Optimization of stochastic combinatorial problem: scheduling maintenance and refueling outages of nuclear power plants

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Context (1) : the original problem (Roadef 2010 challenge)

- A purely combinatorial part (scheduling) :
  - A set of nuclear power plants $i \in I$.
  - Outages to be scheduled.
  - Resource contraints.
- An optimal control part (supply and demand) :
  - For each $i \in I$, evolution of stock $x(i,t)$ is determined by production $p(i,t)$ and refueling $r(i,k)$.
  - Coupling constraints : additional thermal power plants $j \in J$ to balance energy supply and demand.

$$\sum_{i \in I} p(i,t) + \sum_{j \in J} p(j,t) = Dem_t \qquad \forall t \in [0, T-1]$$

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Context (2) : stability of the schedule

- The problem is reoptimized each month
  - Some variables can be adapted easily (production, refuel), other must be decided in advance (dates).
  - Stability : outages don't change two much over time.

$\Rightarrow$ We initially tought to study multistage robust optimization :

  - Robustness : a schedule can withstand changes in the input

- But stability of the whole reoptimization process $\neq$ robustness of the "one-shot" problem.
  - Instances involve scenarios representing possible evolutions.
  - A robust solution is feasible for all these possible variations.
  - Problem : scenarios are modified each month (some have proven to be false).
  - Robustness was garanteed only for those evolutions.
  - Worst : we calibrate the solution w.r. worst cases (potentially unlikely to occur).

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Context (3) : lagrangian relaxation and column generation

- The original model (Roadef 2010) can naturally be handled by a column generation approach :
  - Relaxation of the supply-demand balance constraints to obtain independant subproblems.
- Scenarios represent aleas which affect all the plants together.
- We focus on new aleas specific to each plant, hence handled mostly in the subproblems :
  - Incidents temporarily stop production : binary random variables $\omega_{i,t}$.
  - Outages durations : positive random variables $\psi_{i,k}$.

$\Rightarrow$ Possible adaptations of CG to deal with stochastic subproblems.

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Plan

Relaxation of coupling constraints (1)

- Simplified model : no combinatorial constraints

$$\min \quad \sum_i \left(\sum_k C_{i,k} r(i,k) - C_T x(i,T)\right) + \sum_{j,t} c_{j,t} p(j,t)$$

$$s.t. \quad \boxed{\sum_i p(i,t) + \sum_j p(j,t) = Dem_t}$$

$$x(i,t+1) - x(i,t) = -p(i,t) D^t, \qquad \forall i,k,t \in ec(i,k)$$

$$x(i,t+1) = \frac{Q_{i,k}-1}{Q_{i,k}} (x(i,t) - BO_{i,k-1}) + r(i,k) + BO_{i,k},$$

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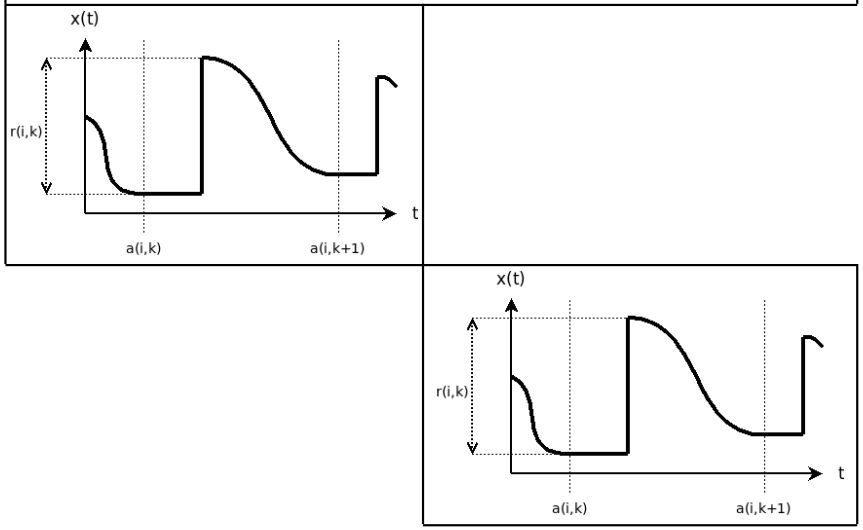
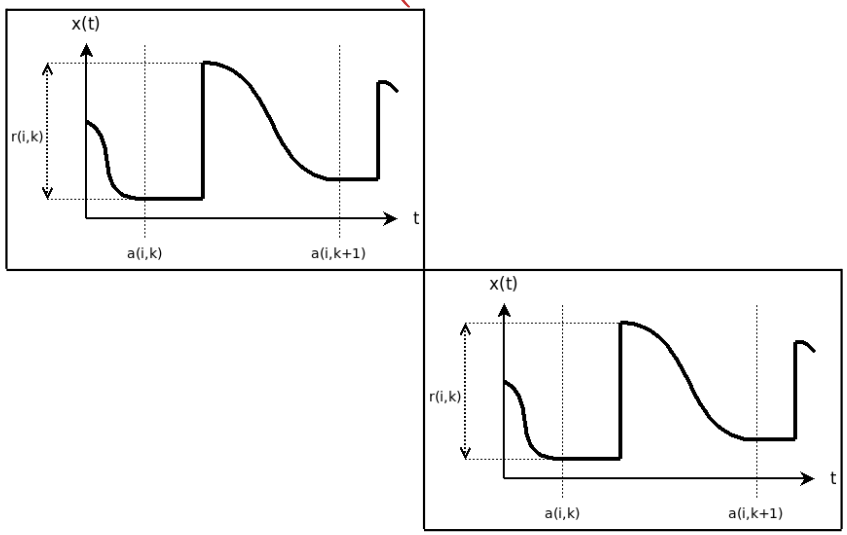


Relaxation of coupling constraints (2)

- Obtaining independant subproblems by relaxing the coupling constraints : price μ_t on each demand constraint.

$$\max_{(\mu_t)} \quad \min \quad \sum_i \left(\sum_k C_{i,k} r(i,k) - C_T x(i,T)\right) + \sum_{j,t} c_{j,t} p(j,t)$$
$$- \sum_t \mu_t \left(\sum_i p(i,t) + \sum_j p(j,t) - Dem_t\right)$$

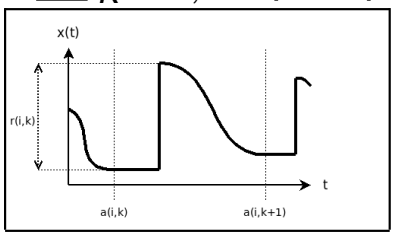
s.t.



