Polynomial time algorithms for the lot-sizing problem under energy constraints

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Context

- Rarefaction of fossil fuels
  $\Rightarrow$ Increase in energy price, price fluctuation of the electricity
- Use of less polluting but less flexible renewable energies
  $\Rightarrow$ Limitation on the available energy
- New environmental standards, quotas, taxes (Kyoto, COP21)
  $\Rightarrow$ Re-organization of industrial activities (to be more ecological, less energy consuming, etc.)

Scope of the talk

- Single-item lot sizing problem
- Identical, parallel and capacitated machines
- Energy limitation for machine start-ups and production
- What to decide : How many machines to start and a feasible production plan for them
- Reduction to a capacity acquisition problem

Problem description

Lot-sizing problem on a time horizon T on parallel machines

- d_t : demand in period t
- c_t, h_t: unit production cost, unit holding cost in t.
  We assume that costs are non-speculative : $c_t + h_t \geq c_{t+1}$
- U: capacity of each machine
- f: start-up cost of a machine

With energy limitations

- E_t: amount of available energy in period t
- p_t: energy consumption to produce one unit in period t
- w_t: energy consumption to start a machine in period t

energy-LSP

$$
\begin{aligned}
\min \quad & \textstyle\sum_{t=1}^{T} f m_t^+ + \sum_{t=1}^{T} c_t x_t + \sum_{t=1}^{T} h_t s_t \\
s.t. \quad & s_{t-1} + x_t = s_t + d_t && \forall t \in \{1..T\} \quad (1) \\
& x_t \leq m_t U && \forall t \in \{1..T\} \quad (2) \\
& p_t x_t + w_t m_t^+ \leq E_t && \forall t \in \{1..T\} \quad (3) \\
& m_t = m_{t-1} + m_t^+ && \forall t \in \{1..T\} \quad (4) \\
& s_t \geq 0, x_t \geq 0, m_t \in \mathcal{Z}^+, m_t^+ \in \mathcal{Z}^+ && \forall t \in \{1..T\} \quad (5)
\end{aligned}
$$

- x_t: quantity produced in t
- s_t: stock level at the end of period t
- m_t: the number of machines running during period t
- m_t^+: the number of machines started-up at the beginning of period t

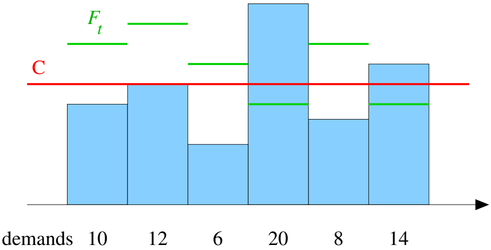
energy-LSP with $w_t = 0$

We consider a much simpler case where $w_t = 0$

- Energy is consumed only by production : define a limit $F_t \equiv E_t/p_t$ on the number of units produced in period t
- We have to decide how many machines m to start at the beginning of the horizon, and a feasible plan for them

$$
\begin{array}{lllr}
\min & fm + \sum_{t=1}^{T} c_t x_t + \sum_{t=1}^{T} h_t s_t & & \\
s.t. & s_{t-1} + x_t = s_t + d_t & \forall t \in \{1..T\} & (1) \\
 & x_t \leq C \equiv mU & \forall t \in \{1..T\} & (2) \\
 & x_t \leq F_t \equiv E_t/p_t & \forall t \in \{1..T\} & (3) \\
 & s_t \geq 0, x_t \geq 0, m \in \mathcal{Z}^+ & \forall t \in \{1..T\} & (5)
\end{array}
$$

energy-LSP with $w_t = 0$



Capacity acquisition problem ($C = mU$) with additional production constraints ($F_t = E_t/p_t$)

energy-LSP with $p_t = 0,\ w_t > 0$

We consider also the case with $p_t = 0$

- Energy is consumed only to start a machine
- Does not limit the production, but the number of machines we can start at a period
- Most probably, we can not start all the machines we wish at the beginning of the horizon !

