Electricity market: analytic approach to the producer's problem

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Modeling of Electricity Markets

- electricity market consists of
  i) generators/consumers respect their own interests in competition with others
  ii) market operator (ISO) who maintain energy generation and load balance, and protect public welfare

- the ISO has to consider:
  i) the market power or participants
  ii) quantities of generated/consumed electricity
  iii) electricity dispatch with respect to transmission capacities

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- since 1990s, Nash equilibrium problem is the most popular way of modeling spot electricity markets

Notation

Let

- $D > 0$ be the overall energy demand of all consumers
- $\mathcal{N}$ be the set of producers
- $q_i \geq 0$ be the production of i-th producer, $i \in \mathcal{N}$
- $A_i q_i + B_i q_i^2$ the true production cost of i-th producer

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Similarly, we assume that producer $i \in \mathcal{N}$ provides to the ISO a quadratic bid function

$$a_i q_i + b_i q_i^2$$

given by non-negative parameters $a_i, b_i \geq 0$.

Aim of the Study

Consider a particular producer $i \in \mathcal{N}$ (EDF, for instance).

Then, knowing the overall demand $D > 0$ and bid vectors $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N}$ provided by other producers, we search for the best response $(a_i, b_i) \in \mathbb{R}_+^2$ of producer i in order to maximize his profit

$$\pi_i(a, b) = a_i q_i + b_i q_i^2 - (A_i q_i + B_i q_i^2)$$

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Following this path, we realized that we can not avoid linear bids $b_i = 0$, and so an extension of ISO problem was necessary.

Generalized Nash Equilibrium Problem

Peculiarity of electricity markets is their bi-level structure:

$$P_i(a_{-i}, b_{-i}) \qquad \max_{a_i, b_i} \max_{q_i} \quad a_i q_i + b_i q_i^2 - (A_i q_i + B_i q_i^2)$$
$$\text{such that} \quad \begin{cases} a_i, b_i \geq 0 \\ (q_j)_{j\in\mathcal{N}} \in Q(a,b) \end{cases}$$

where set-valued mapping $Q(a, b)$ denotes solution set of

$$ISO(a,b) \qquad Q(a,b) = \operatorname*{argmin}_q \quad \textstyle\sum_{i\in\mathcal{N}} (a_i q_i + b_i q_i^2)$$
$$\text{such that} \quad \begin{cases} q_i \geq 0 \ , \ \forall i \in \mathcal{N} \\ \displaystyle\sum_{i\in\mathcal{N}} q_i = D \end{cases}$$

Reduction to Nash Equilibrium Problem

Whenever ISO(a,b) has an unique solution, $Q(a,b) = \{q(a,b)\}$, the problem $P_i(a_{-i}, b_{-i})$ may be restated as

$$\max_{a_i, b_i \geq 0} \left[a_i q_i(a,b) + b_i q_i(a,b)^2 - \left(A_i q_i(a,b) + B_i q_i(a,b)^2 \right) \right]$$

with ISO(a,b) implicitly considered.

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with ISO(a,b) implicitly considered.

However, this approach is only formal if we do not have a formula for $q(a, b)$.

Moreover, uniqueness of ISO(a,b) is also unavoidable when it comes to real-world markets.

Uniqueness of ISO(a,b) Problem

There are at least three ways for obtaining uniqueness of ISO(a,b):

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- to assume equity property (Aussel and Pištěk, 2013)

Equity property assumption reads:

$$(H) \qquad (\forall i, j \in \mathcal{N}) \left((a_i, b_i) = (a_j, b_j) \implies q_i = q_j \right),$$

i.e., the ISO does not make any difference among producers.

Formulation of ISO Problem

Knowing overall demand $D > 0$ and bid vectors $(a, b) \in \mathbb{R}_+^{2N}$ provided by producers, the ISO computes $q \in \mathbb{R}_+^N$ in order to minimize the total generation cost.

$$\min_{q} \sum_{i \in \mathcal{N}} (a_i q_i + b_i q_i^2)$$

$$\text{s.t.} \begin{cases} q_i \geq 0, \ \forall i \in \mathcal{N} \\ \\ \sum_{i \in \mathcal{N}} q_i = D \end{cases}$$

This problem has a unique solution if we assume $b_i > 0$ for all $i \in \mathcal{N}$.

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$$\text{s.t.} \begin{cases} q_i \geq 0, \ \forall i \in \mathcal{N} \\ (a_i, b_i) = (a_j, b_j) \Longrightarrow q_i = q_j, \forall i, j \in \mathcal{N} \quad \text{(H)} \\ \sum_{i \in \mathcal{N}} q_i = D \end{cases}$$

...or if we add an equity property assumption allowing $b_i = 0$ for some $i \in \mathcal{N}$.