Columns generation (1)

- Given prices (μ_t), subproblem i generates production planning :

$$\begin{array}{ll} \min & \sum_k C_{i,k} r(i,k) - C_T x(i,T) - \sum_t \mu_t p(i,t) \\ \text{s.t.} & \end{array}$$



- Plans with negative reduced costs are added in a master problem (new columns)

	$\lambda^1_{i_1}$	$\lambda^2_{i_1}$	$\lambda^1_{i_2}$	$\lambda^2_{i_2}$	$p(j,0)$	$\cdots$	$p(j,T-1)$	
μ_1	$p^1_{i_1}(0)$	$p^2_{i_1}(0)$	$p^1_{i_2}(0)$	$p^2_{i_2}(0)$	1	$\cdots$	0	Dem_0
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
μ_{T-1}	$p^1_{i_1}(T-1)$	$p^2_{i_1}(T-1)$	$p^1_{i_2}(T-1)$	$p^2_{i_2}(T-1)$	0	$\cdots$	1	Dem_{T-1}
	$c^1_{i_1}$	$c^2_{i_1}$	$c^1_{i_2}$	$c^2_{i_2}$	$c_{j,0}$	$\cdots$	$c_{j,T-1}$	

- Master solves relaxation and proposes new prices.

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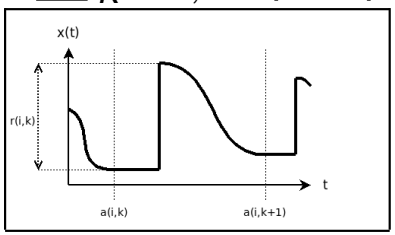

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μ_{T-1}	$p^1_{i_1}(T-1)$	$p^2_{i_1}(T-1)$	$p^1_{i_2}(T-1)$	$p^2_{i_2}(T-1)$	0	$\cdots$	1	Dem_{T-1}
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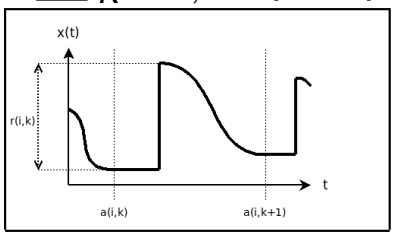

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μ_1	$p^1_{i_1}(0)$	$p^2_{i_1}(0)$	$p^1_{i_2}(0)$	$p^2_{i_2}(0)$	1	$\cdots$	0	Dem_0
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- Master solves relaxation and proposes new prices.

Columns generation (2)

- Occasionally, the master chooses a combination of columns satisfying demand constraints...

$\lambda_{i_1}^1$	$\lambda_{i_1}^2$	$\lambda_{i_2}^1$	$\lambda_{i_2}^2$	$p(j,0)$	$\cdots$	$p(j,T-1)$	
$p_{i_1}^1(0)$	$p_{i_1}^2(0)$	$p_{i_2}^1(0)$	$p_{i_2}^2(0)$	1	$\cdots$	0	Dem_0
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$p_{i_1}^1(T-1)$	$p_{i_1}^2(T-1)$	$p_{i_2}^1(T-1)$	$p_{i_2}^2(T-1)$	0	$\cdots$	1	Dem_{T-1}
$c_{i_1}^1$	$c_{i_1}^2$	$c_{i_2}^1$	$c_{i_2}^2$	$c_{j,0}$	$\cdots$	$c_{j,T-1}$	

- ... with exactly one planning for each plant...

$$\sum_n \lambda_i^n = 1 \quad \forall i$$
$$\lambda_i^n \in \{0,1\} \quad \forall i,n$$

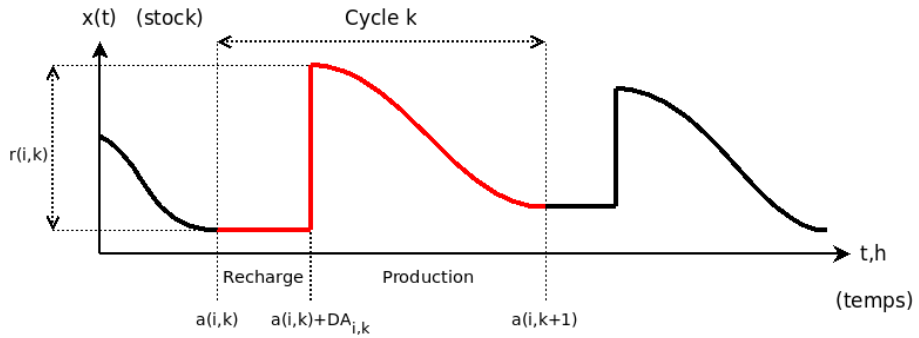
- ... and respecting additional combinatorial constraints.

Subproblem : command of one nuclear plant

- A discret time optimal control problem :
  - Set of equations describing the evolution of stock :

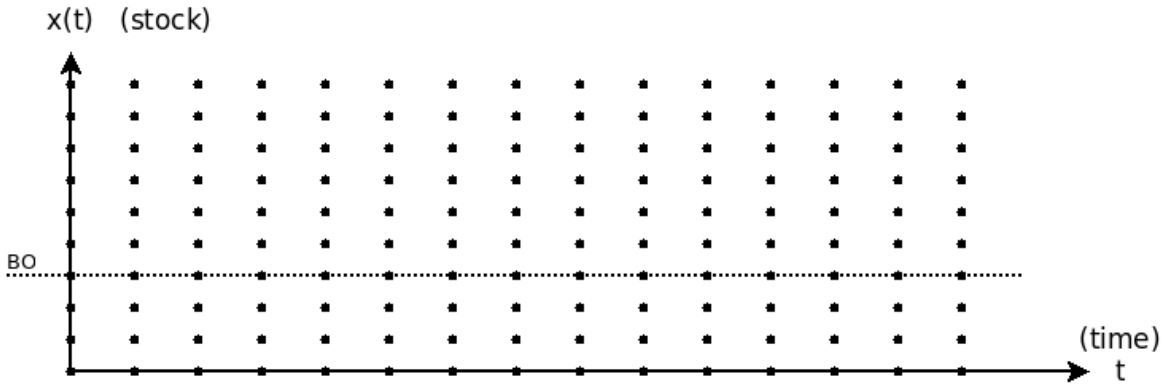
$$x(i,t+1) - x(i,t) = -p(i,t)D^t, \qquad \forall i,k,t \in ec(i,k)$$
$$x(i,t+1) = \frac{Q_{i,k}-1}{Q_{i,k}}(x(i,t) - BO_{i,k-1}) + r(i,k) + BO_{i,k}, \qquad \forall i,k,t = N^t \times ea(i,k)$$
$$RMIN_{i,k} \leq r(i,k) \leq RMAX_{i,k} \qquad \forall i,k$$
$$PMIN_{i,t} \leq p(i,t) \leq PMAX_{i,t} \qquad \forall i,t$$
$$\ldots$$

  - To determine policy (production and refueling) minimizing the cost along the resulting path.



