Shortest Path Problem variants for the Hydro Unit Commitment Problem

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Hydro Unit Commitment Problem

Application problem:

- A time horizon along which the decisions have to be made, sampled at a finite number of time instants.
- Set of generating units with corresponding energy production costs and consumption.
- The objective consists of determining which units must be utilized to satisfy the demand and other operational constraints, with minimum operating cost.

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Elements of a hydro power plant:

- Reservoirs can be either natural or artificial water basins.
- Turbines produce power by using the water's potential energy.
- Pumps transfer water back to the reservoir by using some power.

Main characteristics of the problem

Assumptions:

- Single reservoir - Hydro Unit Commitment problem.
- Multiple (aggregated) Hydro units, i.e, turbines and pumps.
- Short-term time horizon: it is a day or a week.
- Deterministic formulation: the inflows and demands are known and assumed to be the forecast.
- Price-taker : the electricity prices are assumed to be the forecast.

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This is a multi-period problem: in each period we must decide the operations of multiple generating units in order to maximize the profit, satisfying production and technological constraints.

Main characteristics of the problem

Objective:

- Maximize the profit.
- Minimize the water consumption.
- Minimize the costs of start-up and shut-downs of generating units.
- Minimize the pumping costs.
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Constraints:

- Water flow balance equations.
- Forbid of simultaneous pump and turbine mode.
- Final reservoir level.
- Minimum release of water.
- Lower and upper bounds on water levels.
- ...

Resolution Approaches

Challenges:

- Large-scale problem.
- Non-linearities.
- Multiple (conflicting) objectives.
- Non-feasibility of some instances.
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Approaches:

- MINLP.
- MILP.
- Dynamic Programming.
- Decomposition Methods.
- Heuristics.
- Other.

Resolution Approaches

Non Linearity:

- Focus on MINLP with non-linear objective function and linear constraints.
- The production function is a nonlinear function of the water flow and either the water level or (equivalently) the water volume in the reservoir.

$$P_{jt} = \phi(q_{jt}, v_t) \; \forall j \in \mathcal{J}, t \in \mathcal{T}$$

- Accurate description of the hydropower plants characteristics.
- More realistic and feasible results.

Resolution Approaches

VS Linearity:

- MILP solvers are more efficient than MINLP ones and handle large-scale instances.
- Piecewise linear approximation (easily applied for univariate functions).
- Functions of 2 variables (fix one of the variables).

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VS Discrete Operational Points:

- Non-linearities and production difficulties are avoided.
- Operational Points are used in practice.
- Assumption: finite and discrete number of Operational Points.

Operational Points

Sets:

- $\mathcal{T} = \{1 \dots T\}$ = set of time periods.
- $\mathcal{J} = \{1 \dots \bar{\jmath}\}$ = set of turbine/pump units.

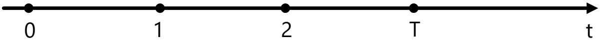
For each period t , we have the three possible cases that can occur relative to turbine/pump unit j :

- if unit j is generating power $\rightarrow q_{jt} > 0$ and $p_{jt} > 0$;
- if unit j is pumping water $\rightarrow q_{jt} < 0$ and $p_{jt} < 0$;
- if unit j is not operating $\rightarrow q_{jt} = 0$ and $p_{jt} = 0$.

where:

- q_{jt} is the water flow in unit j in period t $(j \in \mathcal{J}, t \in \mathcal{T})$
- p_{jt} is the power generated or consumed by unit j in period t $(j \in \mathcal{J}, t \in \mathcal{T})$

