Optimal Control of Storage under Time Varying Electricity Prices

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Problem of finding optimal energy arbitrage

Arbitrage: buying a commodity at low price and sell when it is expensive.

What is energy storage arbitrage?

When end users alter the manner of charging and discharging decisions of a battery considering variations in electricity prices to make profit.

The objective of this work is to find an efficient algorithm for finding optimal energy arbitrage!!
Rich source of literature in the domain.

We present a special case of the problem formulated by Cruise, James, et al. in "Optimal control of storage incorporating market impact and with energy applications."

Our Contributions:

- Computationally efficient algorithm
- Threshold based structure of the optimal decisions
Total energy demand met by the Grid and/or Rooftop Solar PV and/or Battery

Selection of mode of operation will be based on the price of electricity, solar generation, end user consumption level and battery charge level.
Assumptions and Variables

Assumptions:
- Electricity price vary over time
- Buy and sell price is the same at any time instant
- Price taker
- Non-elastic end user load

\[
\text{Net load without storage, } z_i = d_i - r_i
\]

\[
\text{Billable component of electricity } = z_i + s_i
\]
Model of the battery

\[
\begin{align*}
\text{Charging} & : \quad s_i = [x_i]^+ / \eta_{\text{ch}} \\
\text{Discharging} & : \quad s_i = [x_i]^\text{−} \eta_{\text{dis}}
\end{align*}
\]

\[x_i = (\text{Ramp Rate}) \times (\text{Sampling time})\]

\[s_i = \text{Battery output}\]

Energy Constraint of the battery

\[b_i = b_{i-1} + x_i \quad \text{ with } \quad b_{\text{min}} \leq b_i \leq b_{\text{max}}\]

Power Constraint of the battery

\[\delta_{\text{min}} \leq \delta_i \leq \delta_{\text{max}}\]

\[X_{\text{min}} \leq x_i \leq X_{\text{max}}\]

\[X_{\text{min}} = \delta_{\text{min}} h, \quad X_{\text{max}} = \delta_{\text{max}} h\]
Residential Energy Arbitrage Problem: Minimizing Cost of Consumption

\( z_i \) is End user consumption \(- Solar generation \\
\( p_i \) is the Price of electricity in instant \( i \)

**Objective to find:**  \( G_{opt} = \min \sum_{i=1}^{N} (z_i + s_i)p_i \)
Residential Energy Arbitrage Problem: Minimizing Cost of Consumption

\( z_i \) is End user consumption – Solar generation

\( p_i \) is the Price of electricity in instant \( i \)

**Objective to find:** \( G_{opt} = \min \sum_{i=1}^{N} (z_i + s_i) p_i \)

\[
G_{opt} = \min \left\{ \sum_{i=1}^{N} z_i p_i + \sum_{i=1}^{N} s_i p_i \right\}
\]

\[\implies G_{opt} = \sum_{i=1}^{N} z_i p_i + \min \sum_{i=1}^{N} C_{storage}^{(i)} (x_i) \]

Finding \( G_{opt} = \min \sum_{i=1}^{N} C_{storage}^{(i)} (x_i) \) is equivalent to solving for \( G_{opt} \) for equal buy and sell price of electricity.

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Optimal Control of Storage under Time Varying Electricity Prices
\[ C^{(i)}_{\text{storage}}(x_i) = s_i p_i = p_i \left( \frac{1}{\eta_{\text{ch}}} [x_i]^+ - \eta_{\text{dis}} [x_i]^{\text{\textdagger}} \right). \]  \hspace{1cm} (1)

\( C^{(i)}_{\text{storage}}(x_i) \) is a **piecewise linear, continuous increasing function in** \( x_i \) **for all** \( i \).

Thus the objective function for the optimal arbitrage problem is **convex**.
Primal Problem

Minimize \( \sum_{i=1}^{N} C^{(i)}(x_i) \) storage \( (x_i) \) 
subject to \( b_{\min} \leq b_i \leq b_{\max}, \ \forall i, \)  

Lagrangian Dual Problem

\[ \mathcal{L}(x, \alpha, \beta) = \sum_{i=1}^{N} \left( C^{(i)}(x_i) + \alpha_i (b_{\min} - b_i) + \beta_i (b_i - b_{\max}) \right) \]

\[ \max \phi(\alpha, \beta) \quad \text{subject to} \quad \alpha_i, \beta_i \geq 0, \ \forall i \]

where \( \phi(\alpha, \beta) = \inf_{x_i \in [X_{\min}, X_{\max}]} \mathcal{L}(x, \alpha, \beta) \)
We show the existence of strong duality and using Saddle point inequality condition, we propose a mechanism to find a sub-horizon.