Subproblem : command of one nuclear plant

- Discretisation of stock level and Dynamic Programming



  - Cost of a production transition : $\mu_t \times (x(i, t+1) - x(i, t))$.
  - Under a threshold BO_k, transitions are imposed.
  - On some state, we can perform refuel of cycle k.
  - Cost of a refuel transition : $C_k \times (x(i, t + DA_k) - x(i, t))$

- Forward or backward induction : in $O(E) = O(L^2 \times T)$ where L is the number of levels.

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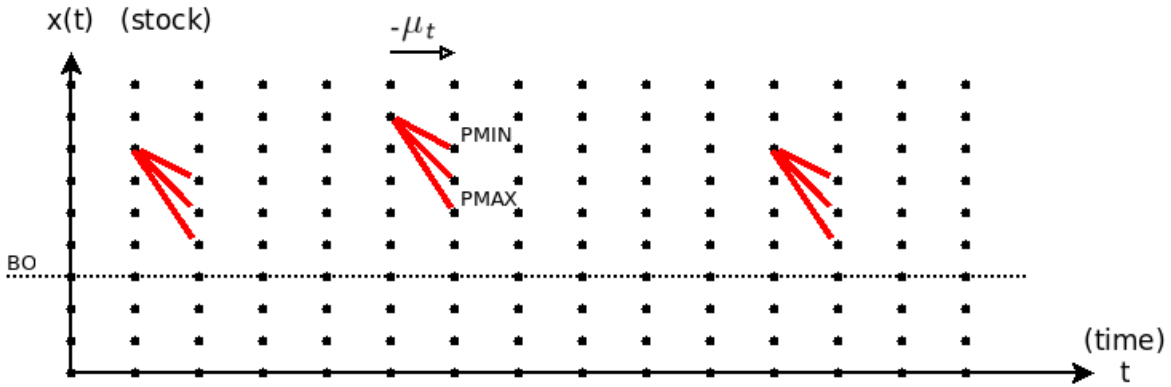

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Subproblem : command of one nuclear plant

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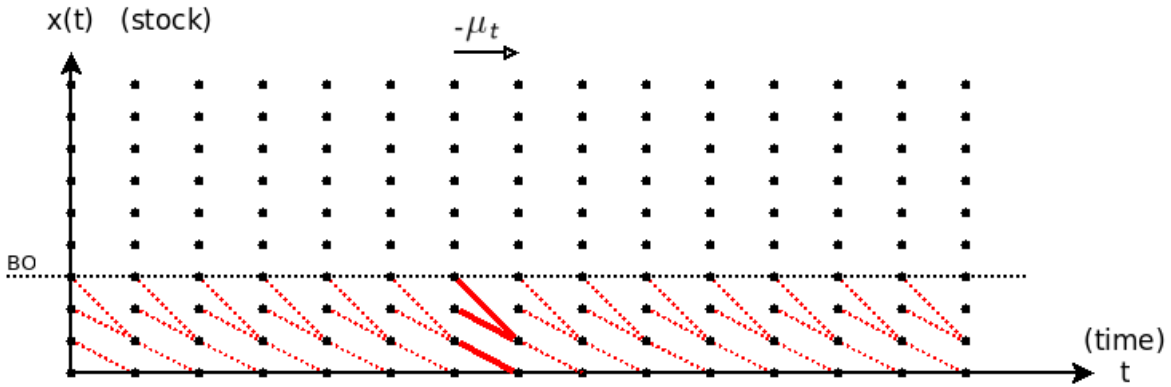

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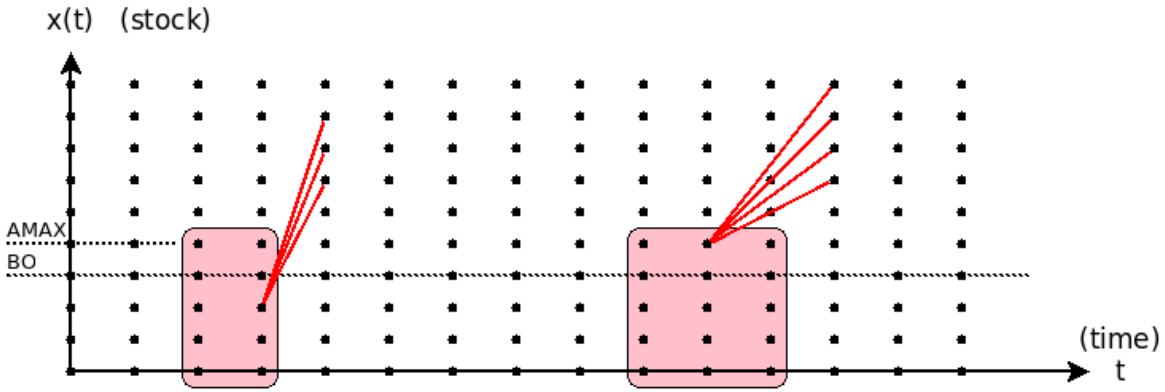

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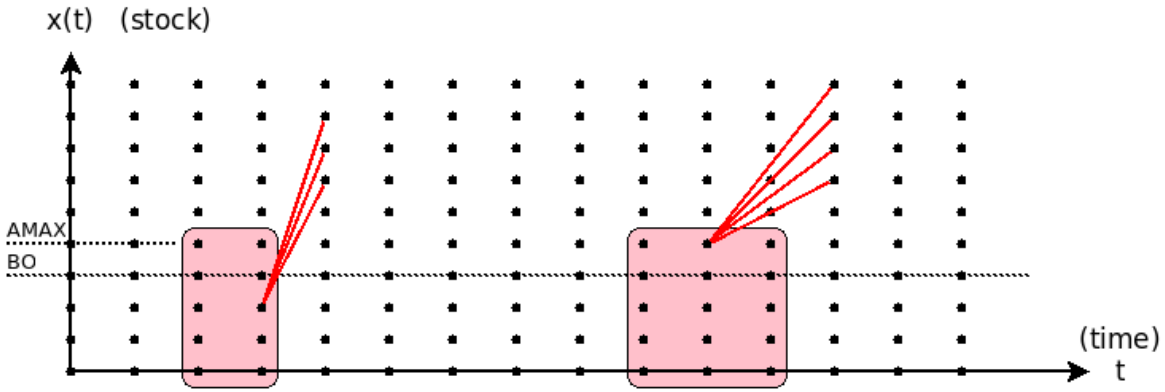