Directed Acyclic Graph $G(N, A)$



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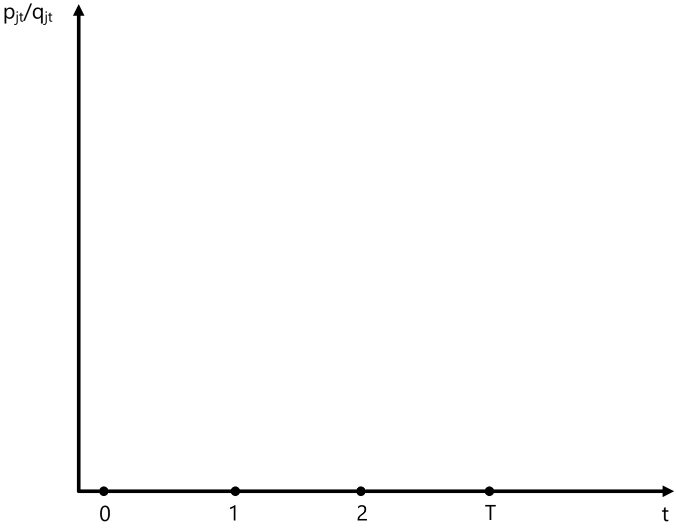

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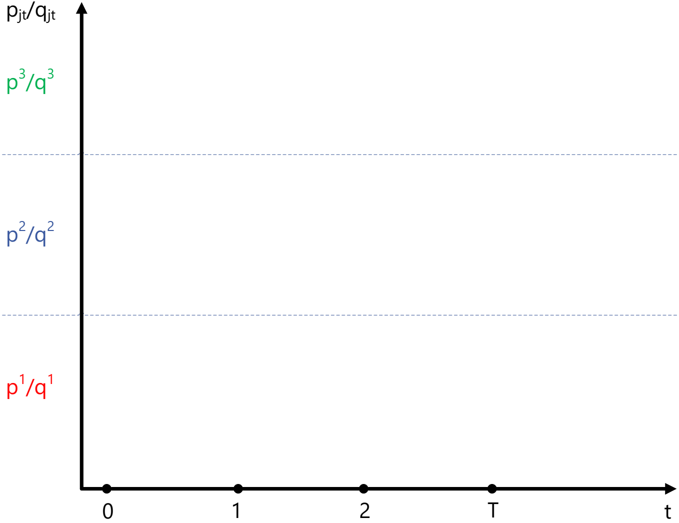

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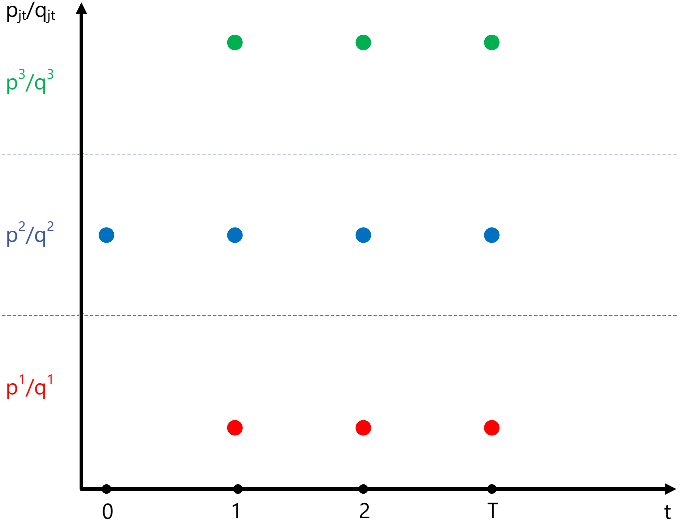

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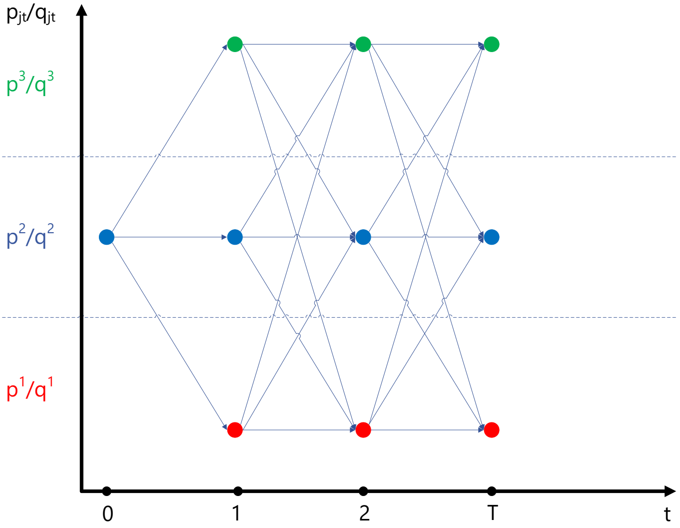