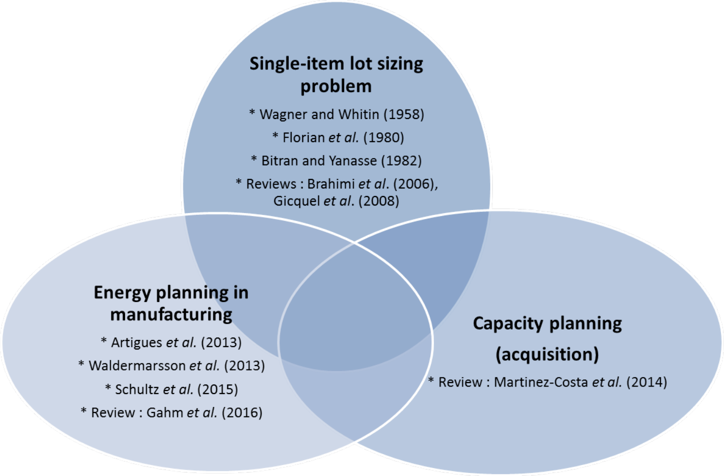
We can formulate the problem as before, with additional production constraints defined by:

$$F_t = \sum_{u=1}^{t} \lfloor E_u / w_u \rfloor$$

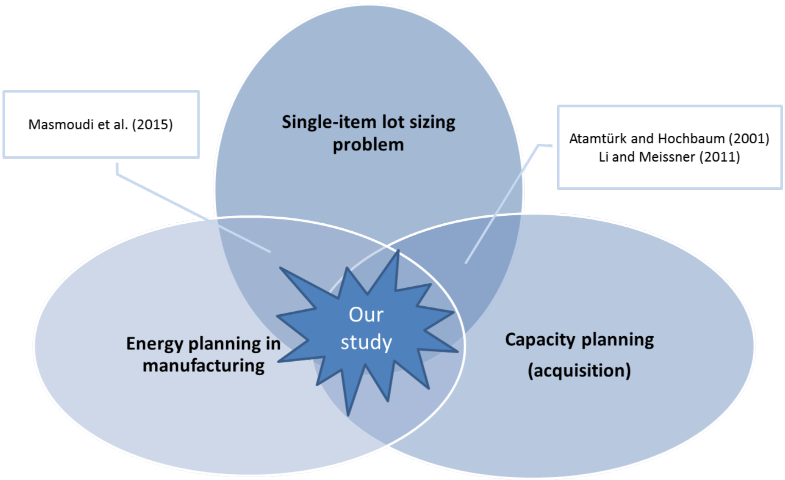
We want to solve a capacity acquisition lot-sizing problem (with additional capacity constraints)

Literature

There are 3 important domains to position our problem :



Literature


Masmoudi et al. (2015)
Single-item lot sizing problem
Atamtürk and Hochbaum (2001)
Li and Meissner (2011)
Our study
Energy planning in manufacturing
Capacity planning (acquisition)

Pure capacity acquisition problem

We consider first the capacity acquisition problem, without additional energy constraints

- Capacity $C = mU$ is considered as continuous
- Problem studied by Atamtürk & Hochbaum (2001) (with subcontracting), Management Science 2001.

The approach of Atamtürk & Hochbaum (2001) :

- For a given capacity C, an optimum solution x can be found in linear time $O(T)$
- There are at most $O(T^2)$ capacities to consider for extreme solutions : Overall time complexity in $O(T^3)$

We improve dramatically their algorithm complexity (but without subcontracting)

No-Slack solution

We only consider the solutions x such that:

- either t is a full production period, that is, $x_t = C$
- or t is a regeneration period, that is, $s_{t-1} = 0$

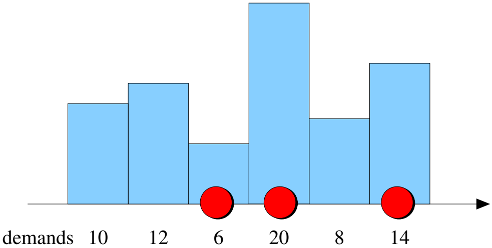
In fact, we are mainly interested in the set of periods at full production capacity

$$\mathscr{B} = \{t | x_t = C\}$$

Idea : forget about solution x, only focus on set $\mathscr{B}$!