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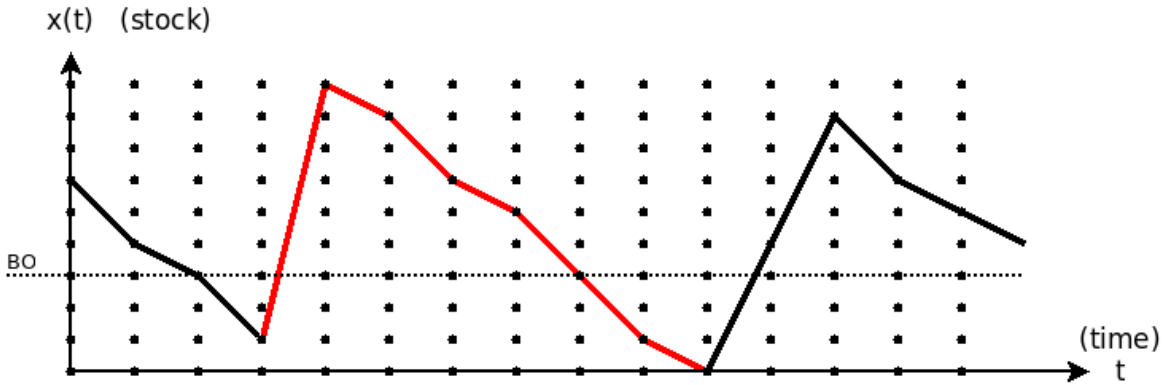

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  - Modulation consists in not producing at PMAX.
  - Modulation is cumulated over a cycle and cannot exceed a certain amount :

$$\sum_{t \in ec(i,k) \wedge x_{i,t} \geq BO_{i,k}} (PMAX_i^t - p(i,t)) \leq MMAX_{i,k}$$

$\Rightarrow$ Searching in a reduced graph (Rozenknop, Wolfler *et al*).

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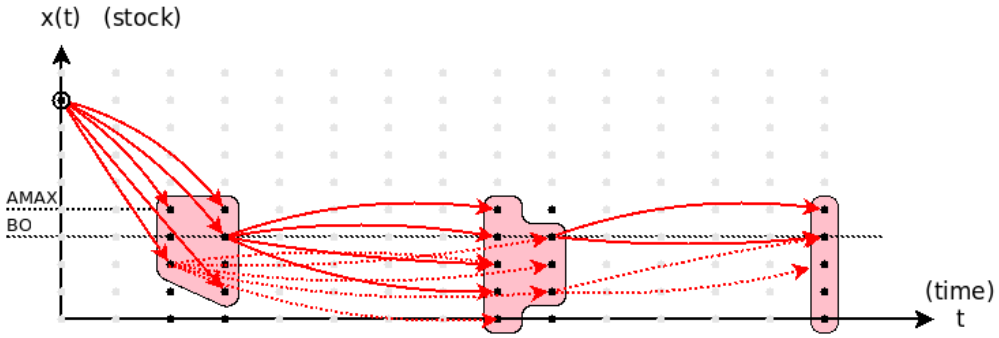

- Optimal policy between nodes (x, t) and (x', t') can be computed greedily.
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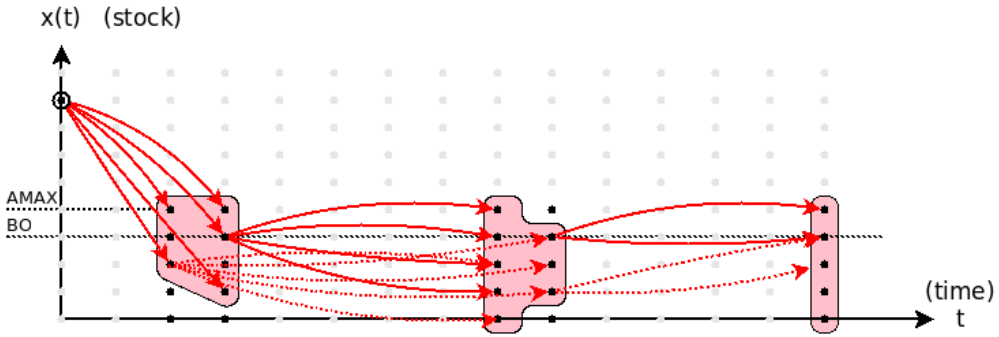

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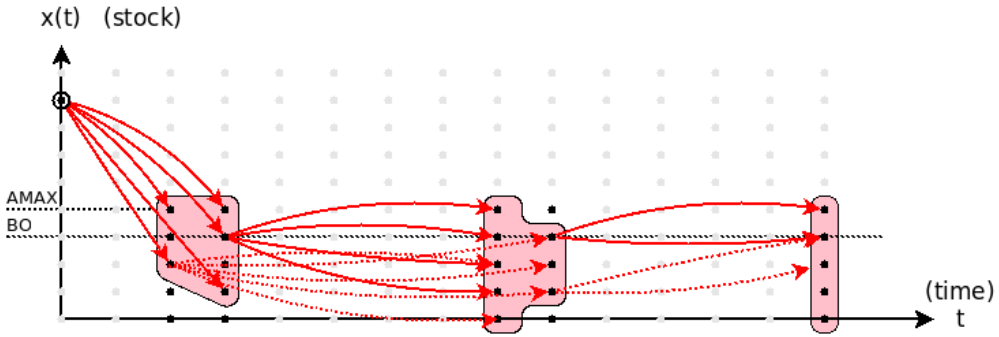

