minus the exports in the area *a*)

Area model

$$V^a(Q^a) = \begin{cases} \min\limits_{U^a, X^a} \quad \mathbf{E}\left[\sum_{t=0}^{T} c_t^a(U_t^a)\right] \\ s.t. \quad D_t^a = P^a U_t^a + Q_t^a & \forall t \\ \quad X_{t+1}^a = f_t^a(X_t^a, U_t^a, W_t^a) & \forall t \\ \quad X_0^a = x_0^a \\ \quad 0 \leq U_t^a \leq u_M^a & \forall t \\ \quad 0 \leq X_t^a \leq x_M^a & \forall t \\ \quad U_t^a \preceq \mathcal{F}_t \quad (\text{not } \mathcal{F}_t^a) \end{cases}$$

Area model

Structure

- Cost is additive: with information on the structure of Q^a (time-independent, Markov,...), we can use dynamic programming.
- 'Hard' to solve in any case (coupled in time).

Temporary hypothesis

- We can solve the area model;
- We can get a subgradient $\lambda^a \in \partial V^a(Q^a)$.

Network model

$$N(Q) = \begin{cases} \min_{\phi} \; \mathbf{E}\left[\sum_{t=0}^{T}\sum_{e\in\mathcal{E}} \ell_t^e(\phi_t^e)\right] & \\ s.t. \; \sum_{e\in\mathcal{E}} \delta_{ae}\phi_t^e = Q_t^a & \forall a, t \\ \quad -\phi_M^e \leq \phi_t^e \leq \phi_M^e & \forall e, t \\ \quad \phi_t^e \preceq \mathcal{F}_t & \forall e, t \end{cases}$$

‘Easy’ to solve:

- Separable for each scenario
- Separable in time

Decomposed global model

The global model is equivalent to:

$$\min_{Q} \quad N(Q) + \sum_{a \in \mathcal{A}} V^a(Q^a)$$

Optimality conditions

$$\begin{cases} \lambda_\sharp + \mu_\sharp = 0 \\ \mu_\sharp \in \partial N(Q_\sharp) \\ \lambda_\sharp^a \in \partial V^a(Q_\sharp^a) \quad \forall a \in \mathcal{A} \end{cases}$$

Idea: use a ‘subgradient’ algorithm.

Global algorithm

Let $Q_{[k]}$ be a quantity process, and compute $\lambda_{[k]}$ such that:

$$\lambda^a_{[k]} \in \partial V^a(Q^a_{[k]}), \forall a \in \mathcal{A}$$

the next quantity $Q_{[k+1]}$ is the solution of:

$$\min_Q \quad N(Q) + \left\langle \lambda_{[k]}, Q - Q_{[k]} \right\rangle + \rho_{[k]} \left\| Q - Q_{[k]} \right\|^2$$

Remarks on the method

Practical difficulties

The process Q_t has no structure (even if W_t is time-independent):

- Cannot to use dynamic programming to solve the area problem.
- How to store (numerically) the process Q_t?
  (On a scenario tree with K values for W_t^a, we need $K^{|\mathcal{A}|t}$ values for Q_t.)

Problem with the model

The decision U_t^a depends on $\mathcal{F}_t$:

- Not reasonable: we don't have a 'perfect' information of the rest of the network

Plan

Observation process

For each area a, we introduce an observation process Y^a

- Represents the information on the rest of the network
- Y^a depends only on the data (not the controls): $Y_t^a \preceq \mathcal{F}_t$
- Example: sum of the demands in the adjacent areas

And the associated filtration:

$$\mathcal{Y}_t^a = \sigma(W_0^a, \dots, W_t^a, Y_0^a, \dots, Y_t^a)$$

Ideally, we want: $U_t^a \preceq \mathcal{Y}_t^a$:

- More 'reasonable' model;
- If Y_t^a is time-independent (or Markov), we can use dynamic programming in the area model

Modifications of the problems

We want $U_t^a \preceq \mathcal{Y}_t^a$ and

$$D_t^a = Q_t^a + P^a U_t^a \quad (Q_t^a \not\preceq \mathcal{Y}_t^a)$$

Two possibilities

- Impose the constraint $Q_t^a \preceq \mathcal{Y}_t^a$;
- Replace the constraint by: $D_t^a = \mathbf{E}\left[Q_t^a \middle| \mathcal{Y}_t^a\right] + P^a U_t^a$

Comparison of the formulations

Mathematical structure

Forget ϕ, and let R be the 'Q' that appears in the area models.

$$(P_0) \quad V_0 = \min \mathbf{E}\left[g(U, Q, R)\right] \quad \text{s.t.} \quad R_t^a = Q_t^a$$

$$(P_1) \quad V_1 = \min \mathbf{E}\left[g(U, Q, R)\right] \quad \text{s.t.} \quad R_t^a = Q_t^a \preceq \mathcal{Y}_t^a$$

$$(P_2) \quad V_2 = \min \mathbf{E}\left[g(U, Q, R)\right] \quad \text{s.t.} \quad R_t^a = \mathbf{E}\left[Q_t^a \middle| \mathcal{Y}_t^a\right]$$

Results: $V_2 \leq V_0 \leq V_1$

- Any solution of (P_1) is solution of (P_0): $V_0 \leq V_1$.
- By convexity of g (variation of Jensen), $V_2 \leq V_0$.

Advantages of the methods

Numerical methods…

- We known the (simple) structure of the process Q
- we can use dynamic programming)
- On scenario trees: storage is possible

…with a reasonable interpretation

- Gives bounds of the original problem
- Reasonably intuitive approximation

Plan

Conclusion

Mathematical results

- ‘Strong’ mathematical results
- With an economical interpretation
- The proof is not complete in infinite-dimensional spaces

Numerical results (TODO)

- Compare the bounds
- Influence of the Y process
- Comparison with SDDP on the original model

Conclusion

Mathematical results

- ‘Strong’ mathematical results
- With an economical interpretation
- The proof is not complete in infinite-dimensional spaces

Numerical results (TODO)

- Compare the bounds
- Influence of the Y process
- Comparison with SDDP on the original model

Thank you for your attention!

Questions?