Set $\mathscr{B}$

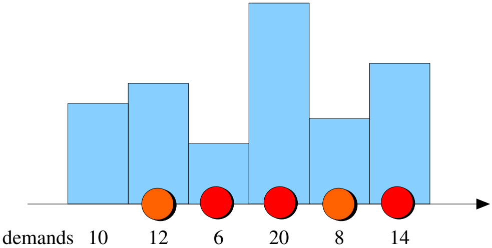
For a set $\mathscr{B}$ of periods :



On the example, $\mathscr{B} = \{3, 4, 6\}$

Set $\mathscr{A}$

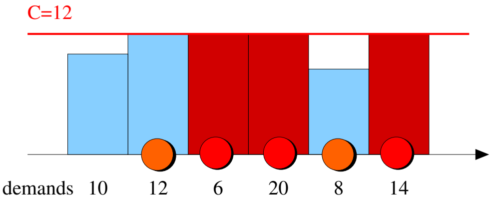
For a set $\mathscr{B}$ of periods :



Define $\mathscr{A} = \{t \mid t \notin \mathscr{B} \text{ and } (t+1) \in \mathscr{B}\}$

Associated solution $x(\mathscr{B}, C)$

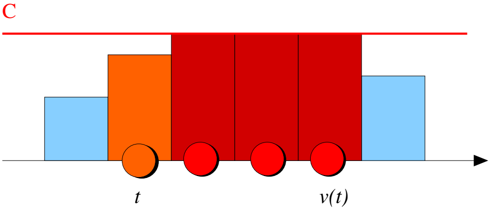
For a set $\mathscr{B}$ of periods and a capacity C



Define the no-slack solution $x_t(\mathscr{B}, C) =$

- Capacity C if $t \in \mathscr{B}$
- Demand d_t if $t \notin \mathscr{A} \cup \mathscr{B}$
- What is needed if $t \in \mathscr{A}$

What is needed ?



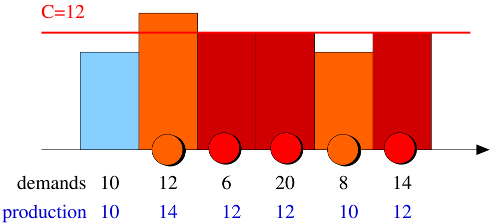
Write flow conservation till the next regeneration point (between periods t and $v(t)$):

$$x_t(\mathscr{B}, C) = D_{t,v(t)} - (v(t) - t)C$$

where $D_{t,v(t)} = \sum_{k=t}^{v(t)} d_k$ is the total demand on $\{t, \ldots, v(t)\}$

Feasible ?

Solution $x(\mathscr{B}, C)$ may be unfeasible : $x_t > C$ in some $\mathcal{A}$-period.



On the example, with $\mathscr{B} = \{3, 4, 6\}$ and $C = 12$, production in period 2 exceeds C

What is needed may exceed what is possible

Optimality

Property

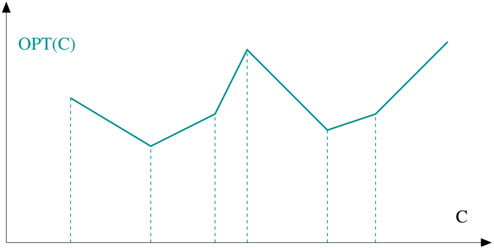
If solution $x(\mathscr{B}, C)$ is feasible, and produces exactly the total demand ($s_T = 0$), then it is an optimal solution for the problem with capacity C

If $x(\mathscr{B}, C)$ is *dominant*, its cost is $\mathrm{OPT}(C)$

Property

If for all capacities C in an interval $[a, b]$ the solution $x(\mathscr{B}, C)$ is *dominant*, then $\mathrm{OPT}(C)$ varies linearly over $[a, b]$.

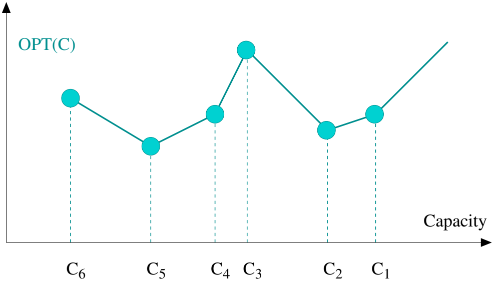
What does optimal value look like?