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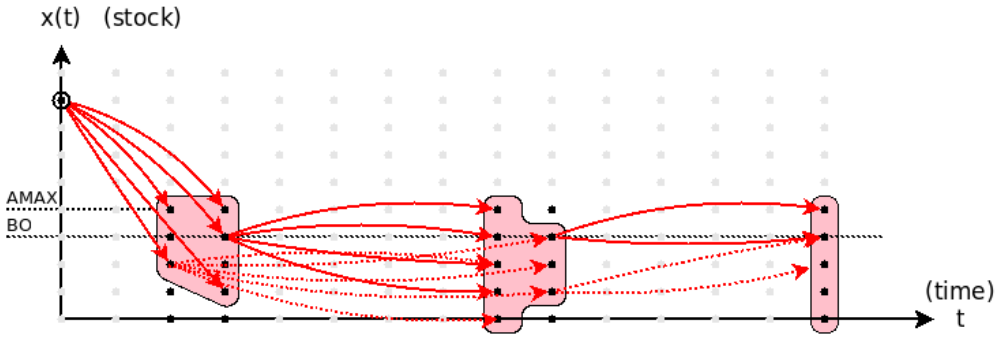

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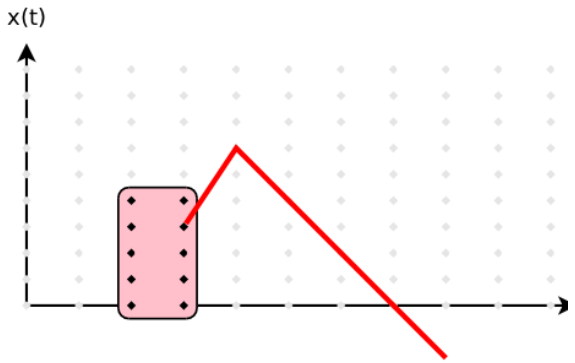

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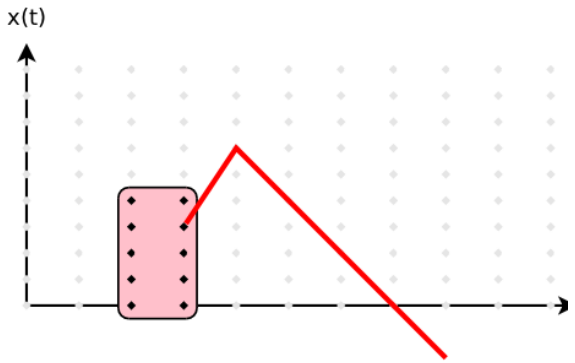

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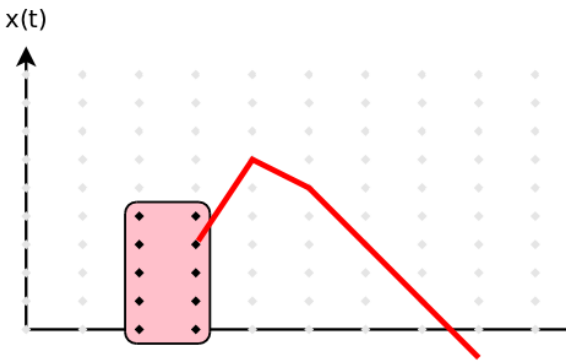

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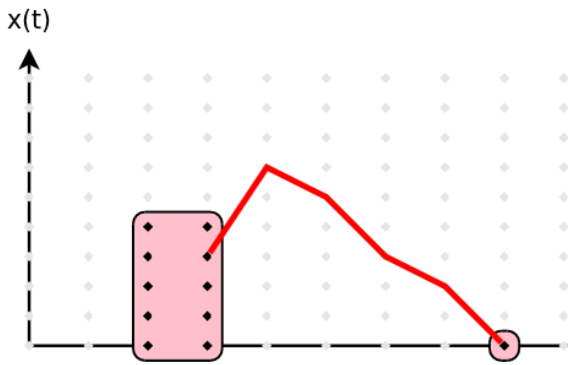

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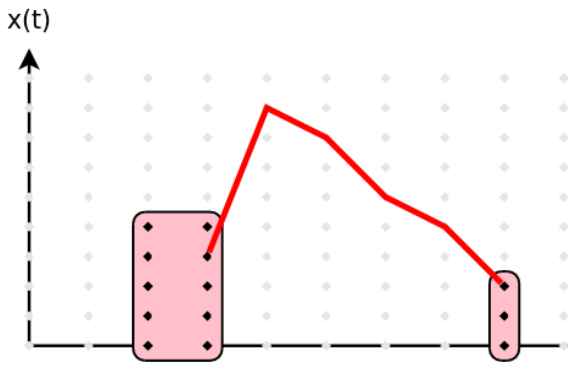

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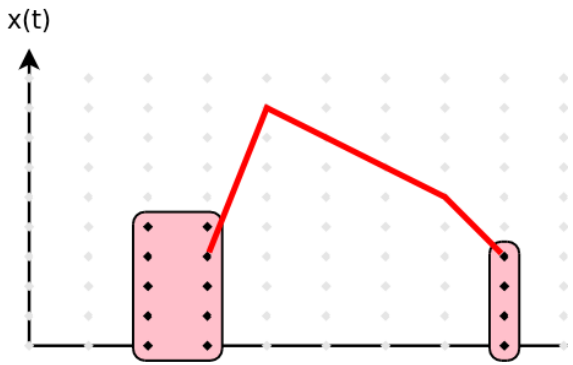

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- Shortest path with a resource constraint :
  - Forward induction : propagation of labels (k, c, m)
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  - There are possibly several labels at each node :
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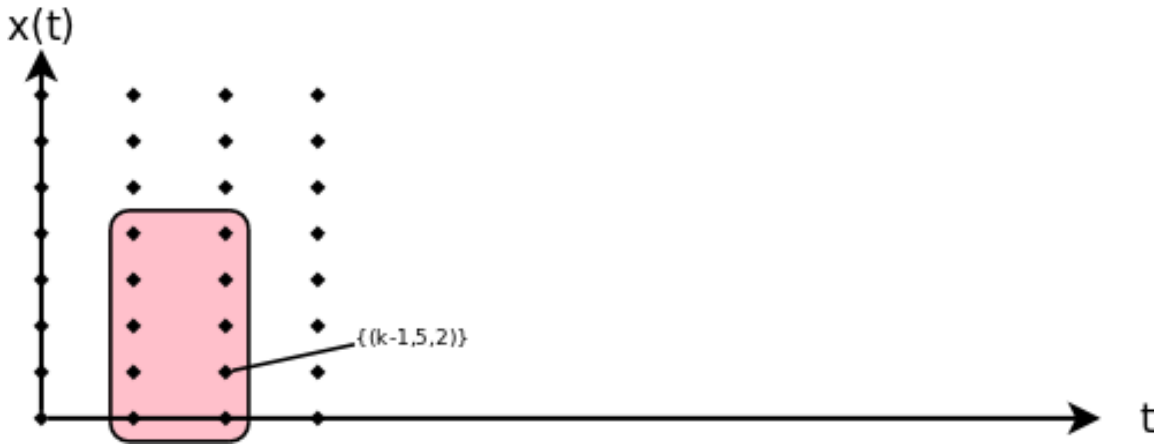

