The (1+λ) Evolutionary Algorithm with Self-Adjusting Mutation Rate

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Backgrounds

- $\textsc{OneMax}$ Problem: find a string $x = x_1x_2\dots x_n$ with $x_i \in \{0,1\}$ that minimize

$$\textsc{Om}(x) = \sum_{i=1}^{n} x_i.$$

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- Black-box complexity (Lehre,Witt (2010)): the smallest expected number of function evaluations needed to solve a problem.
- We only consider unary unbiased variation operators which are symmetric with respect to the bit positions $[n] := \{1, \dots, n\}$ and the bit values 0 and 1.

Structure of (1+λ) EA

Algorithm 1 (1+λ) EA

Select x uniformly at random from $\{0,1\}^n$;
repeat
    for $i = 1$ to λ **do**
        Create x_i by flipping each bit in a copy of x independently with probability r/n;
    end for
    $x^* \leftarrow \arg\min_{x_i} \textsc{Om}(x_i)$;
    if $\textsc{Om}(x^*) \leq \textsc{Om}(x)$ **then**
        $x \leftarrow x^*$
    end if
until $\textsc{Om}(x) = 0$

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- Static mutation rate r (i.e. mutation prob. $p = r/n$) for the $(1+\lambda)$ EA (Gießen, Witt, 2016):

$$(1 \pm o(1)) \left(\frac{1}{2} \cdot \frac{n \log \log \lambda}{\log \lambda} + \frac{e^r}{r} \cdot \frac{n \log n}{\lambda} \right)$$

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- Improvement of a factor of $\Theta(\log \log \lambda)$ can be obtained by using dynamic parameter settings

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- Dynamic mutation rate $r = \max \{\ln \lambda / \ln(en/k), 1\}$ where $k = \textsc{Om}(x)$ for the (1+λ) EA (Badkobeh, Lehre, Sudholt, 2014):

$$O\left(\frac{n}{\log \lambda} + \frac{n \log n}{\lambda} \right)$$

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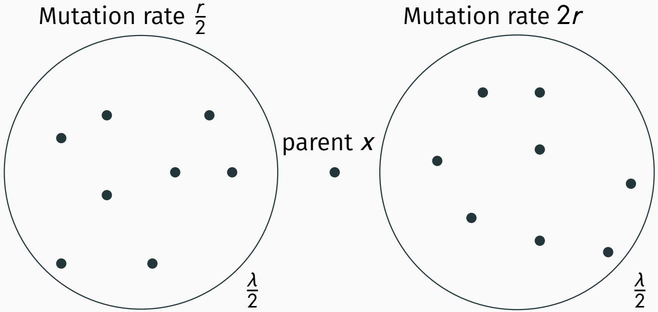
- Mutation rate should depend on the state of the current search process
- Idea: let the algorithm changes parameters automatically according to recent performance (similar to 1/5-rule)

(1+λ) EA with two-rate standard bit mutation



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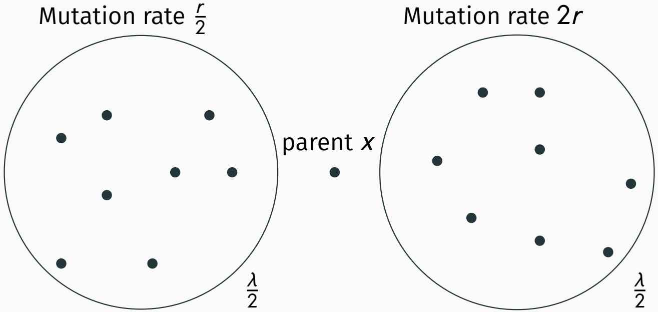

- Replace parent by a best offspring x^* if better or equal (breaking ties randomly, favouring offspring)