**Theorem**

The optimal arbitrage problem is equivalent to

$$\min C_{storage}(x) - \mu_i^* x$$

Optimal accumulated Lagrange multiplier, $\mu_i^* = \sum_{j=i}^{N} (\alpha_j^* - \beta_j^*)$, satisfies:

- $\mu_{i+1}^* = \mu_i^*$, if $b_{\text{min}} < b_i^* < b_{\text{max}}$
- $\mu_{i+1}^* \leq \mu_i^*$, if $b_i^* = b_{\text{min}}$
- $\mu_{i+1}^* \geq \mu_i^*$, if $b_i^* = b_{\text{max}}$
Remark

The optimal control decision $x_i^*$ in the $i$th instant minimizes the function $C_{storage}^{(i)}(x) - \mu_i^* x$ for $x \in [X_{min}, X_{max}]$. The optimal decision $x_i^*(\mu)$ is

$$x_i^*(\mu) = \begin{cases} 
X_{min}, & \text{if } \mu < p_i, \\
[X_{min}, X_{max}], & \text{if } \mu = p_i, \\
X_{max}, & \text{if } \mu > p_i,
\end{cases} \quad (2)$$

Description of the threshold based structure of the optimal decisions.
Finding Sub-Horizon - Algorithm 1 Example

\[ b^*_i(\mu) = \begin{cases} 
X_{\min}, & \text{if } \mu < p_i, \\
[X_{\min}, X_{\max}], & \text{if } \mu = p_i, \\
X_{\max}, & \text{if } \mu > p_i,
\end{cases} \]

(a) \( b^* \) for \( \mu = 0 \)
Finding Sub-Horizon - Algorithm 1 Example

\( b_{\text{max}} \)

\[ \mu = 0 \]

\[ b_{\text{min}} \]

\begin{array}{ccccccccc}
  i = 0 & i = 1 & i = 2 & i = 3 & i = 4 & i = 5 & N-2 & N-1 & i = N \\
  \mu = 0 & b_{\text{max}} & b_{\text{min}} & p_1 & p_2 & p_3 & p_4 & p_5 & p_{N-1} & p_N \\
\end{array}

Time instants  
Price Levels  
Length of Sub-horizon  

\( X_i^*(\mu) = \begin{cases} 
X_{\text{min}}, & \text{if } \mu < p_i, \\
[X_{\text{min}}, X_{\text{max}}], & \text{if } \mu = p_i, \\
X_{\text{max}}, & \text{if } \mu > p_i,
\end{cases} \)

\( (a) \) \( b^* \) for \( \mu = 0 \)

\( (b) \) \( b^* \) for \( \mu = p_1 \)

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Finding Sub-Horizon - Algorithm 1 Example

\( \mu = p_2 \)

\[ b_{max} \]

\( \mu = p_3 \)

\[ b_{max} \]

(c) \( b^* \) for \( \mu = p_2 \)

(d) \( b^* \) for \( \mu = p_3 \)
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Optimal Control of Storage under Time Varying Electricity Prices
Optimal solution is not unique as the cost function is not strictly convex.

Algorithm 1 returns the envelope of possible solutions in the sub-horizon. Algorithm 2 identifies the optimal solution within the feasible envelope.

Violation of lower and upper battery capacity boundary

- The worst case run-time complexity is quadratic in terms of number of time instants for which price values are available.
Optimal Control of Batteries: Numerical Example of Lossless battery

NYISO Price Signal

$\mu^*$ or the Shadow price of the transaction.

Optimal ramp rate.

Optimal battery charge level.
Remark

The optimal control decision $x_i^*$ in the $i$th instant minimizes the function $C_{\text{storage}}^{(i)}(x) - \mu_i^* x$ for $x \in [X_{\min}, X_{\max}]$. The optimal decision $x_i^*(\mu)$ is

$$x_i^*(\mu) = \begin{cases} 
X_{\min}, & \text{if } \mu < p_{\text{dis}}(i), \\
[X_{\min}, 0], & \text{if } \mu = p_{\text{dis}}(i), \\
0, & \text{if } p_{\text{ch}}(i) > \mu > p_{\text{dis}}(i), \\
[0, X_{\max}], & \text{if } \mu = p_{\text{ch}}(i), \\
X_{\max}, & \text{if } \mu > p_{\text{ch}}(i),
\end{cases}$$  \quad (3)

where $p_{\text{ch}}(i) = p_i/\eta_{\text{ch}}$ and $p_{\text{dis}}(i) = p_i \eta_{\text{dis}}$.