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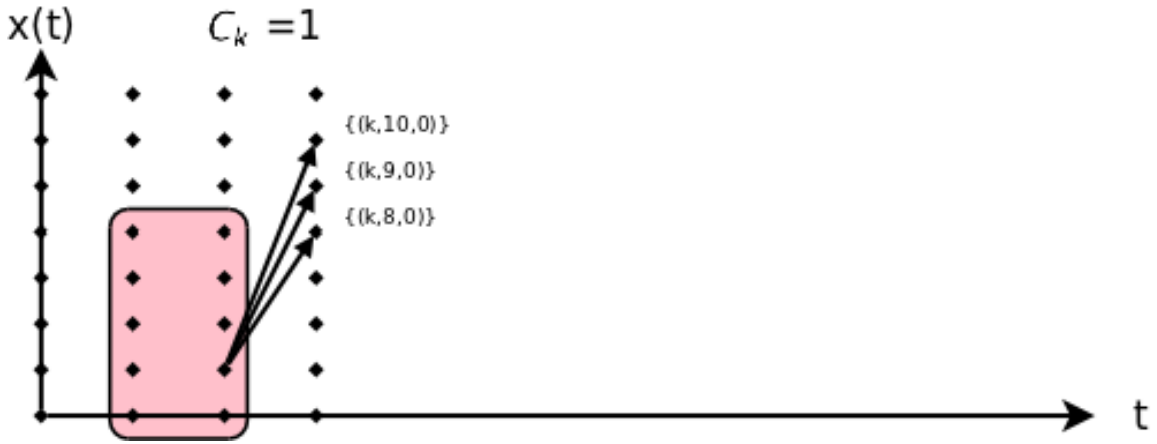

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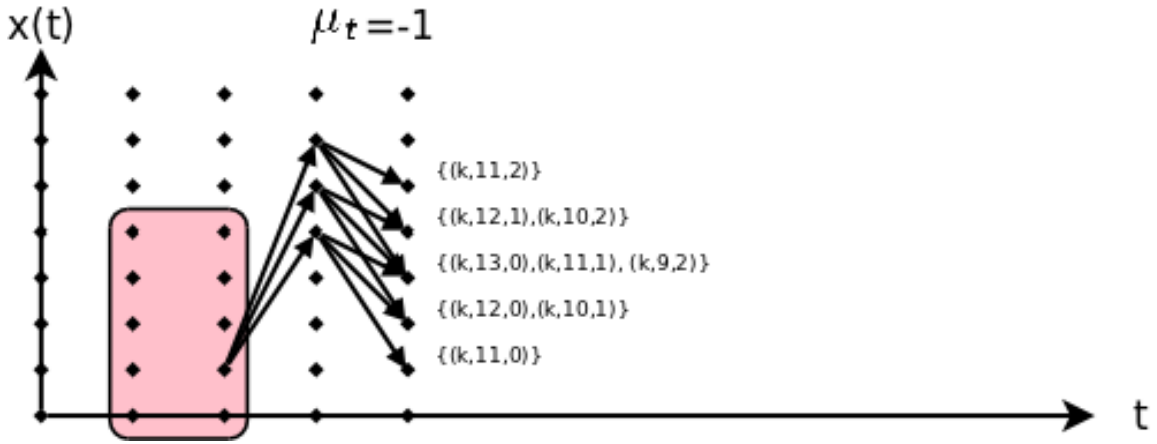

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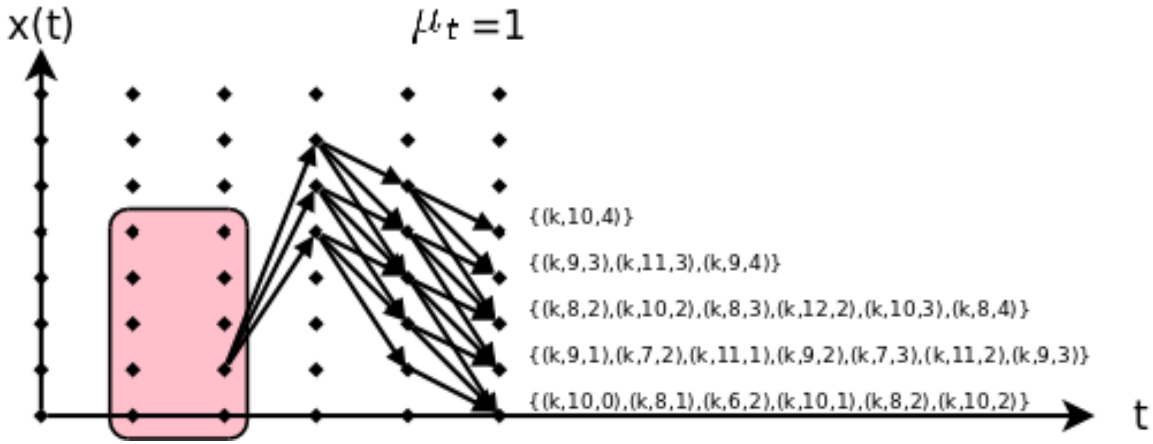

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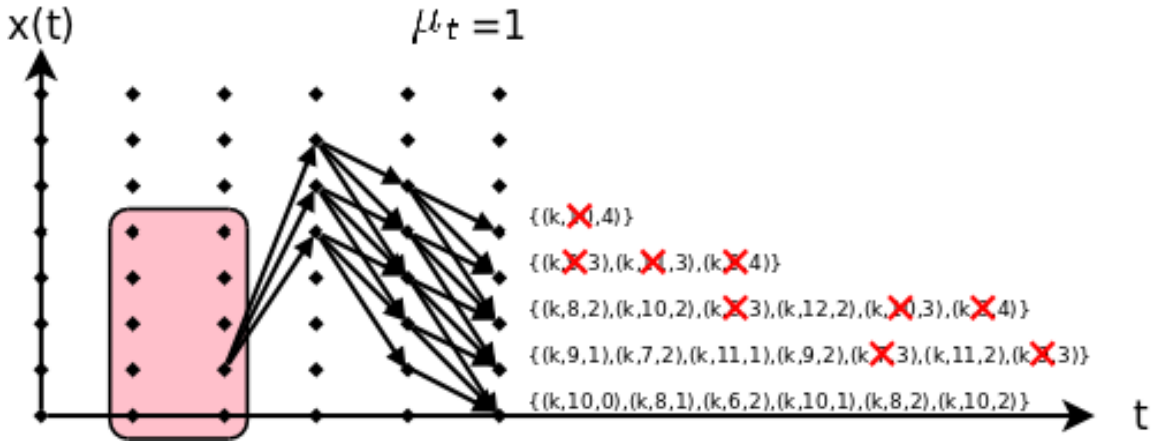