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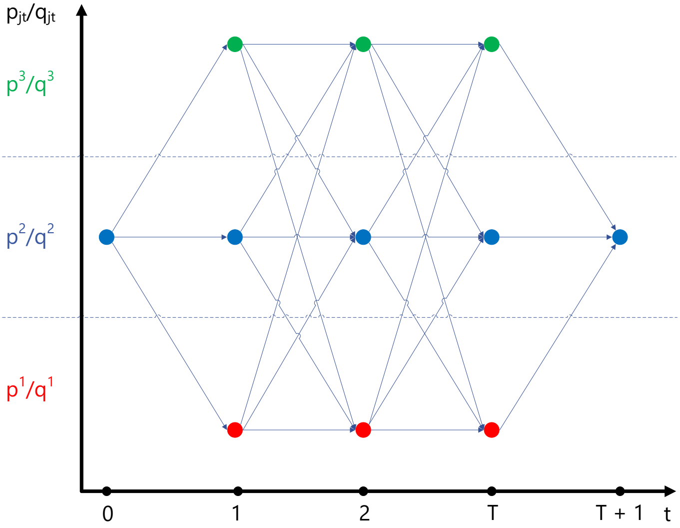

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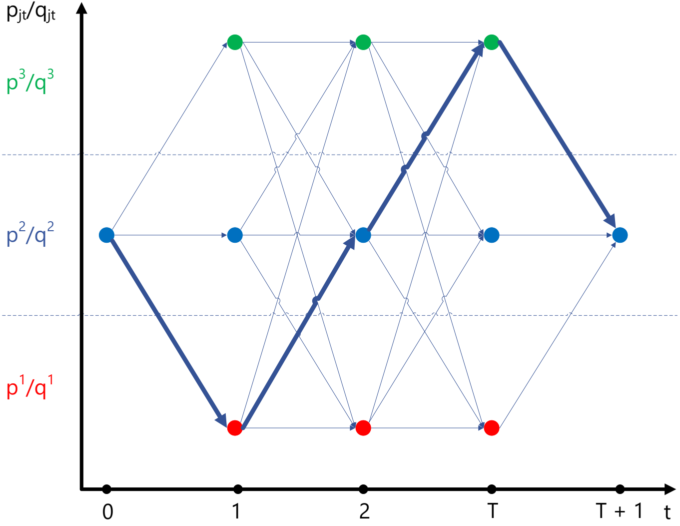


Constrained Shortest Path Problem (CSPP)

- I_t = predicted water inflow in period t $(t = 1, \ldots, T)$ $[\mathrm{m}^3/\mathrm{s}]$.
- Δt = period duration [s].
- $[\underline{V}, \overline{V}]$ = lower and upper bounds on water volume in the reservoir $[\mathrm{m}^3]$.
- V_T = target water volume in reservoir at the end of the time horizon $[\mathrm{m}^3]$.
- V_0 = initial water volume in reservoir $[\mathrm{m}^3]$.
- Q_i = water flow corresponding to node i $(i \in N)$ $[\mathrm{m}^3/\mathrm{s}]$.
- c_{ij} = cost of the arc $i(i, j) \in A$, depending on the cost of power generated or consumed, on the start up costs of units, and on the pumping cost[€].

$$c_{ij} = -\Delta t \Pi_t P_j + \begin{cases} C_j & \text{if startup turbine} \\ (D_j + \Pi_t E_j) & \text{if startup pump} \\ 0 & \text{otherwise} \end{cases}$$