Note for $\mu = p_{\text{ch}}(i)$ or $\mu = p_{\text{dis}}(i)$, $x_i^*(\mu)$ takes an envelope of values and for any other value of $\mu$ it is a singleton set.
Optimal Control of Batteries: Numerical Example for Lossy Battery

![Graph showing battery capacity and $\mu^*$](image)

**Optimal battery capacity and $\mu^*$**

![Graph showing optimal ramp rate](image)

**Optimal ramp rate corresponding to $b_0$, price values, $\eta_{ch} = 0.95$ and $\eta_{dis} = 0.95$.**

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Optimal Control of Batteries: Lossless vs Lossy Ramp Rate Comparison

**Lossless Battery**

**Lossy Battery**

Optimal ramp rate for lossless and lossy battery

- Losses create friction in mode change
- Coupling of optimal actions in a sub-horizon
- Intermediate ramp rates could also be optimal
Comparison of the Proposed Solution

Table: Comparison of performance for lossless battery

<table>
<thead>
<tr>
<th>Algorithm Type</th>
<th>Run Time (sec)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Algorithm</td>
<td>0.1967</td>
<td>0.1403245</td>
</tr>
<tr>
<td>Linear Program</td>
<td>1.4873</td>
<td>0.1403245</td>
</tr>
<tr>
<td>Matlab’s Fmincon</td>
<td>23.0526</td>
<td>0.1402757</td>
</tr>
</tbody>
</table>

Table: Comparison of performance for lossy battery

<table>
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<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Algorithm</td>
<td>0.164189</td>
<td>0.1193289</td>
</tr>
<tr>
<td>Matlab’s Fmincon</td>
<td>23.41217</td>
<td>0.1192396</td>
</tr>
</tbody>
</table>

Such a computationally efficient algorithm will be apt for real time implementation!!
Case Study: Potential of energy arbitrage in contemporary electricity markets

Using the proposed algorithm, we conducted extended simulations for real price data from several ISOs in USA and Europe for the year 2016.

We extrapolate the arbitrage gains for an end user for a five year period, considering detailed losses in the battery.

\[
\eta_{ch} = 0.95 \\
\eta_{dis} = 0.95 \\
\eta_{cycle} = \eta_{ch} \eta_{dis} = 0.9025 \\
\eta_{converter} = 0.95 \\
Average Capacity to Degradation = 0.9
\]
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We extrapolate the arbitrage gains for an end user for a five year period, considering detailed losses in the battery.

\[ \eta_{ch} = 0.95 \quad \eta_{dis} = 0.95 \quad \eta_{cycle} = \eta_{ch} \eta_{dis} = 0.9025 \quad \eta_{converter} = 0.95 \quad \text{Average Capacity to Degradation} = 0.9 \]

- Rated Battery Capacity: 100%
- Optimal SoC Band
  - SoC_{max} = 98%
  - SoC_{min} = 10%
  - Net Capacity = 100 x 0.88 = 88%
- Cycling Losses
  - \eta_{ch} = 0.95
  - \eta_{dis} = 0.95
  - \eta_{cycle} = \eta_{ch} \eta_{dis} = 0.9025
  - 88 x 0.9025 = 79.4%
- Converter Losses
  - \eta_{converter} = 0.95
  - 79.4 x 0.95 = 75.5%
- Performance Degradation
  - Average Capacity to Degradation = 0.9
  - 75.5 x 0.9 = 67.9%
Table: The price signals evaluated are listed below

<table>
<thead>
<tr>
<th>Region/ISO</th>
<th>Pricing</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>NordPool</td>
<td>Real Time</td>
<td>1 hour</td>
</tr>
<tr>
<td>PG&amp;E</td>
<td>ToU</td>
<td>-</td>
</tr>
<tr>
<td>CAISO (Average Price)</td>
<td>Real Time</td>
<td>5 min</td>
</tr>
<tr>
<td>PJM (Zone AEP)</td>
<td>Real Time</td>
<td>1 hour</td>
</tr>
<tr>
<td>ERCOT (Zone LZ-Huston)</td>
<td>Real Time</td>
<td>1 hour</td>
</tr>
<tr>
<td>ISONE (Zone Z.SEMASS)</td>
<td>Real Time</td>
<td>1 hour</td>
</tr>
<tr>
<td>MISO (Zone Michigan Hub)</td>
<td>Real Time</td>
<td>1 hour</td>
</tr>
<tr>
<td>NYISO (Zone N.Y.C.)</td>
<td>Real Time</td>
<td>1 hour</td>
</tr>
</tbody>
</table>
Case Study: Potential of energy arbitrage in contemporary electricity markets