More Notation, Critical Parameters of ISO

To analyse problem ISO(a,b) further, we introduce

$$
\begin{aligned}
\mathcal{N}_a(\lambda) &= \{i \in \mathcal{N} | a_i < \lambda \in \mathbb{R}_+\} \subset \mathcal{N} \\
F(a, b, \lambda) &= \sum_{i \in \mathcal{N}_a(\lambda)} \frac{\lambda - a_i}{2b_i}
\end{aligned}
$$

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$$F(a,b,\lambda) = \sum_{i \in \mathcal{N}_a(\lambda)} \frac{\lambda - a_i}{2b_i} \ldots \lambda(a,b,D) \Leftrightarrow F(a,b,\lambda(a,b,D)) = D$$

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Since we allow $b_i = 0$, we need to introduce several more variables

$$\lambda^c(a,b) = \min_{i \in \mathcal{N}, b_i = 0} a_i$$

$$D^c(a,b) = F(a,b,\lambda^c(a,b))$$

$$\mathcal{N}^c(a,b) = \{i \in \mathcal{N} |\, a_i = \lambda^c(a,b), b_i = 0\}$$

Their meaning will be clarified soon.

Market Marginal Price

The previous observation justifies the following definition

$$\lambda(a, b, D) = \begin{cases} \lambda \in \mathbb{R}_+ \text{ s.t. } F(a, b, \lambda) = D \text{ if } D \in]0, D^c(a, b)[\\ \lambda^c(a, b) \text{ if } D \geq D^c(a, b) \end{cases} \tag{1}$$

For any $(a, b) \in \mathbb{R}_+^{2N}$ function $\lambda(a, b, D)$ is continuous and piece-wise linear in D.

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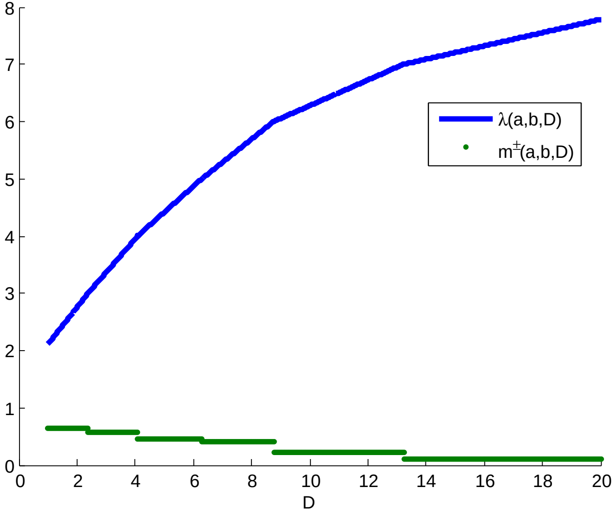
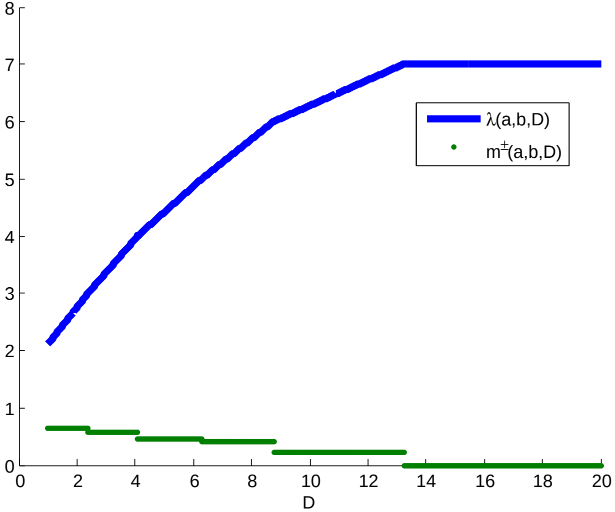
For any $(a,b) \in \mathbb{R}_+^{2N}$ function $\lambda(a,b,D)$ is continuous and piece-wise linear in D. We denote $m^{\pm}(a,b,D) := \partial_D^{\pm}\lambda(a,b,D)$.

$$\begin{aligned} \frac{1}{m^-(a,b,D)} &= \sum_{i \in \mathcal{N}_a(\lambda(a,b,D))} \frac{1}{2b_i} && \text{if } D \leq D^c(a,b) \\ m^-(a,b,D) &= 0 && \text{if } D > D^c(a,b) \end{aligned}$$

Example, $\lambda^c(a, b) = +\infty$



Example, $\lambda^c(a, b) = 7$



Analytic Solution to ISO(a,b) Problem

Theorem

Let $D > 0$ and $(a, b) \in \mathbb{R}_+^{2N}$, then $ISO(a, b)$ admits a unique solution obeying the equity property (H) with $q(a, b)$ given by

$$q_i(a,b) = \begin{cases} \frac{\lambda(a,b,D)-a_i}{2b_i} & \text{if } a_i < \lambda(a,b,D) \\ \frac{D-D^c(a,b)}{N^c(a,b)} & \text{if } a_i = \lambda(a,b,D),\, b_i = 0 \\ 0 & \text{if } a_i > \lambda(a,b,D),\ \text{or } a_i = \lambda(a,b,D),\, b_i > 0 \end{cases}$$

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Moreover, we know all directional derivatives now!