CSPP - ILP formulation

$$\min_x \sum_{(i,j)\in A} c_{ij} x_{ij}$$

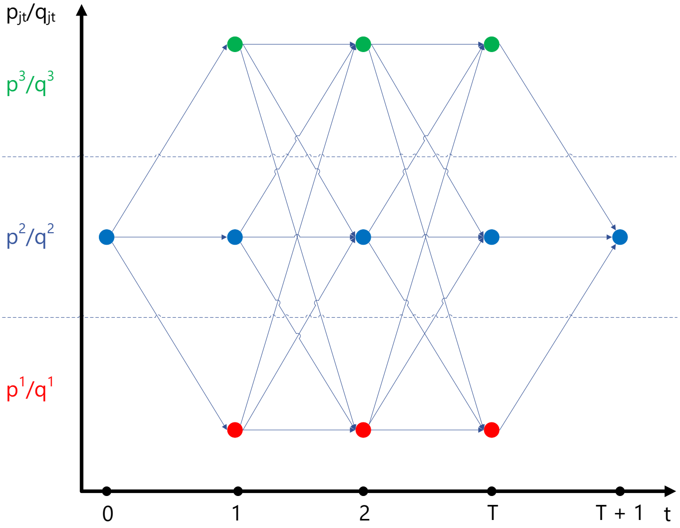
$$\sum_{(j,i)\in A} x_{ji} - \sum_{(i,j)\in A} x_{ij} = \begin{cases} -1 & \text{if} \quad i = s \\ 0 & \text{if} \quad i \in N \setminus \{s,d\} \\ 1 & \text{if} \quad i = d \end{cases} \qquad \forall i \in N$$

$$\underline{V} \leq v_0 + \Delta t \sum_{k=1}^{t} \left(I_k - \sum_{j\in N_k} \sum_{(i,j)\in A} Q_j x_{ij} \right) \leq \overline{V} \qquad \forall t \in 1, \ldots, T$$

$$v_0 + \Delta t \sum_{k=1}^{T} \left(I_k - \sum_{j\in N_k} \sum_{(i,j)\in A} Q_j x_{ij} \right) \geq V_T$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A$$

Directed Acyclic Graph $G^*(N^*, A^*)$



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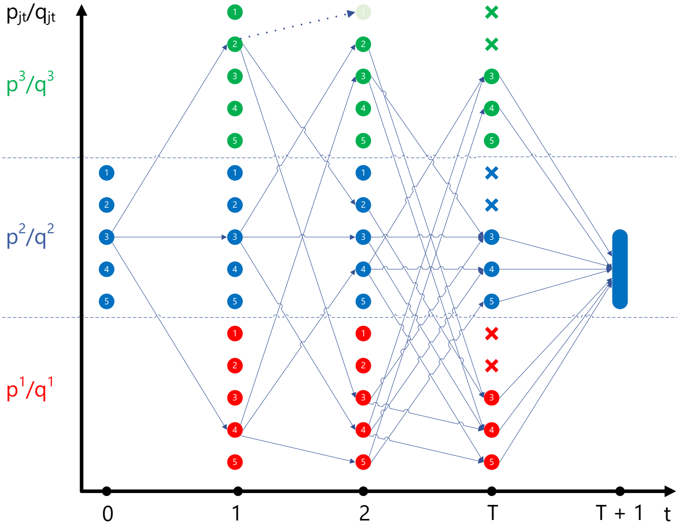