Table: **Battery 1**: $\delta_{\text{max}}=0.26 \text{ kW}$, $\delta_{\text{min}}=-0.52 \text{ kW}$

<table>
<thead>
<tr>
<th>Region or ISO</th>
<th>Cumulative Gains in 2016</th>
<th>Operational Cycles in 2016</th>
<th>Discounted Gains in 5 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>NordPool</td>
<td>€0.991</td>
<td>1748</td>
<td>€4.3</td>
</tr>
<tr>
<td>PG&amp;E</td>
<td>$4.38</td>
<td>184</td>
<td>$18.7</td>
</tr>
<tr>
<td>CAISO</td>
<td>$37.9</td>
<td>914</td>
<td>$162.0</td>
</tr>
<tr>
<td>PJM</td>
<td>$11.2</td>
<td>573</td>
<td>$47.9</td>
</tr>
<tr>
<td>ERCOT</td>
<td>$18.6</td>
<td>430</td>
<td>$79.5</td>
</tr>
<tr>
<td>ISONE</td>
<td>$15.3</td>
<td>687</td>
<td>$65.4</td>
</tr>
<tr>
<td>MISO</td>
<td>$10.5</td>
<td>595</td>
<td>$44.9</td>
</tr>
<tr>
<td>NYISO</td>
<td>$23.3</td>
<td>700</td>
<td>$99.6</td>
</tr>
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</table>
**Case Study: Potential of energy arbitrage**

**Table: Battery 2:** $\delta_{\text{max}} = 1kW, \delta_{\text{min}} = -1kW$

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<tr>
<td>NordPool</td>
<td>€1.09</td>
<td>2836</td>
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<tr>
<td>CAISO</td>
<td>$73.2</td>
<td>2008</td>
<td>$312.9</td>
</tr>
<tr>
<td>PJM</td>
<td>$16.1</td>
<td>825</td>
<td>$68.8</td>
</tr>
<tr>
<td>ERCOT</td>
<td>$25.02</td>
<td>534</td>
<td>$107.0</td>
</tr>
<tr>
<td>ISONE</td>
<td>$23.51</td>
<td>1082</td>
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<td>NYISO</td>
<td>$36.32</td>
<td>1225</td>
<td>$155.3</td>
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**Table: Battery 3:** $\delta_{\text{max}} = 2kW, \delta_{\text{min}} = -2kW$

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<td>CAISO</td>
<td>$125.03</td>
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Case Study: Potential of energy arbitrage

Table: **Battery 2**: $\delta_{\text{max}} = 1\text{kW}, \delta_{\text{min}} = -1\text{kW}$

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Table: **Battery 3**: $\delta_{\text{max}} = 2\text{kW}, \delta_{\text{min}} = -2\text{kW}$

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**Batteries can become viable with:** Subsidies & / or More volatile price signal & / or Battery with more cycle life, lower cost and fast ramp rate
Conclusion

- Computationally efficient algorithm
  - **Threshold based structure** of the optimal solution \((x^*)\) based on the level of \(\mu^*\)
  
  \[\downarrow\text{Proved through LD}\]

- **Shadow price**: Accumulated Lagrange multiplier \((\mu^*)\) acts as the ”Shadow Price” of the transaction.

- **Property of a sub-horizon**: Optimal storage control decisions are independent of prices beyond concerned sub-horizon.

  \[\downarrow\]

  Implying energy arbitrage is a finite horizon optimization problem.

- Case study indicates that only arbitrage cannot create positive net present value for storage.

- Generally applicable for any system performing arbitrage with finite buffer capacity.
And its over!!

Link to the paper:
https://www.researchgate.net/publication/319109988_Optimal_Control_of_Storage_under_Time_Varying_Electricity_Prices

Speaker: Md. Umar Hashmi (PhD Student)
https://www.linkedin.com/in/umar-hashmi-8823a325/

Thank You!
Conclusion of the Case Study: Potential of energy arbitrage in contemporary electricity markets

Numerical evaluation indicates that only arbitrage cannot create positive net present value for storage.

⇓

Batteries can become viable with:

- Subsidies & / or
- More volatile price signal & / or
- Battery with more cycle life, lower cost and fast ramp rate
Future Work

- Simultaneous considering forecasting and optimal arbitrage to analyze effect on gains.
- Extension to generalized analysis of unequal buy and sell price of electricity.
- Energy storage arbitrage is equivalent to price based demand response using flexibility derived from energy storage battery. Can we use this algorithm for price based demand response?
The electricity price varies over time, the storage needs to be optimally controlled to maximize gains using storage!
Accommodating Forecast Errors

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