Piece-wise affine function

What does optimal value look like?

The optimum $\text{OPT} = \min\{\text{OPT}(C) \mid C \geq 0\}$ is reached at one breakpoint



We want to construct this cost function

Idea of the algorithm

- $\mathrm{OPT}(C)$ is a piece-wise affine function
- We establish that it admits at most T breakpoints
- The optimal is reached at one of the breakpoints
- For a given value C, the optimal solution can be constructed in linear time $O(T)$

We find iteratively the breakpoints $C_1, C_2, \ldots, C_k$ **:**

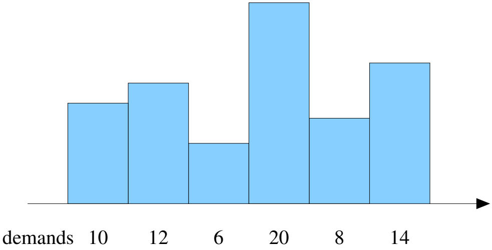
- From a breakpoint C_i and a set $\mathscr{B}_i$.
- We search the lowest capacity $C \leq C_i$ for which solution $x(\mathscr{B}_i, C_i)$ is feasible
- This value defines a new breakpoint C_{i+1}, and a new set $\mathscr{B}_{i+1}$

As $\mathscr{B}_i \subset \mathscr{B}_{i+1}$*, we have at most* T *iterations*

First step

Initialize the algorithm with

- $C_1 = \max_t\{d_t\}$ and $\mathscr{B}_1 = \{t | d_t = C_t\}$
- Clearly $x(\mathscr{B}_1, C_1)$ is feasible (hence optimal for C_1)



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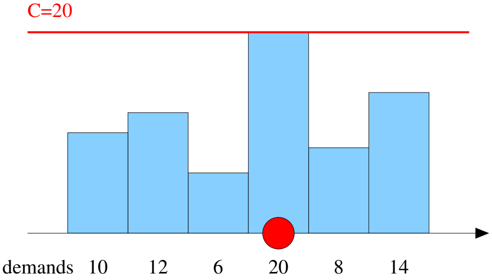
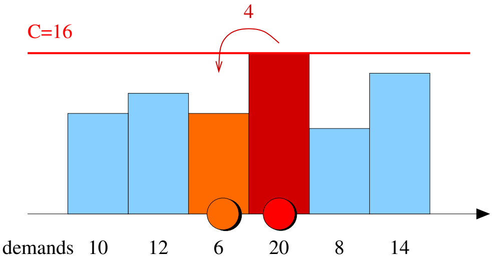


What if we decrease C ?

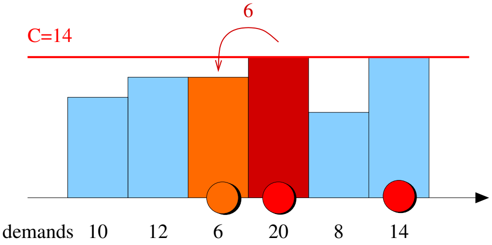
For $C = 16$, solution $x(\mathscr{B}_1, C_1)$ remains feasible



How much can we decrease C ?

First case

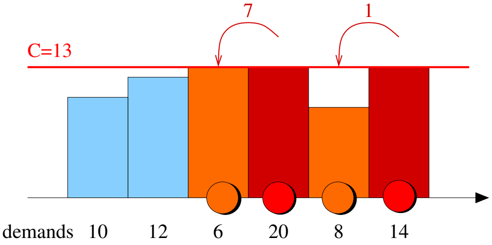
C may hit a demand, here at period 6



- $C_2 = 14$
- $\mathscr{B}_2 = \mathscr{B}_1 \cup \{6\}$

Second case

A period of set $\mathscr{A}$ may become saturated, here period 3

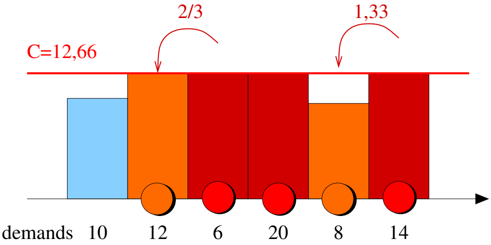


- $C_3 = 13$
- $\mathscr{B}_3 = \mathscr{B}_2 \cup \{3\}$

What is the next capacity to consider ?