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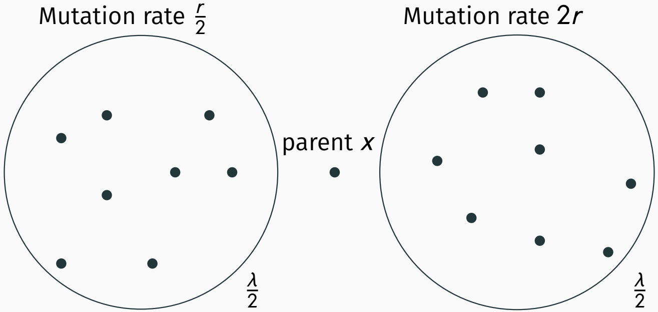

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  - Heads: Replace r by the rate x^* has been created with
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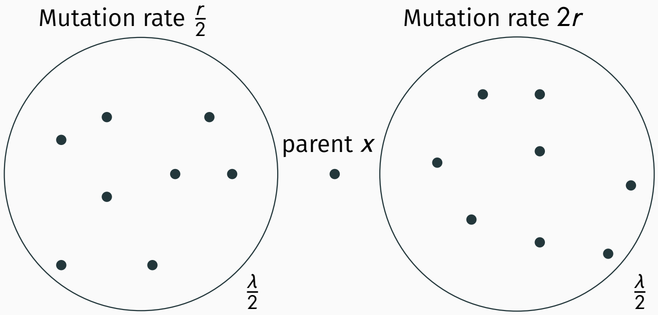

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- Cap r at 2 and $n/4$

Rough estimation of good r

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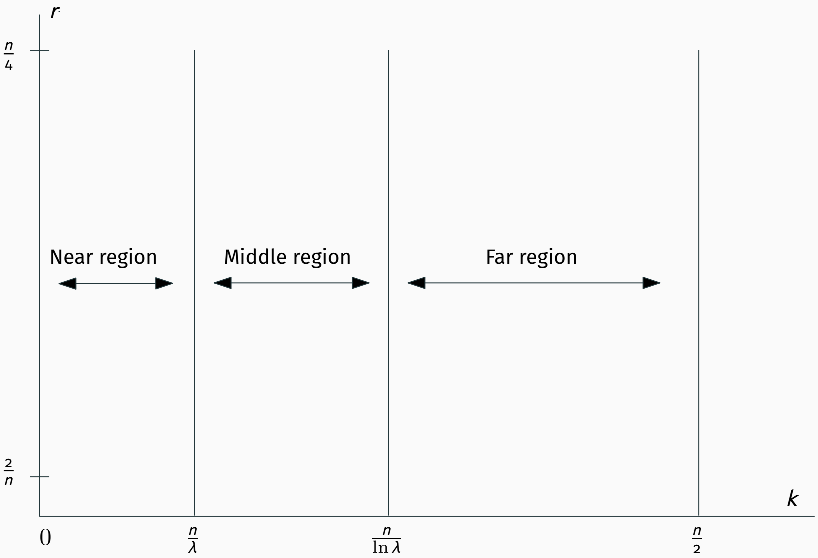
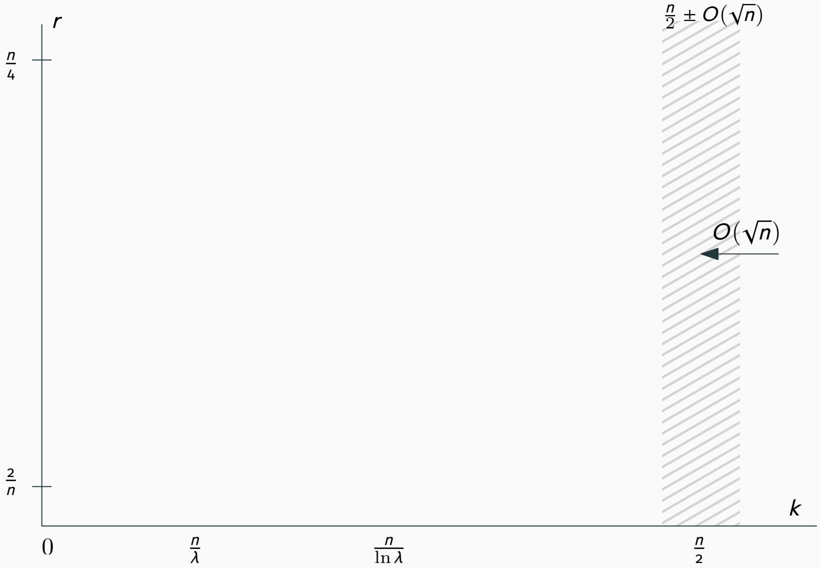
- The best zero-keeping offspring (probably) makes progress.