Meaning of $\lambda(a, b, D)$

Now we may compute the overall cost of production D

$$C(a, b, D) = \sum_{i \in \mathcal{N}} a_i q_i(a, b, D) + b_i q_i(a, b, D)^2$$

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Corollary

Consider the setting of the previous theorem, than we have

$$C(a, b, D) = \sum_{i \in \mathcal{N}_a(\lambda)} \frac{\lambda(a, b, D)^2 - a_i^2}{4 b_i} \quad \text{if} \quad D < D^c(a, b)$$

$$C(a, b, D) = D\lambda^c(a, b) - \sum_{i \in \mathcal{N}_a(\lambda^c)} \frac{(\lambda^c(a, b) - a_i)^2}{4 b_i} \quad \text{if} \quad D \geq D^c(a, b).$$

Moreover, it holds $\partial_D C(a, b, D) = \lambda(a, b, D)$.

Problem of Producer i, $P_i(a_{-i}, b_{-i})$

Once formula for $q_i(a, b)$ is achieved, for profit $\pi_i(a, b)$ we have

$$\pi_i(a, b) = a_i q_i(a, b) + b_i q_i(a, b)^2 - (A_i q_i(a, b) + B_i q_i(a, b)^2)$$

and thus for fixed $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$ problem $P_i(a_{-i}, b_{-i})$ reads

$$\max_{a_i, b_i \geq 0} \pi_i(a_i, a_{-i}, b_i, b_{-i})$$

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Further, we assume

- quadratic real cost, quadratic bid (including $b_i = 0$)
- there is no producer producing for free, $A_i + B_i > 0$
- there are no production bounds, $q_i \in \mathbb{R}_+$
- the 'pay-as-bid' problem setting

Theorem

Assume $D > 0$, and for $i \in \mathcal{N}$ consider $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$ and $q(a, b)$ a unique solution to ISO(a,b). Then, the i-th player profit $\pi_i(a, b)$ satisfies one of the following statements:

(a) *for $a_i < \lambda(a_{-i}, b_{-i}, D)$ and $b_i > 0$ we have*

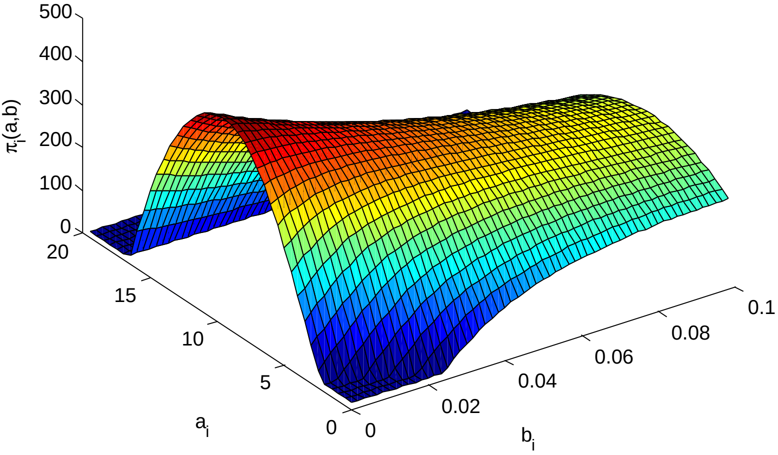
$$\pi_i(a, b) = \frac{\lambda(a, b, D) - a_i}{4b_i^2}\left[a_i b_i - 2A_i b_i + a_i B_i + \lambda(a, b, D)(b_i - B_i)\right]$$

(b) *for $0 < a_i \leq \lambda(a_{-i}, b_{-i}, D)$ and $b_i = 0$ (and so $a_i = \lambda^c(a, b)$)*

$$\pi_i(a, b) = (\lambda^c(a, b) - A_i)\frac{D - D^c(a, b)}{N^c(a, b)} - B_i\left(\frac{D - D^c(a, b)}{N^c(a, b)}\right)^2,$$