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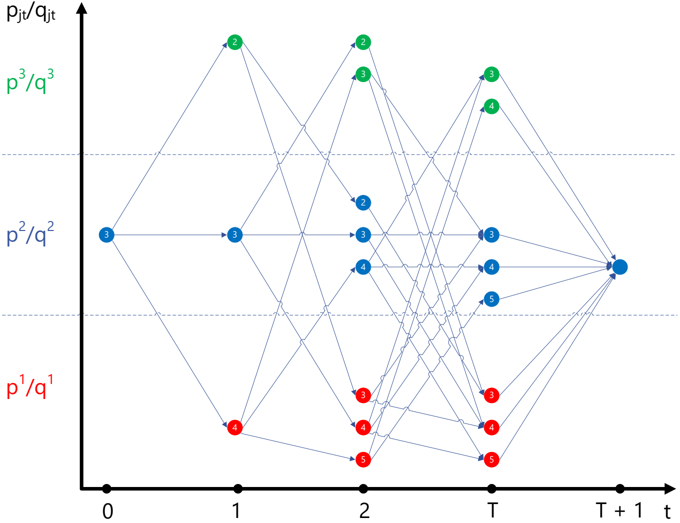

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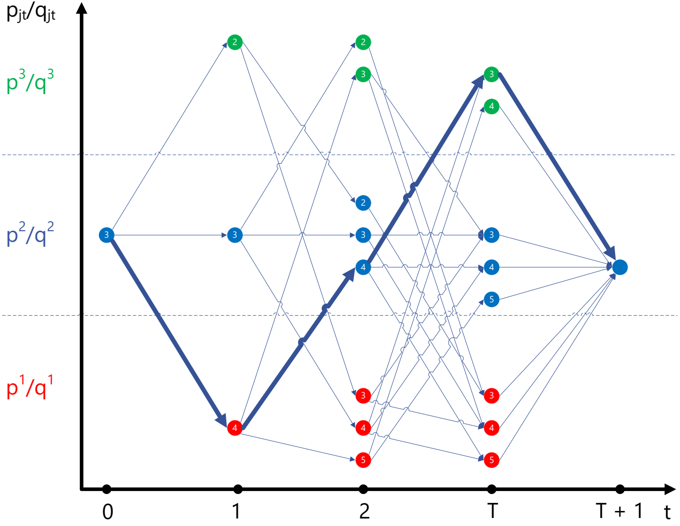

Shortest Path Problem (SPP)

- The graph $G^*(N^*, A^*)$ is weighted, directed, and acyclic.
- Single source and single destination.
- Pruning of the nodes:
  for a given $t \in \{1, \dots, T-1\}$, if

$$v_t + \Delta t \sum_{k=t+1}^{T} I_k - \Delta t(T-t)Q^- < V_T,$$

  where $Q^- = \min_j\{Q_j\}$, then the node corresponding to v_t and the arcs entering it can be removed from the graph.
- Topological Sorting Algorithm through a Dynamic Programming approach while generating the graph.

Test instances

Hydro Unit Commitment:

- We use data from [Borghetti et al., 2008].
- We tested different combinations of final volumes.
- Operational points are from 5 to 17. For instance 5 o.p. correspond to 3 o.p. of turbine on , 1 o.p. where everything is off and 1 o.p. with pump on.
- Forecast data for the months of April, June and December.
- The planning horizon covers at most one week.

Models:

- [Borghetti et al., 2008] MILP Model without considering head effect, using CPLEX 12.6.
- MILP formulation of the Constrained Shortest Path Problem, using CPLEX 12.6.
- Dynamic Programming Approach for generating and solving the expanded graph.