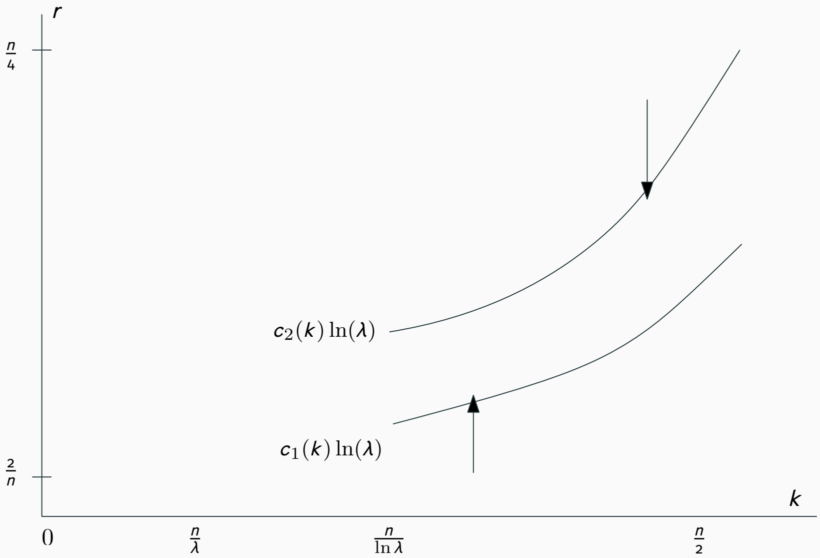
Three regions



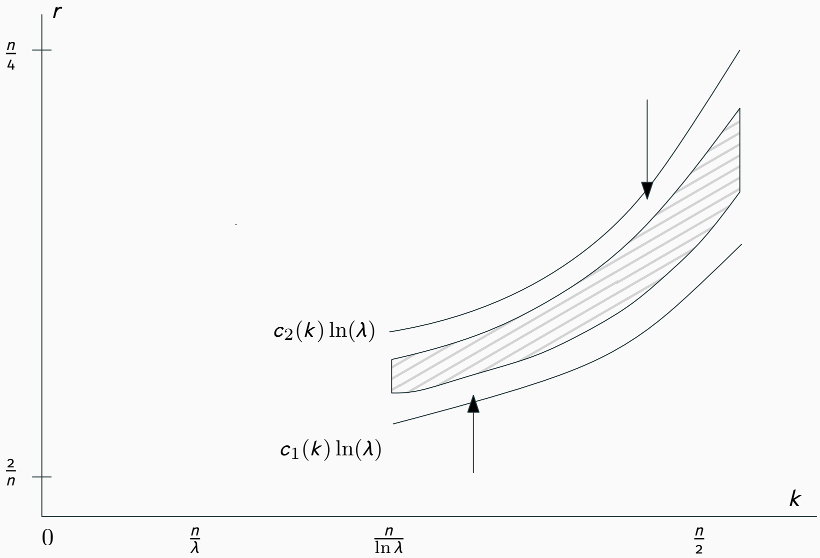
$\sqrt{n}$ drift on distance at the beginning