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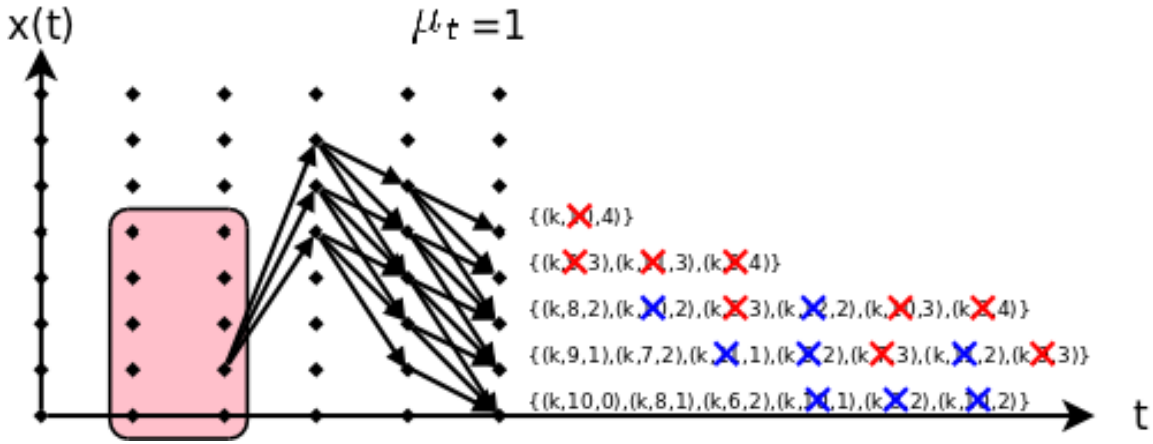

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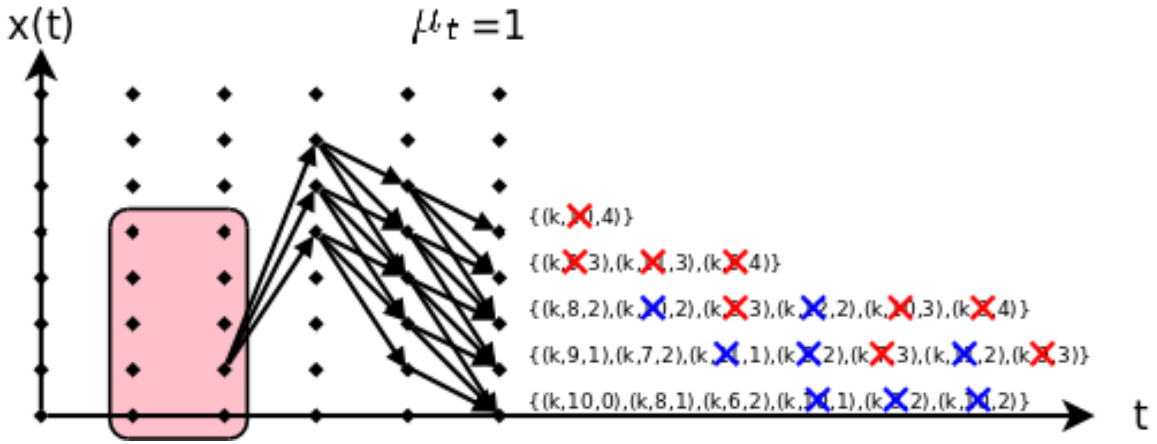

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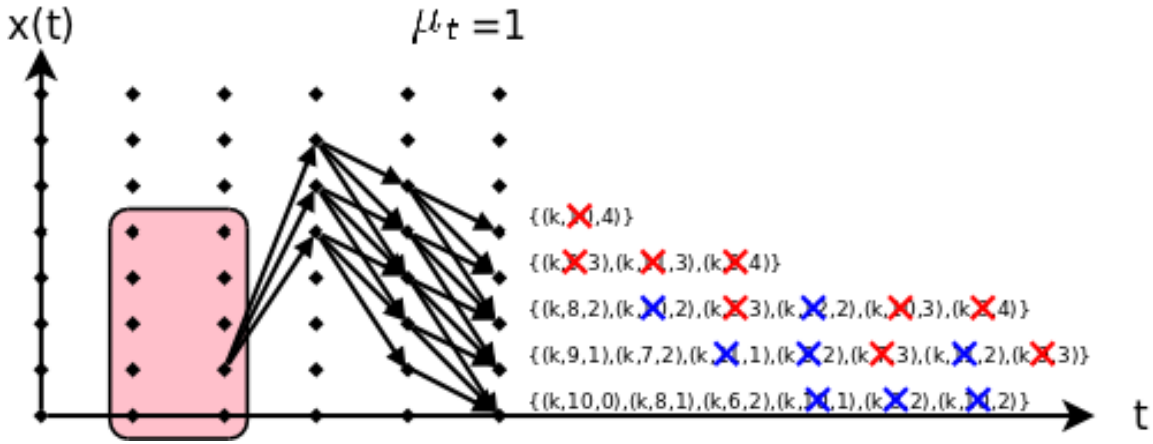