Computational Experiments

		Borghetti et al.					CSPP			
o.p.	T	Time (s)	Gap	Var	% Int.	Constr.	Time (s).	Gap	Int. Var	Constr.
5	24	4.7	0.0	346	63.9	366	5.4	0.0	14,884	171
5	48	6.1	0.0	682	64.1	726	13.8	0.0	58,564	339
5	96	7.2	0.0	1,354	64.2	1,446	14.0	0.0	232,324	675
5	168	5.6	0.0	2,362	64.2	2,526	9.5	0.0	708,964	1,179
7	24	4.6	0.0	396	68.4	366	29.5	0.0	28,900	219
7	48	5.0	0.0	780	68.6	726	6.1	0.0	114,244	435
7	96	5.3	0.0	1,548	68.7	1,446	17.6	0.0	454,276	867
7	168	7.9	0.0	2,700	68.7	2,526	109.3	0.0	1,387,684	1,515
12	24	7.9	0.0	521	76.0	366	19.4	0.0	84,100	339
12	48	6.6	0.0	1,025	76.1	726	13.1	0.0	334,084	675
12	96	8.1	0.0	2,033	76.1	1,446	t.l.	0.1	1,331,716	1,347
12	168	t.l.	0.1	3,545	76.2	2,526	t.l.	0.1	4,072,324	2,355
17	24	41.2	0.0	646	80.7	366	148.9	0.0	168,100	459
17	48	6.2	0.0	1,270	80.7	726	18.8	0.0	669,124	915
17	96	254.7	0.0	2,518	80.7	1,446	t.l.	0.1	2,669,956	1,827
17	168	4.6	0.0	4,390	80.8	2,526	t.l.	0.1	8,168,164	3,195

Computational Experiments

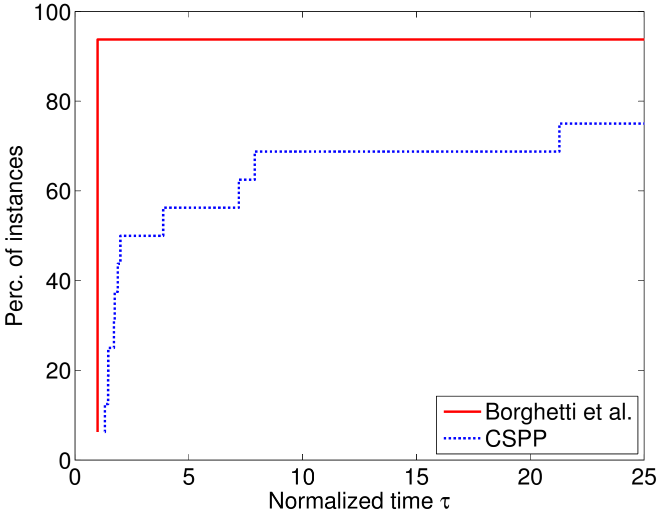

Figure: Performance profiles for 16 instances.

Computational Experiments

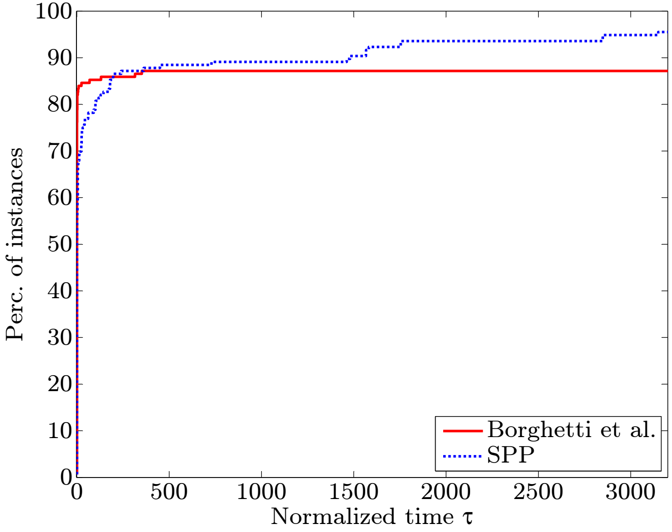

Figure: Performance profiles for 156 instances.

Conclusions

- Solving the expanded graph as a SPP, is time-convenient, however the size of the graph increases exponentially as the number of periods.
- The size of the CSPP graph is convenient, however the most common techniques available in literature are not directly applicable.

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Future work:

- Testing different instances.
- Reformulating the problem in order to ensure monotonicity.
- Applying Labeling techniques for solving the Constrained Shortest Path Problem.

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Thank you for your attention!

HUCP library

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We want your instances!

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