Next iteration

$C = 12,66$: period 2 (in set $\mathscr{A}$) becomes saturated

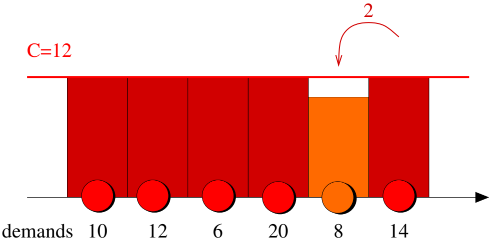


- $C_4 = 12.66$
- $\mathscr{B}_4 = \mathscr{B}_3 \cup \{2\}$

When do we stop ?

Stopping criteria

If the first period enters set $\mathscr{B}$

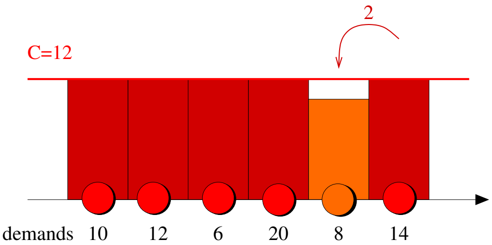


- $C_5 = 12$
- $\mathscr{B}_5 = \mathscr{B}_4 \cup \{1\}$

no feasible solution exists for a lower capacity

Stopping criteria

If the first period enters set $\mathscr{B}$ for capacity C (here $C = 12$), no feasible solution exists for a lower capacity



- A necessary condition to be feasible for a capacity C' is that

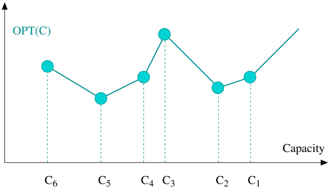
$$D_{1,t} \leq tC' \quad \text{for all periods } t$$

- We have $D_{1,v} = vC$, with here $v = 4$

Complexity of the algorithm

- There are at most $O(T)$ breakpoints :
  $(12,\ 12.66,\ 13,\ 14,\ 20)$ on the example
- Each breakpoint can be found and evaluated in linear time

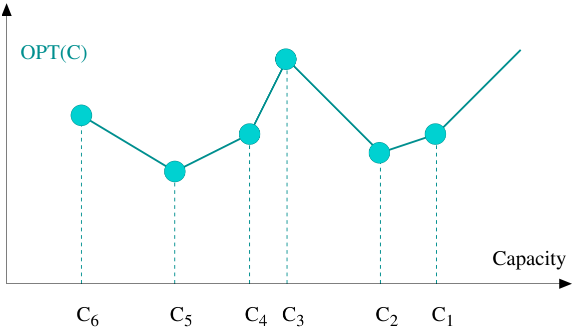
Hence we can build function $\mathrm{OPT}(C)$ in time complexity $O(T^2)$



Complexity of the algorithm

Theorem

The optimum value can be found in time $O(T \log T)$



- Instead of building the optimal planning to evaluate a breakpoint, we compute iteratively the slope of each segment, keeping track of the block structure

With energy constraints ?

- The algorithm is essentially the same : formulas only get more involved to determine the next capacity breakpoint
- Some energy limitations F_t may also become a breakpoint
- We have now at most $2T$ breakpoints

Theorem

The optimum capacity value can still be found in time $O(T \log T)$ with energy constraints (that is, if either $w_t = 0$ or $p_t = 0$)

Conclusion

- The algorithm still works if start-up cost $f()$ is a piecewise concave/convex function
- *C. Rapine, B. Penz, C. Gicquel, A. Akbalik, Capacity acquisition for the single-item lot sizing problem under energy constraints. Omega. To appear*
- Perspective : how to solve the problem with both energy consumption on production AND start-up ?
  Far more challenging, since one has to arbitrate in each period how to use the available amount of energy : either to anticipate the demand (produce to stock) or to increase the capacity of the system (start new machines).

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