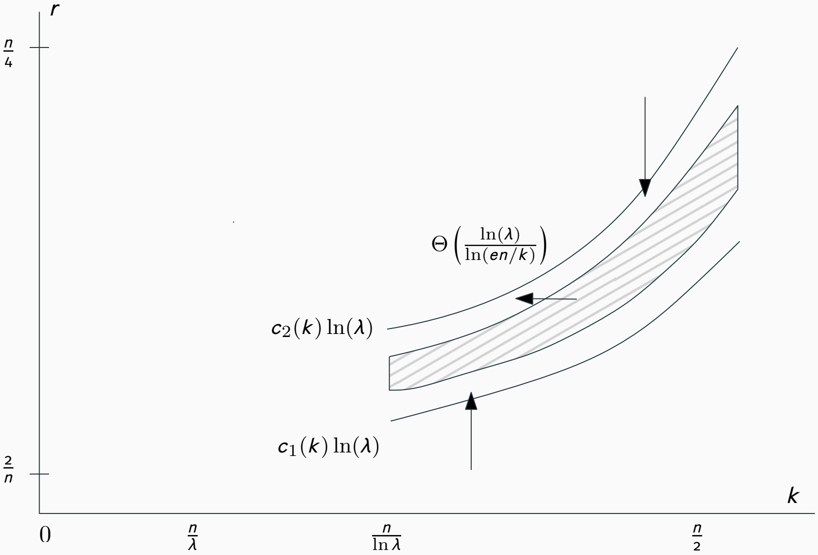
drift on r in far region



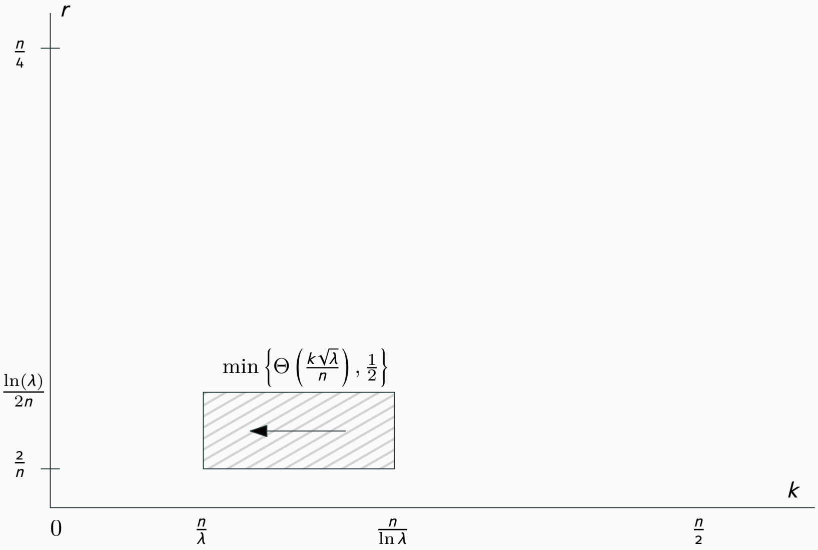
good rate r in far region



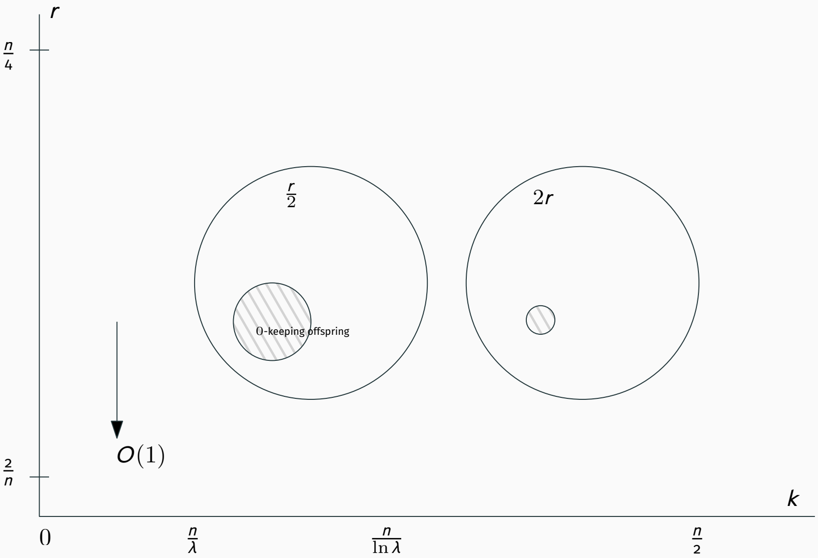
Runtime: $O\left(\frac{n}{\ln(\lambda)}\right)$ generations in far region