(c) $\pi_i(a, b) \leq 0$ *otherwise*

Example: $\pi_i(a_i, b_i)$



Partial Directional Derivatives of $\pi_i(a, b)$

Now, we may calculate partial directional derivatives:

$$\partial^{\pm}_{a_i}\pi_i(a, b, D) = \frac{1}{4b_i^3} \times \Big[(\lambda(a, b, D) - A_i)(m^{\pm}(a, b, D)b_i - 2b_i^2)$$
$$-(\lambda(a, b, D) - a_i)(m^{\pm}(a, b, D)B_i - 2b_iB_i - 2b_i^2)\Big]$$
$$\partial^{\pm}_{b_i}\pi_i(a, b, D) = \frac{\lambda(a,b,D)-a_i}{4b_i^4} \times \Big[(\lambda(a, b, D) - A_i)(m^{\pm}(a, b, D)b_i - 2b_i^2)$$
$$-(\lambda(a, b, D) - a_i)(m^{\pm}(a, b, D)B_i - 2b_iB_i - b_i^2)\Big]$$

The Best Response of Producer $i \in \mathcal{N}$

Theorem

Let $(a, b) \in \mathbb{R}_+^{2N}$ and $D > 0$. If (a_i, b_i) is the i-th producer's best response such that $\pi_i(a, b) > 0$, then $b_i = 0$.

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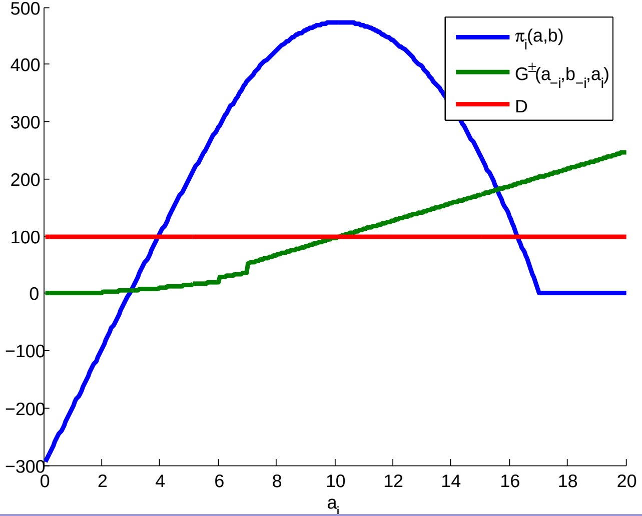
Theorem

Let $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$ such that $\mathcal{N}^c(a_{-i}, b_{-i}) = \emptyset$, $D > F(a_{-i}, b_{-i}, A_i)$ and $b_i = 0$. Then, the best response $(a_i, 0)$ of producer $i \in \mathcal{N}$ yielding $\pi_i(a, b) > 0$ is a unique solution to $G^+(a_{-i}, b_{-i}, a_i) \geq D \geq G^-(a_{-i}, b_{-i}, a_i)$, with

$$G^{\pm}(a_{-i}, b_{-i}, \lambda) = \frac{\lambda - A_i}{2B_i + m^{\pm}(a_{-i}, b_{-i}, F(a_{-i}, b_{-i}, \lambda))} + F(a_{-i}, b_{-i}, \lambda).$$

Example: $\pi_i(a, b)$ with $b_i = 0$ and $\mathcal{N}^c(a_{-i}, b_{-i}) = \emptyset$



Main Achievements

- we found the analytic solution $q(a, b)$ of ISO problem, including linear bids $b_i = 0$ and assuming a newly introduced equity property
- we conclude that the best response of producer i is a linear bid, $a_i > 0$ and $b_i = 0$
- we derived implicit formula for optimal a_i under quite general conditions, we shown existence and uniqueness of such bid
- we deduced explicit formula for directional derivatives of optimal profit π_i

Further Extensions

There are several possible extensions of the proposed model/technique

- to characterize all Nash Equilibria of the proposed model
- to include transmission network constraints
- to add production bounds $q_i \leq \overline{q}_i$

Thank you for your attention.

Selected References

- X. Hu and D. Ralph, *Using EPECs to Model Bilevel Games in Restructured Electricity Markets with Locational Prices*, Operations Research 55 (2007), 809-827.
- D. Aussel, M. Cervinka and M. Marechal, Day-ahead electricity market with production bounds, (2012), 24 pp.
- D. Aussel, R. Correa and P. Marechal *Spot electricity market with transmission losses*, J. Indust. Manag. Optim. (2012), 18 pages.
- R. Henrion, J.V. Outrata and T. Surowiec, Analysis of M-stationary points to an EPEC modeling Oligopolistic Competition in an Electricity Spot Market, ESAIM: COCV 18 (2012) 295-317