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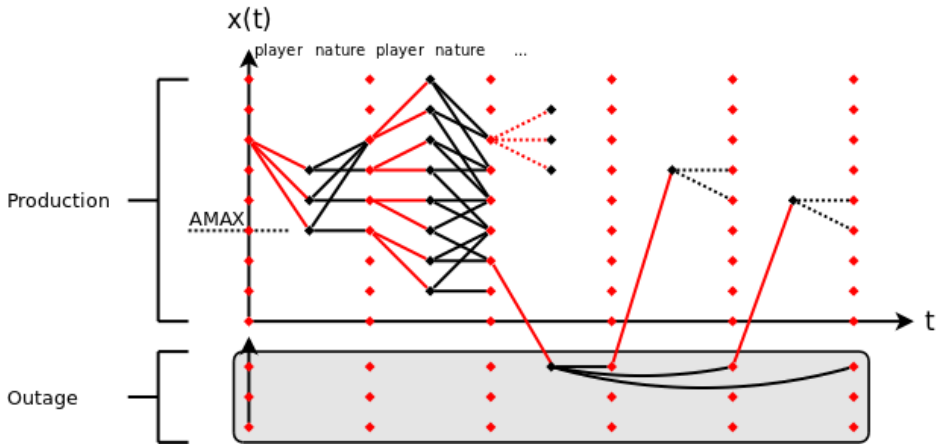
Stochastic subproblem

- Game against nature :
  - During production : player decides production level, nature decides to produce or not (incident)
  - During outage : player decides an outage, nature decides the duration, player decides the refuel and the first production, nature decides to produce or not (incident)
- Duplicate nodes to make explicite the current phase of the cycle (outage or production) :



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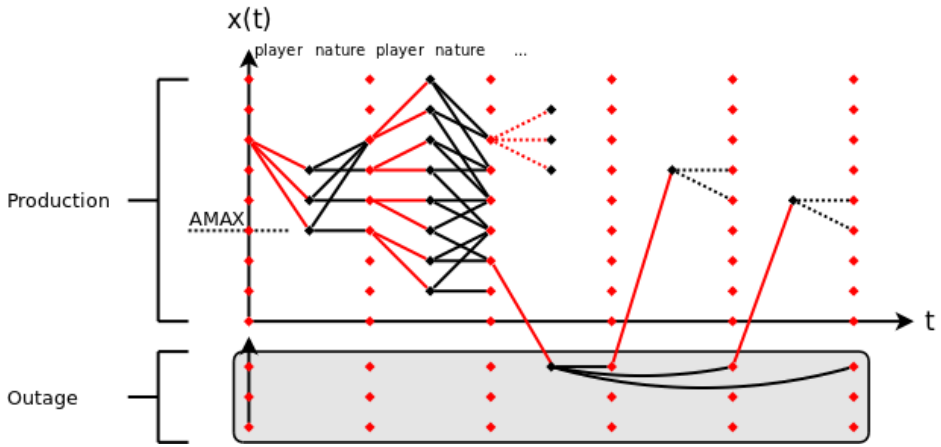

Backward induction for stochastic process

- Backward induction immediately generalizes to minimize expected cost :

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- How should we count modulation when there is incident ?
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- Backward induction of labels (c, m) :
  - c is the expected cost needed to reach the horizon.
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- A solution minimizes expected cost and satisfies maximum modulation in all the cases.
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- A solution minimizes expected cost and satisfies maximum modulation in all the cases.
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  - Too strong $\Rightarrow$ relax this contraint as a chance contraint.

Backward induction and maximum modulation

- How should we count modulation when there is incident ?
  - Hypothesis : a zero production does not count as modulation
- Backward induction of labels (c, m) :
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Stochastic master

- Problem : a planning is now defined as a closed-loop policy :

$$p(i, t, x)$$

- Considering only incidents, can be seen as a policy :

$$p(i, t, (\omega_i^l)^{l=0..t-1})$$

- Hypothesis : thermal plants can adapt instantaneously :

$$p(j, t, (\omega_i^l)_{i \in I}^{l=0..t})$$

- How can we satisfy energy balance exactly ?

$$\sum_{i \mid \omega_i^t = 1} p(i, t, (\omega_i^l)^{l=0..t-1}) + \sum_{j \in J} p(j, t, (\omega_i^l)_{i \in I}^{l=0..t}) = Dem_t, \ \forall (\omega_i^l)_{i \in I}^{l=0..t}$$

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- Subproblem gives only the mean production during each time periods : $\overline{p}(i,t)$.
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- When the master combines planning for each subproblem, the probability distribution on total nuclear production is given by the convolution $*_i \rho(i,t)$ (since the aleas are independant).
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- Forward induction does not generalize easily.
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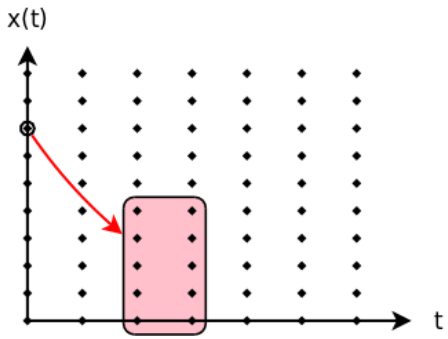

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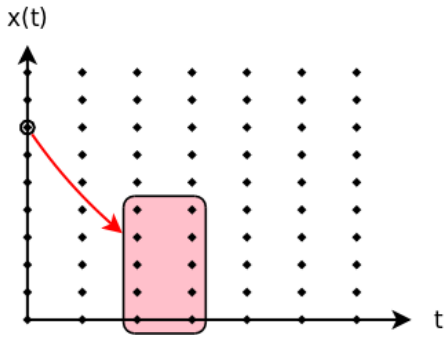

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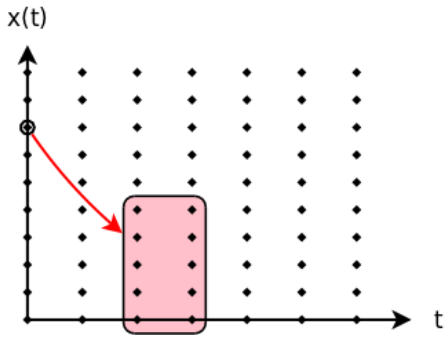

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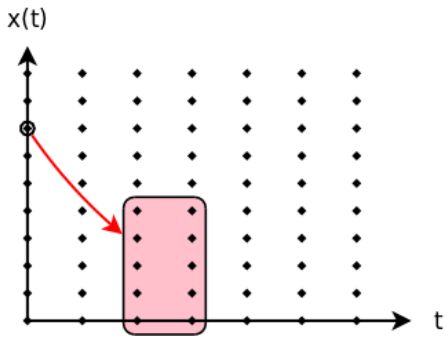

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Subproblem and chance constraints

- Determine a solution which is a compromise between the classical reduced cost and the success probability of the solution :

$$\min \sum_{i,j} -\mu_t p_t - \gamma \log(Pr(success))$$

- Why log ?
  - In the master, we want to garantee that the solution satisfies a global success probability $Pr(success) \geq \alpha$.
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