Runtime: $O\left(\frac{n}{\ln(\lambda)}\right)$ generations in middle region



Runtime: $O\left(\frac{n\ln(n)}{\lambda}\right)$ generations in far region



Conclusion

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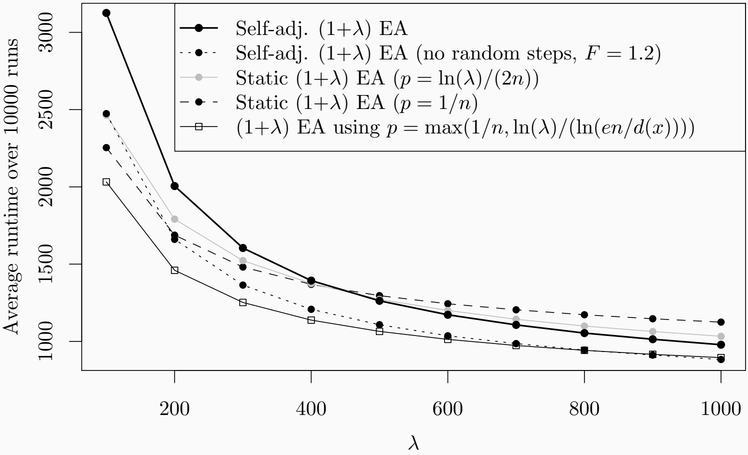
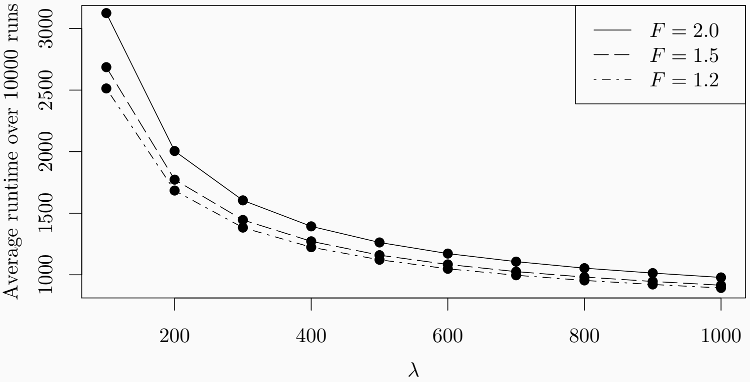
- Matches the lower bound for every parallel unbiased black-box algorithm
- Side-result: fixed rate of $r = \ln\lambda/2$ yields

$$O(n/\log\lambda + n\log(n)/\sqrt{\lambda})$$

(also optimal for λ not too small)



Self-adj. (1+λ) EA
Self-adj. (1+λ) EA (no random steps, $F = 1.2$)
Static (1+λ) EA ($p = \ln(\lambda)/(2n)$)
Static (1+λ) EA ($p = 1/n$)
(1+λ) EA using $p = \max(1/n, \ln(\lambda)/(\ln(en/d(x))))$
Average runtime over 10000 runs
1000
1500
2000
2500
3000
200
400
600
800
1000
λ



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1000
1500
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λ
$F = 2.0$
$F = 1.5$
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